Combinatorial Auctions with Item Bidding: Equilibria and Dynamics

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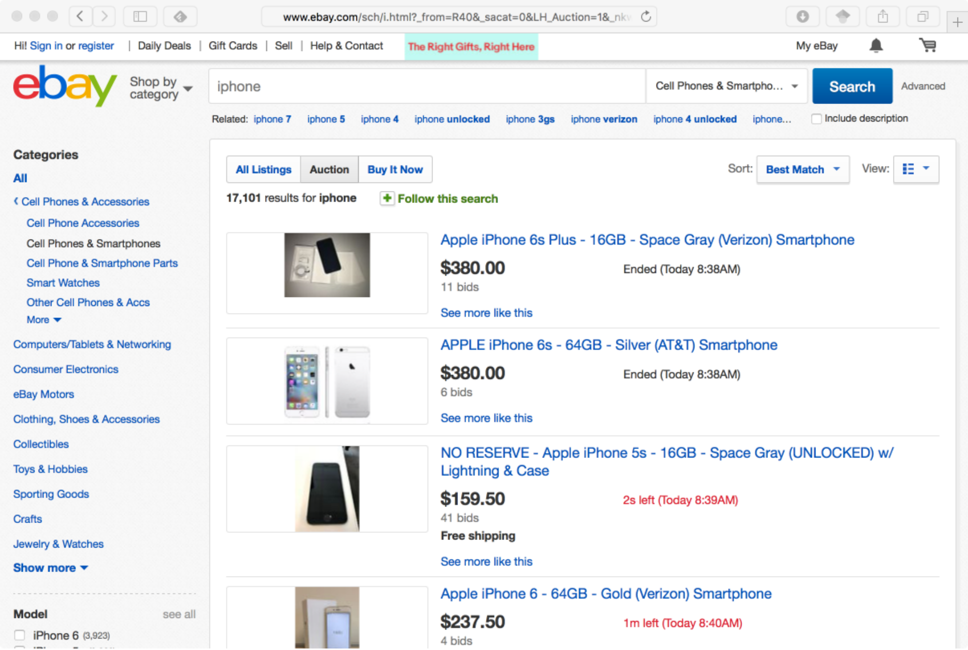
Based on joint work with Paul Dütting

WINE 2016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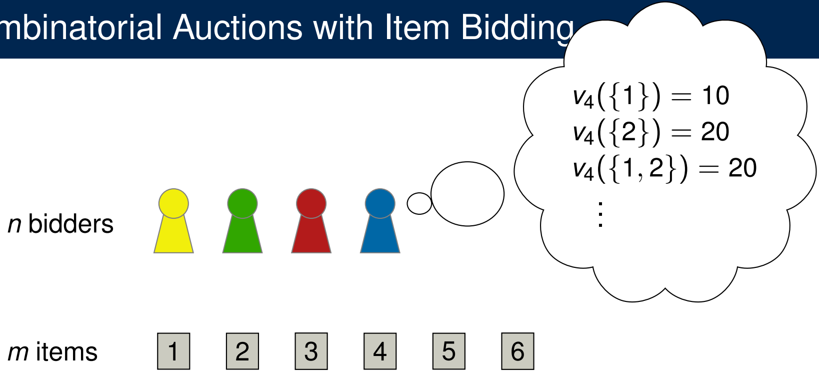


Each item is sold in a separate second-price auction.

- Bidders usually cannot express their preferences.
- Might have to pay for multiple items although they only want one.

Setting

[Christodoulou/Kovács/Schapira, JACM 2016]

- Set of n bidders N, set of m items M
- Each bidder i has valuation function $v_i \colon 2^M \to \mathbb{R}_{\geq 0}$
- Each bidder i reports a bid $b_{i,j} \geq 0$ for every item j
- Each item j is sold to bidder i that maximizes $b_{i,j}$
  Has to pay 2nd highest bid: $\max_{i' \neq i} b_{i',j}$
- Each bidder i tries to maximize his/her utility

$$u_i(b) = v_i(S_i) - \sum_{j \in S_i} \max_{i' \neq i} b_{i',j},$$

where S_i is the set of items bidder i wins under b

Example

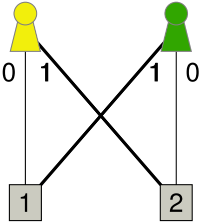
Two bidders, two items

$v_1(\{1\}) = 2$, $v_1(\{2\}) = 1$, $v_1(\{1,2\}) = 2$

$v_2(\{1\}) = 1$, $v_2(\{2\}) = 2$, $v_2(\{1,2\}) = 2$

$b_{1,1} = 0$, $b_{1,2} = 1$

$b_{2,1} = 1$, $b_{2,2} = 0$



Bidder 1 wins item 2; bidder 2 wins item 1.

