

Excursions in Computing Science:
Book 8d. Rocket Science.
Part IV Spaceship Earth.

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39. Speeds. What can provide all the protection and sustenance we've been discussing in the previous Part?

Our planet has a surface gravity of 1 gee. It has a magnetic field and an atmosphere which protect us from solar and cosmic radiation. We can breathe the atmosphere. Seventy percent of the surface is water and although very little of that is fresh water the weather systems provide enough. The ecosystems provide food and shelter. The mineral resources provide shelter and energy.

The Earth is also speeding through space. It orbits the Sun, 1 AU (astronomical unit) out, once a year. That is 2π AU per year.

We are 8 light minutes from the Sun, so $1 \text{ AU} = 8 \times 60 \text{ s} \times 300 \text{ K Km/s}$ and 1 AU per year is this divided by $3600 \times 24 \times 365 \text{ s}$, giving 4.75 Km/s. (Think 5 Km/s for short. I've actually used $1 \text{ AU} = 0.15 \text{ Tm}$ because the 8 light minutes is also approximate.)

The circumference of $2\pi \text{ AU}$ we travel in a year makes our speed 30 Km/s around the Sun. That's bigger than the rocket delta-Vs we discussed in Parts I and II.

And that's not all. The Solar System in turn orbits the centre of the Milky Way galaxy. We are about 26,000 light years from the galactic centre and orbit it about every 225 million years. That's a speed of $2\pi \times 26/225 = 7.3 \text{ light years per } 10,000 \text{ years}$.

I picked that time unit because 1 light year per 10,000 years is 1/100 of 1 percent of lightspeed: $300,000 \text{ Km/s}$ divided by 10,000 is 30 Km/s. It is also the speed of Earth in our orbit around the Sun. Since a *myriad* was an ancient military formation of 10,000 soldiers, we can call 10,000

years a *myriennium* (abbreviated *myr*—not to be confused with a million years, sometimes Myr but conventionally Ma).

So in addition to our 30 Km/s around the Sun, we are also moving at 220 Km/s around the centre of the galaxy.

unit	approx Km/s	our speed	in Km/s
AU/year	5	1 AU/year	30
ly/myr	30	7 ly/myr	220

These two velocities are more or less perpendicular to each other in direction (*velocities* have *direction*; *speeds* have only *magnitudes*) so they don't simply add or subtract but combine in a more complicated way.

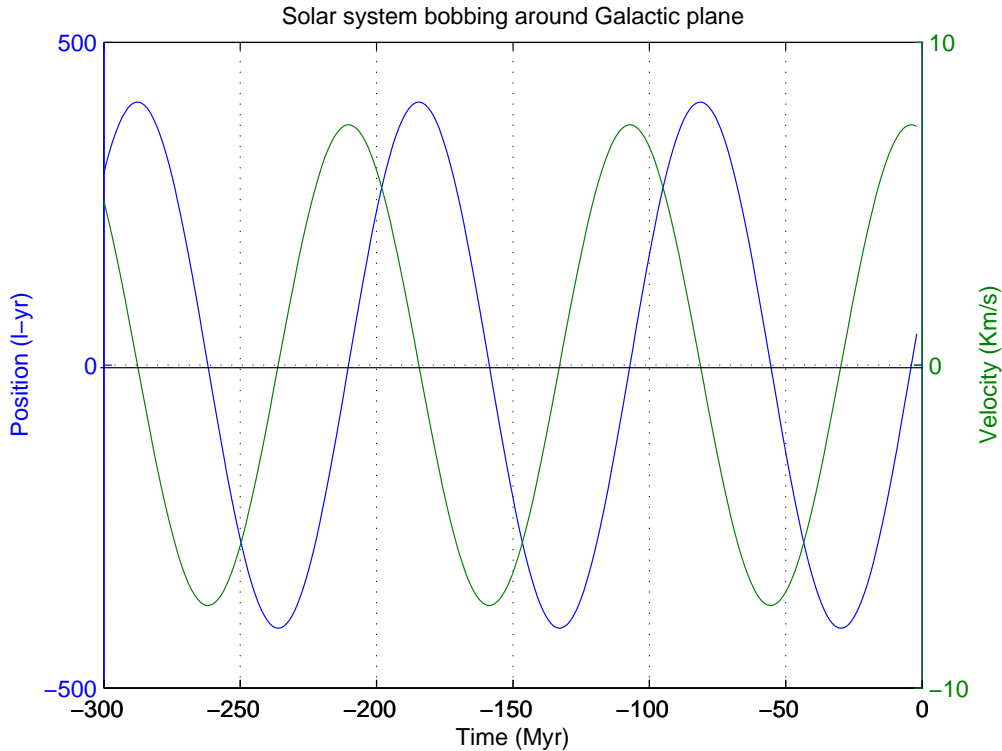
As it orbits the galaxy, our Solar System does not keep following a circle. The galaxy revolves in a more complicated way than planets orbit a star. So there are other components of motion.