No bidder wants to unilaterally deviate $\Rightarrow$ pure Nash equilibrium

Equilibrium Concepts

Definition

A bid profile b is a pure Nash equilibrium if for all bidders i and all b'_i

$$u_i(b) \geq u_i(b'_i, b_{-i})$$

Other equilibrium concepts:

- mixed Nash
- correlated
- Bayes-Nash

Questions

- How good are (pure Nash, mixed Nash, correlated, Bayes-Nash, . . .) equilibria?
- Do they always exist?
- If so, can they be computed in polynomial time?
- If so, can they be reached by simple dynamics?

Outline

Outline

Price of Anarchy

- Given b call $SW(b) = \sum_{i \in N} v_i(S_i)$ social welfare of b
- Compare to $OPT(v) = \max_{(S_1^*, \ldots, S_n^*) \text{ is partition}} \sum_{i \in N} v_i(S_i^*)$

Price of Anarchy

$$PoA = \max_{v_1, \ldots, v_n} \max_{b \in PNE} \frac{OPT(v)}{SW(b)}$$

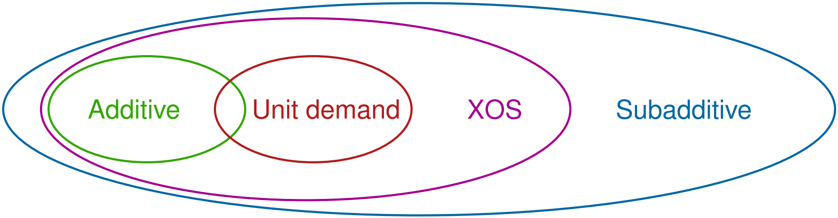
Two bidders, one item: $v_1 = 0$, $v_2 = 1$
$b_1 = 1$, $b_2 = 0$ is pure Nash equilibrium, $SW(b) = 0$, $OPT(v) = 1$

Therefore restrict attention to equilibria with *weak no-overbidding*:
$\sum_{j \in S} b_{i,j} \leq v_i(S)$ if bidder i wins set S

Classes of valuation functions

A function $v_i \colon 2^M \to \mathbb{R}_{\geq 0}$ is ...

- *additive* if $v_i(S) = \sum_{j \in S} v_{i,j}$ for some $v_{i,j} \geq 0$
- *unit demand* if $v_i(S) = \max_{j \in S} v_{i,j}$ for some $v_{i,j} \geq 0$
- *fractionally subadditive* or *XOS* if
  $v_i(S) = \max_\ell \sum_{j \in S} v_{i,j}^\ell$ for some $v_{i,j}^\ell \geq 0$
- *subadditive* if $v_i(S \cup T) \leq v_i(S) + v_i(T)$



Examples

- $v_i(\{1\}) = 2$, $v_i(\{2\}) = 1$, $v_i(\{1,2\}) = 2$
  is unit demand

- Every submodular function is XOS,
  e.g. $v_i(S) = \min\{c_i, \sum_{j \in S} v_{i,j}\}$

- $v_i(S) = \begin{cases} 0 & \text{if } |S| = 0 \\ 1 & \text{if } |S| = 1 \text{ or } |S| = 2 \\ 2 & \text{if } |S| = 3 \end{cases}$
  is subadditive but not XOS

Price of Anarchy: Bound for XOS Valuations

[Christodoulou/Kovács/Schapira, JACM 2016]

Theorem

Consider XOS valuations v. Let b be a pure Nash equilibrium. Then $SW(b) \geq \frac{1}{2} OPT(v)$.

Proof for unit-demand valuations:

Let j_i be the item that bidder i gets in $OPT(v)$.

Bidder i could deviate to $b'_{i,j}$ such that $b'_{i,j} = v_{i,j}$ if $j = j_i$ and 0 otherwise.

$u_i(b) \geq u_i(b'_i, b_{-i}) \geq v_{i,j_i} - \max_{i'} b_{i',j_i}$.

$\Rightarrow \sum_{i \in N} u_i(b) + \sum_{j \in M} \max_{i'} b_{i',j} \geq \sum_{i \in N} v_{i,j_i} = OPT(v)$

$\sum_{i \in N} u_i(b) \leq SW(b)$ by definition,
$\sum_{j \in M} \max_{i'} b_{i',j} \leq SW(b)$ by no-overbidding $\square$

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$u_i(b) \geq u_i(b'_i, b_{-i}) \geq v_{i,j_i} - $ n

$\Rightarrow \sum_{i \in N} u_i(b) + \sum_{j \in M} \max_{i'}$

$\sum_{i \in N} u_i(b) \leq SW(b)$ by defi

$\sum_{j \in M} \max_{i'} b_{i',j} \leq SW(b)$ by no-overbidding $\square$

"Smoothness" proof:
Deviation does not depend on b
$\Rightarrow$ extends to mixed Nash, correlated, Bayes-Nash equilibria

Bound is tight

Two bidders, two items

$v_1(\{1\}) = 2$, $v_1(\{2\}) = 1$, $v_1(\{1,2\}) = 2$

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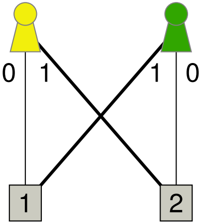
$b_{1,1} = 0$, $b_{1,2} = 1$

$b_{2,1} = 1$, $b_{2,2} = 0$

$SW(b) = 2$, $OPT(v) = 4$



Further Results

- Roughgarden, STOC 2009, Syrgkanis/Tardos, STOC 2013, ...:
  General smoothness framework for Price of Anarchy
- Bhawalkar/Roughgarden, SODA 2011:
  Subadditive valuations: $PoA = 2$ for pure Nash,
  $PoA = O(\log m)$ via smoothness
- Feldman/Fu/Gravin/Lucier, STOC 2013:
  Subadditive valuations: constant PoA for Bayes-Nash equilibria, not a smoothness proof

More results on simultaneous *first-price* auctions, generalized second price, greedy auctions, ...

Outline

Complexity of Equilibria (1/3)

[Dobzinski/Fu/Kleinberg, SODA 2015]

- Submodular valuations: Computing an equilibrium with good welfare is essentially as easy as computing an allocation with good welfare.
- Subadditive valuations: Computing an equilibrium requires exponential communication.
- XOS valuations: “If there exists an efficient algorithm that finds an equilibrium, it must use techniques that are very different from our current ones.”

Complexity of Equilibria (2/3)

[Cai/Papadimitriou, EC 2014]

One unit-demand bidder, others additive:

- Computing Bayes-Nash equilibrium in such auctions is PP-hard
- Finding an approximate Bayes-Nash equilibrium is NP-hard
- Recognizing a Bayes-Nash equilibrium is intractable

Complexity of Equilibria (3/3)

[Daskalakis/Syrgkanis, FOCS 2016]

- Unit-demand valuations: There are no polynomial-time no-regret learning algorithms, unless RP $\supseteq$ NP
  Reason: Huge strategy spaces
- Alternative concept: No-envy learning. Only decide which items to buy but not the bids

Outline

Best-Response Dynamics

b_i is best response to b_{-i} if

$$u_i(b_i, b_{-i}) \geq u_i(b_i', b_{-i}) \qquad \text{for all } b_i'$$

Best-Response Dynamics with Round-Robin Activation

Activate bidders in order $1, 2, \ldots, n, 1, 2, \ldots, n, 1, 2, \ldots$
Every bidder switches to a best response

Best responses usually not unique:

Two bidders, one item.

If $b_1 = 1$ and $v_2 = 2$, then every $b_2 > 1$ is a best response to b_1

Potential Procedure

[Christodoulou/Kovács/Schapira, JACM 2016]

All valuation functions are XOS, that is,
$v_i(S) = \max_\ell \sum_{j \in S} v_{i,j}^\ell$ for some $v_{i,j}^\ell \geq 0$

When bidder i gets activated:

- Determine S that maximizes $v_i(S) - \sum_{j \in S} \max_{k \neq i} b_{k,j}$
- Let ℓ be such that $v_i(S) = \sum_{j \in S} v_{i,j}^\ell$.
- $b_{i,j} = v_{i,j}^\ell$ if $j \in S$ and 0 otherwise

Note: Updates fulfill strong no-overbidding:
For every $S \subseteq M$ and every i and t: $\sum_{j \in S} b_{i,j}^t \leq v_i(S)$.