The most significant one is a bobbing motion perpendicular to the “plane of the galaxy”. The galaxy is a disc 100,000 light years in diameter, but it is not 2-dimensional, nor is it uniform. It has a bar with two spiral arms of stars. But a flat disc bisecting this third dimension can be thought of as a plane which gravitationally attracts stars lying off the plane. These stars and their planetary systems will then “bob” back and forth through the plane of the galaxy, the way a cork might bob in a sink or a buoy on the ocean.

As far as I know, such bobbing has not been measured. But it can be calculated given a certain degree of understanding of the galaxy and given certain assumptions. The most recent study I've found argues for a period of 4×25.79 million years for the Solar System, which amounts to just over two complete cycles in a “galactic year” of 225 million years.

That study also says that we are now 48.9 light years “above” the Galactic plane and moving away from it at 7.4 Km/s with the furthest distance to be 407.5 light years away before we turn around and start back.

If we suppose the solar system is really bobbing like a cork, we can picture its motion over the past 300 megayears.



Here the blue curve gives our position and the green curve our speed.

With a little trigonometry and a little calculus of a new type from what we’ve mentioned before, we don’t need to be told about the speed (although we do need to be told that we are now moving away from rather than towards the Galactic plane).

The blue curve has the form

$$407.5 \sin(\omega t) = 407.5 \sin\left(\frac{\pi t}{2 \times 25.79}\right)$$

—it’s a “sine curve”, the mathematical name for a wave. The sine function, $\sin()$, is a trigonometric function.

The green curve can be found from this using calculus.

$$407.5\omega \cos(\omega t) = \frac{407.5\pi}{2 \times 25.79} \cos\left(\frac{\pi t}{2 \times 25.79}\right)$$

The fraction in front of $\cos()$ gives the maximum velocity in light years per megayear. To convert that to kilometers per second we need to reduce megayears by 100 to myriennia and then multiply light years per myriennium by 30 to get kilometers per second:

$$\frac{407.5\pi}{2 \times 25.79} \frac{30}{100} = 7.44 \text{ Km/s}$$

This is the maximum speed, reached just as we cross the Galactic plane, and pretty close to our current speed of 7.4 Km/s—because we’re pretty close to that midpoint. (And the only reason I kept the third significant figure, that last 4, which is not actually justified by the precision of the other terms I’ve used.)

This is only an exercise based on one claim a quarter century ago. The calculations are very tricky,

as are the observations, and although what we are saying is probably qualitatively true the numbers are almost certainly not correct.

We are interested in the speed. The maximum speed we got of 7 Km/s does not contribute a whole lot to the 30 and 220 Km/s we found earlier. As for direction, it would probably add to and subtract from the 30 Km/s orbital speed of the Earth.

40. Extinctions. Species on Earth do not last very long. They are going extinct all the time. But there have been five notable periods in which not only very large numbers of species but whole *families* (the next level up grouping of species) have disappeared.

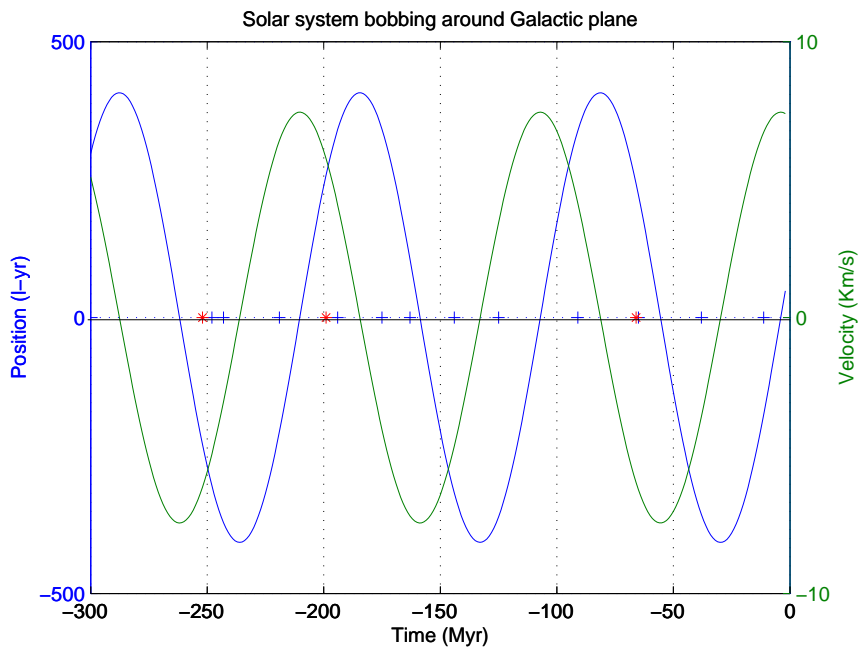
Myr	period	possible cause
-66	end Cretaceous	Mexico asteroid
-199	late Triassic	Atlantic volcanos
-252	end Permian	Siberia volcanos
-378	late Devonian	not known
-447	late Ordovician	global cooling

The most recent of these was the famous extinction of the dinosaurs. The probable cause was an extraterrestrial body—asteroid or comet—which hit the Yucatan peninsula and deposited a world-wide layer of iridium enabling it to be dated. There is still disagreement about this cause, and arguments that the dinosaurs hung in for hundreds of thousands of years after that date. Geology and paleontology are intricate sciences.