Potential Procedure: Convergence

[Christodoulou/Kovács/Schapira, JACM 2016]

Theorem

The Potential Procedure reaches a fixed point (pure Nash equilibrium) after finitely many steps.

Core Lemma

Define *declared welfare*: $DW(b) = \sum_{j \in M} \max_{i \in N} b_{i,j}$.

Lemma

If i makes an improvement step from b^t to b^{t+1}, then $DW(b^{t+1}) - DW(b^t) \geq u_i(b^{t+1}) - u_i(b^t)$.

Proof. Suppose i previously won set S, now wins S'.

By choice of updates: $\sum_{j \in S} b^t_{i,j} \leq v_i(S) \qquad \sum_{j \in S'} b^{t+1}_{i,j} = v_i(S')$

$$
\begin{aligned}
&DW(b^{t+1}) - DW(b^t) \\
&= \sum_{j \in S'} \left(b^{t+1}_{i,j} - \max_{i' \neq i} b^{t+1}_{i',j}\right) - \sum_{j \in S} \left(b^t_{i,j} - \max_{i' \neq i} b^{t+1}_{i',j}\right) \\
&\geq v_i(S') - \sum_{j \in S'} \max_{i' \neq i} b^{t+1}_{i',j} - \left(v_i(S) - \sum_{j \in S} \max_{i' \neq i} b^{t+1}_{i',j}\right) \\
&= u_i(b^{t+1}) - u_i(b^t)
\end{aligned}
$$

$\square$

Potential Procedure: Convergence

[Christodoulou/Kovács/Schapira, JACM 2016]

Theorem

The Potential Procedure reaches a fixed point (pure Nash equilibrium) after finitely many steps.

Proof.

Define *declared welfare*: $DW(b) = \sum_{j \in M} \max_{i \in N} b_{i,j}$.

If i makes an improvement step from b^t to b^{t+1}, then $DW(b^{t+1}) - DW(b^t) \geq u_i(b^{t+1}) - u_i(b^t)$.

Every increase in utility is lower-bounded by some $\epsilon > 0$. $\square$

Potential Procedure: Convergence

[Christodoulou/Kovács/Schapira, JACM 2016]

Theorem

The Potential Procedure reaches a fixed point (pure Nash equilibrium) after finitely many steps.

Theorem

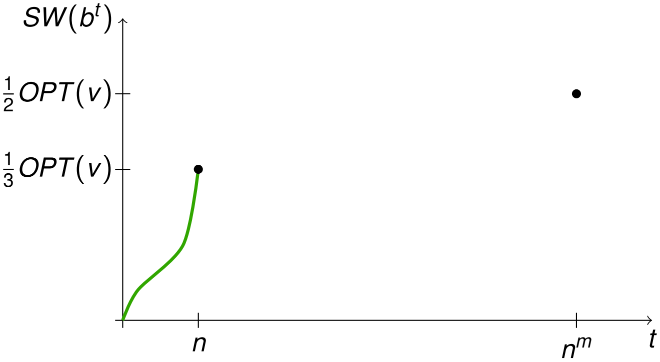
It may take an exponential number of steps (in m) to reach a fixed point.

Potential Procedure: Welfare Guarantee

[Dütting/K., SODA 2017]

Theorem

Let bidders be activated in order $1, 2, \ldots, n, 1, 2, \ldots, n, 1, 2, \ldots$. *Let* b^t *denote bid vector after* t*-th update.*
Then $SW(b^t) \geq \frac{1}{3} OPT(v)$ *for all* $t \geq n$.



Lemma

$\sum_{i=1}^{n} u_i(b^i) \leq DW(b^n)$.

Proof. Suppose bidder i's update buys him the set of items S'

$$u_i(b^i) = \sum_{j \in S'} \left(b^i_{i,j} - \max_{k \neq i} b^i_{k,j} \right) \quad .$$

Define: $z^i_j = \max_{k \leq i} b^i_{k,j}$ for all j.