But let's go with the collision hypothesis. We already discussed (in Note 25 of Part II) what might cause an asteroid to leave the belt between Mars and Jupiter and come zinging towards Earth. What would send a comet our way?

The home of comets is the Oort cloud, which forms a sphere around the Sun at 2,000 to 100,000 AU. That goes as far as 1.5 light years.

One event which could disturb the Oort cloud would be the Solar System passing through the Galactic midplane, which is more densely populated than the outskirts, with both other star systems and dust clouds. Indeed a dozen extinctions over the past 250 Myr, including three of the above big five, have been observed to have a periodicity of 26 Myr, which is pretty close to the 25.79 Myr quarter-period of “bobbing” discussed in the previous Note.



The plot shows the dozen extinctions (black ticks) and the three big ones (red asterisks) superimposed on the periodic sine waves of the previous Note. (My data is from two different sources. The red asterisks should coincide with the ticks nearest them, but do so in only one of three cases. The chronology of the fossil record is very difficult to pin down.)

We note that there is a sort of correlation. The first two red asterisks indeed occur a few million years after a mid-plane crossing, and the third a similar lag after the peak position has been reached and the Sun starts to turn back. Maybe the disturbance takes this long to generate an Earth-crossing comet.

This is all pretty much speculation. There are other reasons for extinctions, some of which are listed in the table above for the big five. “Nonlinearities” in the dynamics of populations—akin to the nonlinearities that produce chaos—can spontaneously lead to population collapse. Indeed, we are apparently at the start of a “sixth extinction” which we are causing ourselves and has nothing to do with close comets (not even Shoemaker-Levy which hit Jupiter in 1994).

The point is that Spaceship Earth, despite its size and advantages, is also vulnerable and we must look after it.

41. Herd science. What must we learn to be successful stewards of Spaceship Earth? Our evolution as social animals has given us benefits we could not have enjoyed as solitary individuals. We have gained remarkable achievements through cooperation. But there is more. Beyond herdthink there is education.

Our brief discussion of swarms in Note 31 of Part III made two major assumptions. First, communications are local (although my own swarm there was so small that I simply included it all). Second, individuals are identical.

“Local communications” can mean a variety of things. If the swarm is a flock of birds or a school of fish, the communication is observing the directions and magnitudes of motion of the nearest so many neighbours in three dimensions and reacting accordingly. If it is a herd, we normally think of it as moving in two dimensions (with obstacles and varied terrain). But communications can transcend spatial dimensions, as, for humans, with telephones and the Internet, when we must begin to describe it as topological, as in a network.

We use the metaphor “being in touch” for kinships and friendships. If you’ve ever had the experience of being put in the middle of a flock (herd) of sheep by sheepdogs instructed to surround you, you will know that sheep prefer literally to be in touch. The metaphor is suggestive. The head of the Clarendon Laboratory when I was a physics student knew what to do with the kind of people who would write at length on what was wrong with physicists and on what they *should* be thinking: he put them in touch with each other. The Internet now does this routinely and without intervention.

So the first component of herdthink is, if you have a (big) decision to make, talk to people you know. When I had a flat tire on a weekend in a place where I don’t know the garages very well but do know the community, as well as using a search engine, I telephoned a friend whom I would expect to have had some garage experience.

But when your friends are not in a position to know about the issue, their advice is less helpful. For instance, I write in the midst of a plague. Not the Black Plague that sent Newton home from university to build mechanical devices and to connect the gravity that makes apples fall with the gravity that keeps the Moon in its orbit, but a pandemic for which we have vaccines.

If I were hesitant to be vaccinated, because of false information or because I was just concerned by the speed with which the vaccine had been developed, pooling my anxieties with those of my friends would not be so constructive. We would need to break out of the herd.

This can be hard to do. We are social animals. Ideologies can be remarkably resistant to reason. Indeed we could define ideology to be socialized opinion—reinforced by talking to friends—and

opinion to be a set of ideas reinforced by evidence to the contrary.

That's because the evidence to the contrary can be seen as an attack on the ideology. So one cannot "correct" an ideology from the outside but must come from the inside. That is, the converter must become a friend of the herd.

The second assumption we made about swarms in Note 31 of Part III was that there are no individuals. All birds or fish are interchangeable.

But that is not true for humans or most animals or even birds or fish. An active topic of zoological research is "dominance". In the first instance this is about who gets most of the resources—say, food at the kill or mates in breeding season. The domination-submission order is also called the "pecking order" because it was first investigated in domestic chickens. Note that it distinguishes two individuals, the dominant and the submissive, and so is a binary relationship which may extend into a hierarchy. It is easy to see this aspect of dominance in human society. Just think of income.