We have: $\sum_{j \in S'} (b^i_{i,j} - \max_{k \neq i} b^i_{k,j}) \leq \sum_{j \in M} (z^i_j - z^{i-1}_j)$

Reason:

- For $j \notin S'$: $z^i_j \geq z^{i-1}_j$ by definition.
- For $j \in S'$, $b^i_{i,j} = z^i_j$ and
  $\max_{k \neq i} b^i_{k,j} \geq \max_{k < i} b^i_{k,j} = \max_{k < i} b^{i-1}_{k,j} = z^{i-1}_j$.

Lemma

$\sum_{i=1}^{n} u_i(b^i) \leq DW(b^n)$.

Proof. Suppose bidder i's update buys him the set of items S'

$$u_i(b^i) = \sum_{j \in S'} \left(b^i_{i,j} - \max_{k \neq i} b^i_{k,j} \right) \quad .$$

Define: $z^i_j = \max_{k \leq i} b^i_{k,j}$ for all j.

We have: $\sum_{j \in S'} (b^i_{i,j} - \max_{k \neq i} b^i_{k,j}) \leq \sum_{j \in M} (z^i_j - z^{i-1}_j)$

Overall:

$$\sum_{i \in N} u_i(b^i) \leq \sum_{i \in N} \sum_{j \in M} (z^i_j - z^{i-1}_j) = \sum_{j \in M} (z^n_j - z^0_j)$$
$$= \sum_{j \in M} \max_k b^n_{k,j} = DW(b^n) \qquad \square$$

Lemma

Let $S_1^*, \ldots, S_n^*$ be any feasible allocation. We have $\sum_i u_i(b^i) \geq \sum_{i \in N} v_i(S_i^*) - DW(b^n) - DW(b^0)$.

Proof. Bidder i could have bought the set of items S_i^*.

$$u_i(b^i) \geq v_i(S_i^*) - \sum_{j \in S_i^*} \max_{k \neq i} b^i_{k,j}$$

Define $p^t_j = \max_i b^t_{i,j}$ for all items j. We have: $\max_{k \neq i} b^i_{k,j} \leq p^n_j + p^0_j$. Thus

$$u_i(b^i) + \sum_{j \in S_i^*} (p^n_j + p^0_j) \geq v_i(S_i^*) \enspace .$$

And therefore

$$\sum_{i=1}^n u_i(b^i) + \sum_{i=1}^n \sum_{j \in S_i^*} (p^n_j + p^0_j) \geq \sum_{i=1}^n v_i(S_i^*) \enspace . \qquad \square$$

Potential Procedure: Welfare Guarantee

[Dütting/K., SODA 2017]

Theorem

Let bidders be activated in order $1, 2, \dots, n, 1, 2, \dots, n, 1, 2, \dots$. *Let* b^t *denote bid vector after* t*-th update.*
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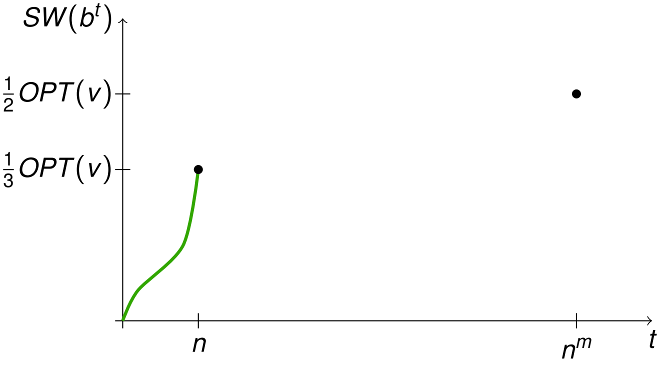

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Theorem

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Then $SW(b^t) \geq \frac{1}{3} OPT(v)$ *for all* $t \geq n$.

Proof.

- $\sum_i u_i(b^i) \leq DW(b^n)$
- $\sum_i u_i(b^i) \geq OPT(v) - DW(b^n) - DW(b^0)$
- $DW(b^0) \leq DW(b^n) \leq DW(b^t)$

$\Rightarrow$ $DW(b^t) \geq \frac{1}{3} OPT(v)$

Let $S_1, \ldots, S_n$ be allocation in b^t, then $DW(b^t) = \sum_i \sum_{j \in S_i} b^n_{i,j}$.
By strong no-overbidding: $\sum_{j \in S_i} b^t_{i,j} \leq v_i(S_i)$. So $DW(b^t) = \sum_i \sum_{j \in S_i} b^t_{i,j} \leq \sum_i v_i(S_i) = SW(b)$. $\square$

How to bid if valuations are only subadditive?

$$v_i(S) = \begin{cases} 0 & \text{if } |S| = 0 \\ 1 & \text{if } |S| = 1 \text{ or } |S| = 2 \\ 2 & \text{if } |S| = 3 \end{cases}$$

How to best respond to $(0, 0, 0)$?