Understandably, then, a dominant individual is a focus of attention from the submissive. That can become leadership.

Leadership can be significant for humans, too. During this pandemic we have depended on our leaders. Societies with corrupt or disorganized government have fared badly; those with honest, open, trusted leaders have done relatively well.

But we notice in this example that the leaders who have done well have also taken scientific advice. They didn't just watch their social media pages but appointed advisors who are in a position to know. These are people who are outside the herd in a significant way.

They are people who have been educated (*e ducere* = to lead out).

What it means to be led out of herdthink is to have been liberated into being able to say "I was wrong" (or, more scarily, "I am wrong"). In particular, that is how science works. We've seen that in the scientists' responses to the pandemic: very little has been sure right away; only as evidence trickles in does the fluidity of the situation stabilize.

The uncertainty of experts can be very hard to take for people for whom to learn is to lose face. It is more acceptable to those educated in science, who know that scientific statements deliberately court error because error found is error contained, and error can thereby eventually be eliminated. (That is why a scientific theory is never "just a theory": theories are systems of ideas that contain their own potential disproof. If it doesn't tell you how to try to disprove it, it is not a theory but a much lesser thing.) The touchstone built into science for detecting error is disagreement with observation or with repeatable experiment.¹² Truth is hard-earned, after many mistakes.

Attention, influence and leadership are power. Power is the opportunity to mess up. Education speaks truth to power and so is threatening and liable to be resisted.

So the zoologists are studying herd behaviour and dominance and leadership—scientifically—but we must go beyond being just in the herd when it comes to pandemics and climate change - looking after Spaceship Earth.

42. Climate. Herd thinking has been especially prevalent on the topic of global warming—climate change. This is understandable since the process is immensely complicated, and educated thought, by its nature, has not leaped to conclusions but has been converging gradually since first inklings in 1824 and 1896.

Apart from climate change there are many less likely and less predictable threats to Spaceship

¹For example, the theory that the seasons are caused by Earth being alternately closer to and further from the Sun in its elliptical orbit, is refuted by the observation that we don't get two winters every year. The theory that the phases of the Moon are caused by Earth's shadow is refuted by any daytime observation of a phased Moon and the Sun together in the sky.

²However, if you want to unmask a scientific charlatan, you might do well also to be a trained magician: Houdini was enlisted to check spiritualists.

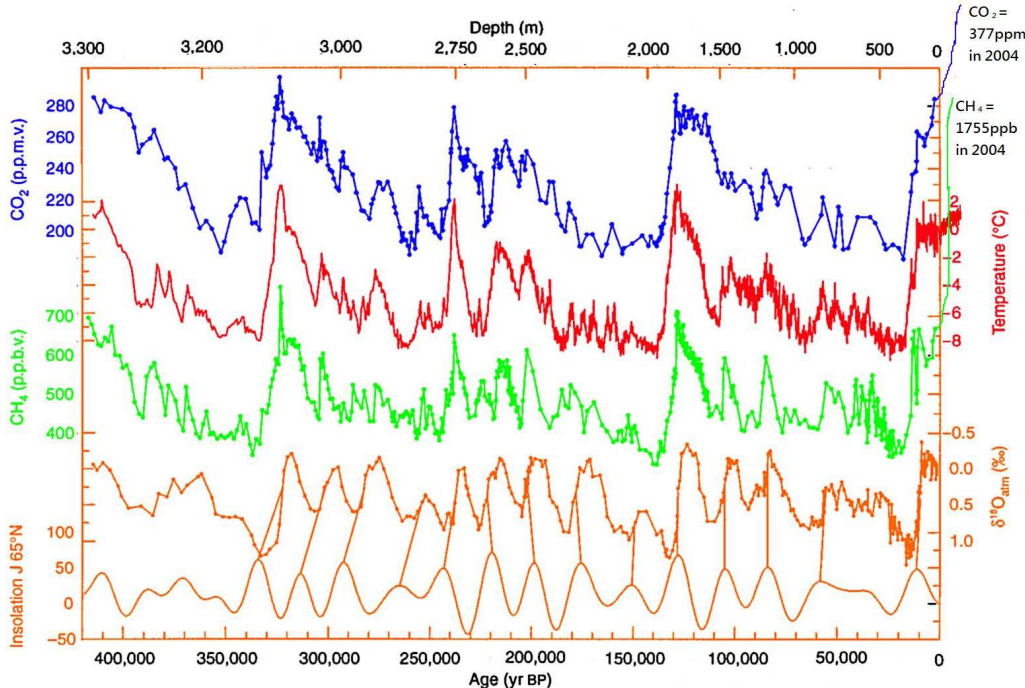
Earth: nearby supernova or gamma-ray burst, supervolcano eruption, or asteroid or comet strike. The eruption of the Laki fissure in Iceland for eight months in 1783–4 has been blamed for triggering the French Revolution by darkening European skies long enough to devastate the harvest. This was by no means a supervolcano such as the tuff and dome eruptions in New Mexico’s Valles Caldera from 1.25 million to only about 40,000 years ago. Comet Shoemaker-Levy hit Jupiter on July 16 1994 at 61 Km/s. Its mass has been estimated at 4–40 teragrams (Tg, 10^{12} g). So the energy released $mv^2/2 = 7.5\text{--}75 \times 10^{18}$ joules. This is an explosion of 2 to 20 thousand megatonnes of TNT. The biggest man-made explosion was the Soviet Tsar Bomba of 58 megatonnes.

But climate change is certain, immediately upon us and man-made.

Jean Baptiste Joseph Fourier calculated in the 1820s that the Earth’s average temperature should be -18°C to keep it in equilibrium with radiation from the Sun. So he wondered if the fact that Earth is 15°C on average was due to atmospheric trapping of heat, like a greenhouse. Svante Arrhenius in 1896 identified CO_2 , carbon dioxide, as a heat trapper and pointed out that coal burning since the industrial revolution in the 1750s would increase the gas in the atmosphere and warm the globe. Guy Callendar in 1938 published data showing a half-degree warming since 1890 and that CO_2 levels had risen ten percent. Charles David Keeling started detailed monitoring of atmospheric CO_2 in 1958, which has continued to this day.

To distinguish science from opinion in this discussion we must constantly ask *how do we know?* And we must recognize that knowing is doing—the checking of what we think we know, increasingly by calculation. The healthy attitude is to rejoice in what we (know we) don’t know, especially when the means exist to find out.³

How do we know what temperatures and CO_2 levels have been, before we started monitoring? Ice cores are one way. Glaciers and ice caps go back thousands of years. They trap bubbles of atmospheric gases which can be analyzed. And they are built in layers of annual snowfall which can be counted for dates and whose thicknesses give snowfall and hence temperature. Here is 420,000 years of data from ice cores from the Vostok region of Antarctica.



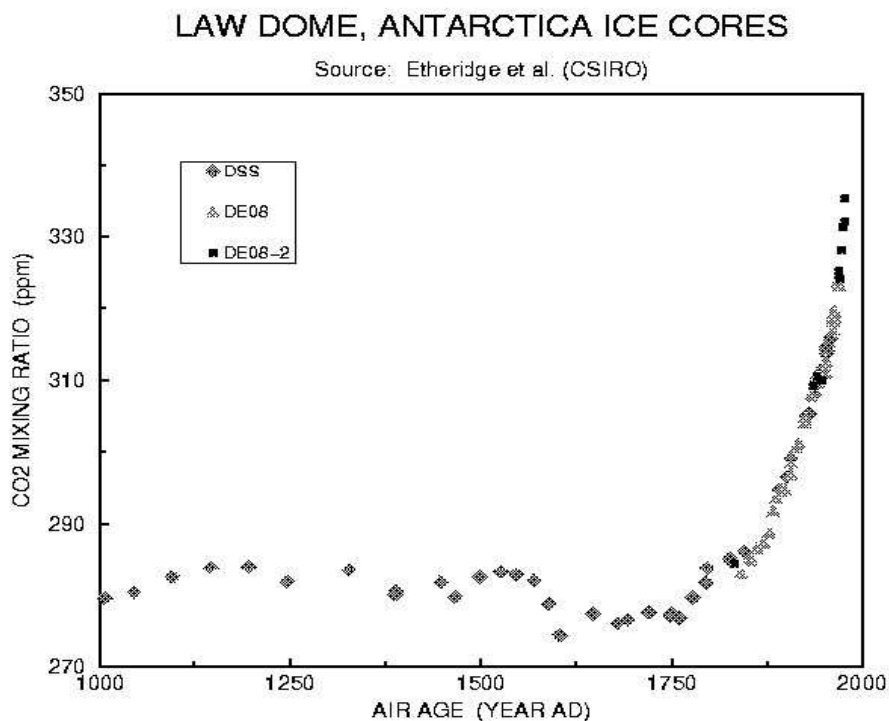
³The opposite of intellectual modesty is intellectual arrogance. That is risky, like the mouse which has lost its fear of cats: *Toxoplasma gondii* is a parasite which reproduces in cat intestines; it gets into a mouse brain and causes it to decide to go and kill a cat. Or at least cuddle. If you ever experience that kind of existential self-certainty, beware!

420,000 years of ice core data from Vostok, Antarctica research station. Current period is at right. From top to bottom: Levels of carbon dioxide (CO₂) in parts per million by volume; Relative temperature; Levels of methane (CH₄) in parts per billion by volume; ¹⁸O isotope of oxygen per mil (‰); Solar variation at 65°N due to Milankovitch cycles (connected to ¹⁸O).