- $(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$ bids $\frac{4}{3} > 1$ on $\{1, 2\}$ (i.e. overbidding)
- $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is strongly no-overbidding but bids only $\frac{3}{2}$ on $\{1, 2, 3\}$

Generally: No pure Nash equilibria that fulfill strong no overbidding
[Bhawalkar/Roughgarden, SODA 2011]

Aggressive and Safe Bids

Declared utility: $u_i^D(b) = \sum_{j \in S} b_{i,j} - \max_{k \neq i} b_{k,j}$, if i wins S under b

- We call bid b_i by bidder i against bids b_{-i} α-*aggressive* if $u_i^D(b) \geq \alpha \cdot \max_{b_i'} u_i(b_i', b_{-i})$.
- A best response dynamic is β-*safe* if it ensures that $u_i^D(b) \leq \beta \cdot u_i(b)$ for all players i and reachable bid profiles b.

Theorem

In β-safe round-robin bidding dynamic with α-aggressive bid updates at any time step $t \geq n$

$$SW(b^t) \geq \frac{\alpha}{(1 + \alpha + \beta)\beta} \cdot OPT(v).$$

Best Response Dynamics for Subadditive Valuations

Use: $S \mapsto v_i(S) - \sum_{j \in S} \max_{k \neq i} b_{k,j}$ is subadditive

Implies: Can be approximated by XOS function

Consequence: $\alpha = \frac{1}{\log m}$-aggressive, $\beta = 1$-safe dynamics

Theorem

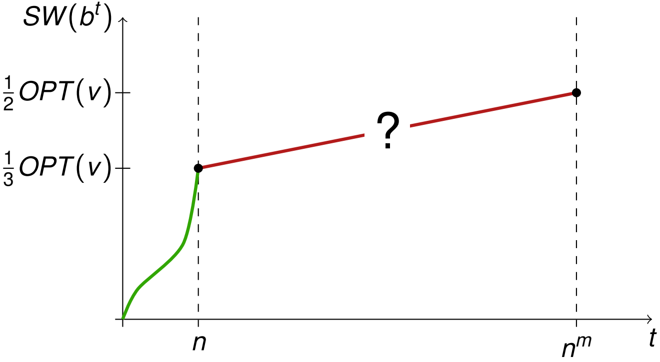
For subadditive valuations, there is a round-robin best-response dynamic such that at any time step $t \geq n$
$SW(b^t) = \Omega\left(\frac{1}{\log m}\right) \cdot OPT(v)$.

Theorem

For subadditive valuations, for every best-response dynamic there is an instance such that for infinitely many t
$SW(b^t) = O\left(\frac{\log \log m}{\log m}\right) \cdot OPT(v)$.

Outline

Convergence rate after n-th step



In case of XOS valuations:

- Reach $\frac{1}{3}OPT(v)$ after n steps (tight)
- Reach $\frac{1}{2}OPT(v)$ eventually (tight)
- How fast is convergence in between?

What about single-minded valuations?

Valuation functions of the form

$$v_i(S) = \begin{cases} c_i & \text{if } S \supseteq T_i \\ 0 & \text{otherwise} \end{cases}$$

for $|T_i| \leq k$.

More generally: MPH-k valuations

Relation to Smoothness?

So far: Techniques similar to price-of-anarchy analyses via smothness

Is there a general connection?

How do no-regret dynamics converge?

So far: Mainly use convergence to correlated equilibria, analyze those.

How fast? How difficult are single steps?

Can we guarantee any better approximation than $O(\log m)$ in case of subadditive functions?

Other auction formats?

Design mechanisms with better price of anarchy

Limitations: [Roughgarden, FOCS 2014]

Design mechanisms that are easier to play

Example: [Devanur/Morgenstern/Srygkanis/Weinberg, EC 2015]

Consider other settings than combinatorial auctions

Thank you!

Questions?

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