As well as CO₂ and temperature, levels of methane (a greenhouse gas 30 times more potent than carbon dioxide) are reported, and the heavier isotope of oxygen (which we'll come to).

CO₂ in parts per million does not seem much. But 1 ppm is somewhere between 7.8 and 12 gigatonnes (GtCO₂). We must be careful in quoting gigatonnes. These figures are GtCO₂ as opposed to GtC. Carbon dioxide consists of a carbon atom (atomic mass 12) and two oxygen atoms (atomic mass 16 each)—normally: when we are not considering isotopes ¹³C, ¹⁷O or ¹⁸O—so CO₂ masses 12+16+16 = 44 while atomic carbon masses 12. Thus 1 GtCO₂ is 44/12 GtC.

Running off the chart are plots from contemporary monitoring of CO₂, CH₄ (methane) and temperature. If the scales were the same as the rest of the plots the first two lines would be vertical. Here is CO₂ for the last thousand years, from an ice core at Law Dome, Antarctica.



What happened after 1750 is clear. Human industry is increasing CO₂ immensely.

The effect on temperature is harder to tease out. The Vostok ice core is not dramatic about temperature the way it and Law Dome are about CO₂. We need sophisticated statistics and a wider range of data—tree rings and coral reefs, which also have temperature-dependent growth layers—as well as ice cores.

The now famous “hockey stick” curve was published [MBH98] in 1998. The implication of their results is that temperature was also horizontal from 1400 to 1750, then turned sharply upwards. You can imagine a hockey stick on the ground. I don’t reproduce their graphs here because they need the context of the paper to be understood. Instead I set the precedent of citing a reference directly in the text of these Notes rather than in an Excursion. It is worth at least skimming some

of the basic original papers, if only to appreciate how involved the work of finding these things out really is. (The authors did not use the phrase “hockey stick”. If their curve on p.783 is a hockey stick, we’re playing hockey using a living flamingo.)

For even more involvement there is, for instance, the 2018 IPCC report of 630 pages [IPC19]. The Intergovernmental Panel on Climate Change reports attempt to cover all the scientific literature on climate change, which is now immense. At the risk of taking this out of context I’ll quote only one result, from p.31 (p.45 in the .pdf):

Human-induced warming reached approximately 1°C (likely between 0.8°C and 1.2°C) above pre-industrial levels in 2017, increasing at 0.2°C (likely between 0.1°C and 0.3°C) per decade (high confidence).

Note the qualified language: these qualifications have precise ranges incorporating scientific agreement and confidence in the results.

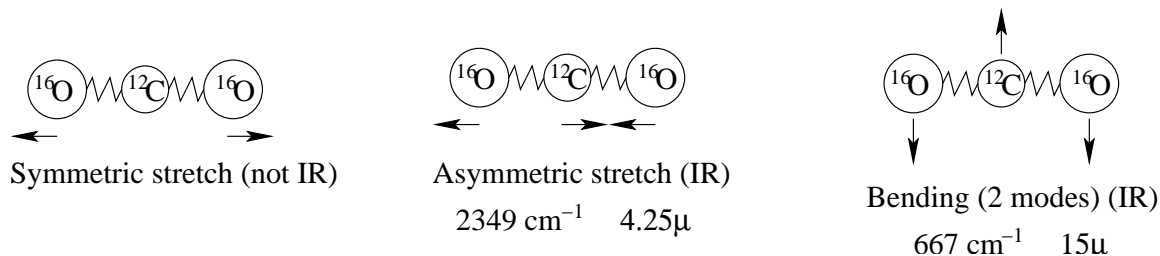
The *basic* science is fairly easy to understand.

A greenhouse (glass house, hothouse) works by allowing energy from the Sun in the form of visible light through the glass into the plants and soil inside. Absorbing this energy warms them up, and they in turn re-radiate the energy, but at infrared wavelengths. The glass does not transmit these wavelengths, so that re-radiated energy stays inside, warming the whole interior, by absorption by the glass among other mechanisms.

“Greenhouse gases” do the same thing as the glass—although without also providing walls the wind (and snow) can’t get through. They absorb infrared energy because their molecules vibrate internally at those frequencies.

Although you will see, if you have looked at the IPCC report, that there are other significant man-made greenhouse gases, such as methane and nitrous oxide (N₂O) (from agriculture, for instance) and all the “Montreal protocol” gases (which include exceedingly strong absorbers), carbon dioxide is the dominant one and illustrates the processes.

CO₂ has four modes of vibration, three of which can be triggered by infrared photons.



(The bending modes can be in two directions, up-and-down as shown, or back-and-forth, by rotating the whole molecule around its axis.)

The numbers give the frequencies at which the molecule vibrates and so absorbs, or rather the wavelengths, λ , of those frequencies, f , related by $f = c/\lambda$ where c is the speed of the wave, in this case lightspeed. The wavelength is measured in microns, $\mu = 10^{-6}\text{m}$; the cm^{-1} gives $1/\lambda$ as so many waves per centimeter.

We can ask how much the temperature of the molecule rises when it absorbs a photon of infrared radiation. The temperature change T is related to the change in energy E by Boltzmann’s constant $k_B = 1.38 \times 10^{-23}$ joules per Kelvin degree.

$$E = k_B T$$

The energy of a wave is proportional to its frequency by Planck’s constant $h = 6.626 \times 10^{-34}$ joule-seconds.

$$E = hf$$

So the temperature change for the more energetic 4.25μ absorption works out to be ⁴

$$T = \frac{E}{k_B} = \frac{hf}{k_B} = \frac{hc}{\lambda k_B} = \frac{h \times 71\text{THz}}{k_B} = \frac{47 \times 10^{-21}\text{J}}{k_B} = 3390\text{K}^\circ$$

And the temperature change for the less energetic 15μ absorption is

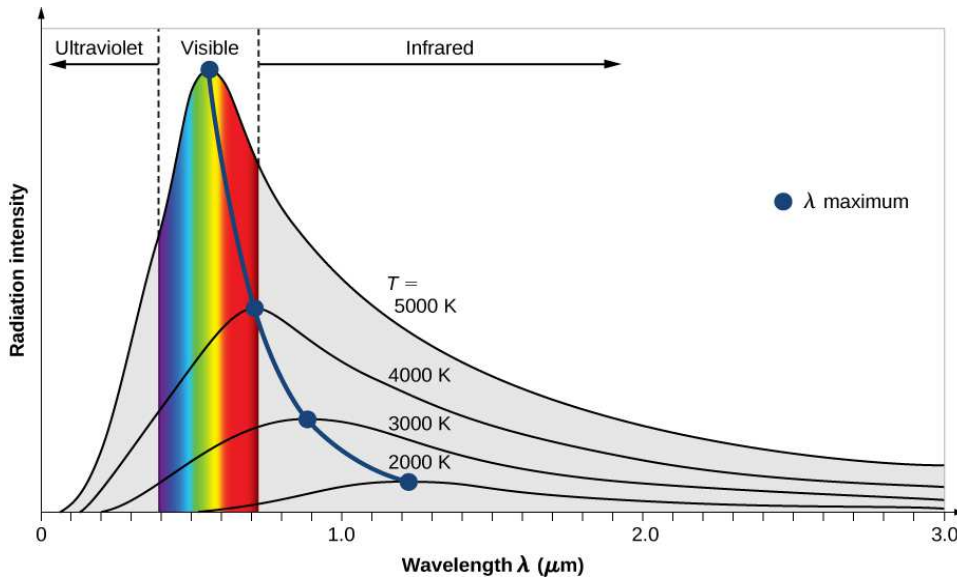
$$T = \frac{E}{k_B} = \frac{hf}{k_B} = \frac{hc}{\lambda k_B} = \frac{h \times 20\text{THz}}{k_B} = \frac{13 \times 10^{-21}\text{J}}{k_B} = 910\text{K}^\circ$$

But that’s only one CO_2 molecule mixed in with lots of nitrogen N_2 , oxygen O_2 and other, trace, gases. Crudely speaking, if the CO_2 is at 280 parts per million (the pre-industrial concentration of CO_2) and shares this heat energy equally, that temperature change amounts to just under 1 degree.

If we suppose that the pre-industrial atmosphere was in equilibrium, radiating back into space all that absorbed energy, then if industrial activity were to add another 280 ppm, we might loosely expect the atmospheric temperature to go up by one degree.

To improve this very faulty argument we must actually consider radiation, the way Fourier did.

Here, from [cnx21], are the “blackbody” spectra at four quite hot temperatures, including 5000°K , the approximate temperature of the Sun. The horizontal axis shows a range of wavelengths and the figure dramatically presents the range of visible electromagnetic radiation—light. The 5000°K spectrum peaks at yellow light, which is the colour of the Sun. (We can see that the 4.25μ and the 15μ wavelengths at which CO_2 absorbs are well into the infrared.)



The vertical axis is a bit more subtle and scale is not given. It is essentially given in terawatts/meter², power per unit area, but Planck’s distribution also includes “per steradian” to take into account radiation that is not perpendicular to the surface of the “black body”.⁵

This figure also shows “Wien’s law” governing the relationship between the wavelength λ_{max} at the

⁴The symmetric stretching mode does not absorb electromagnetic energy—it is not “IR -active” (although it is “Raman-active” but that’s another topic)—because the two positively-charged oxygen atoms are moving in opposite direction. The other modes are IR-active because equal charges move in the same direction and opposite to the opposite charge.

⁵Max Planck calculated the blackbody distribution in 1900 by supposing that radiation is emitted in “quanta”—units of hf where h is the Planck constant we’ve just encountered—as opposed to continuously, an error which led classical physics to immense discrepancies from observation especially in the ultraviolet parts of the spectra. Thereby began a significant era in physics.

NB. Fourier preceded Planck so didn’t have the complete blackbody law. But so did Wien, Stefan and Boltzmann: their laws were not originally derived from theory but empirical.

peak of the distributions, and the temperature. If you eyeball the graph you'll see that the product of wavelength in microns and the temperature in Kelvin is somewhere around 2800 (except for the 2000°K peak which may be misplaced). The actual number is almost 2898, so

$$\lambda_{\max}T = 2.898 \times 10^{-6} \text{ °K}$$

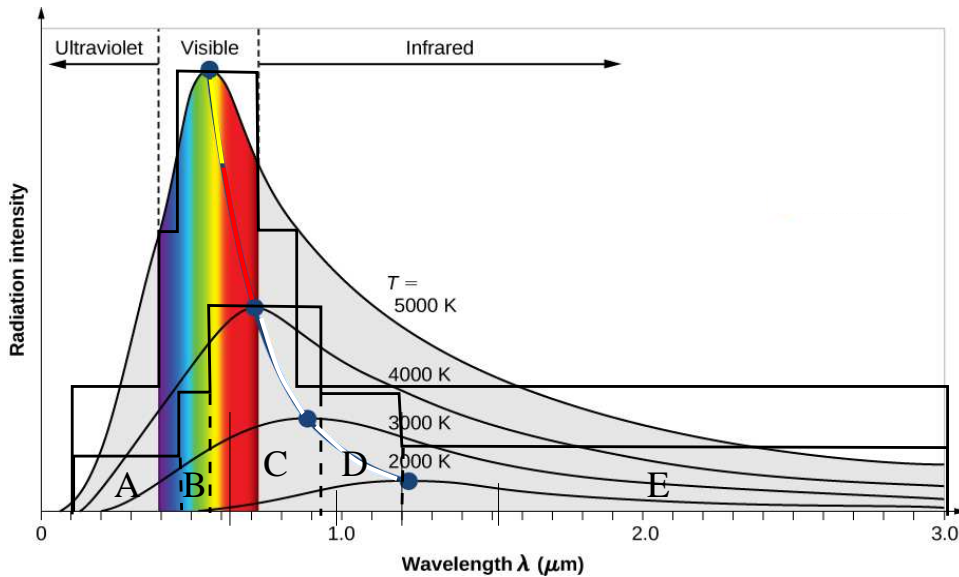
Wien's law enables us to determine the temperatures of stars from their colours.

The *Stefan-Boltzmann* law is more useful for our present need. It relates the total radiation per unit area ("flux", F) to the temperature

$$F = \sigma T^4$$

with $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$. This fourth-power dependence on temperature is remarkable and we should at least persuade ourselves from the plot that it is right.

We get the total radiated power from the area under the spectrum curve. Calculus calls this "integrating" but we can approximate it by the five rectangles shown for each of the 5000°K and the 4000°K distributions. (It was too messy to try to do all four curves.) I "eyeballed" the rectangles in such a way as to gain more or less as much area where they go beyond the curve as they lose where they fall short: as I say, crude.



Since I didn't have the vertical scale, I just measured the areas using sixteenths of an inch in both directions, on my screen display.

	A	B	C	D	E	total	total $\times t^4$
4000°K	8 \times 17	17 \times 4	29 \times 16	17 \times 11	9 \times 77	1548	6.0
5000°K	18 \times 12	40 \times 3	61 \times 11	40 \times 6	18 \times 91	2896	4.6

The A,B,C,D,E rectangles are shown in the figure for the 4000°K curve; they have the corresponding meaning for the 5000°K distribution.

The t is the temperature divided by 1000: 4 and 5 respectively. The numbers 6.0 and 4.6 should be the same. But note that I've left off the infrared tails of the distributions, a serious omission, especially for the higher temperature. So the T^4 dependence seems acceptable at the level of this bit of junior physics.

So if we accept the Stefan-Boltzmann law we can calculate flux from temperature or vice-versa.

The *effective temperature* of the Earth is the temperature it would have as a black body radiating all of the solar input. That we saw in Note 6 of Part I is 1366 watts/meter². The Earth absorbs

70% of this power (it reflects 30%, its *albedo*) across an area πr^2 where r is Earth's radius. That heats up a surface area of $4\pi r^2$ which radiates almost like a black body. So the net radiated power is

$$\frac{1366 \times 0.7}{4} \text{ W/m}^2$$

and the effective temperature is

$$\left(\frac{1366 \times 0.7}{4 \times 5.67 \times 10^{-8}} \right)^{1/4} = 255 \text{ }^\circ\text{K}$$

That is a chilly -18°C .

(The average temperature of Earth is actually 288°K or $+15^\circ\text{C}$. This is the discrepancy that worried Fourier. The difference— $33 \text{ }^\circ\text{C}$ —is due partly to the greenhouse effect and partly to the radioactivity-induced heat that keeps our iron core molten (and generates our protective magnetic field).)

Knowing the effective temperature, T , we can also use the Stefan-Boltzmann law to find the effect of a small bit of “forced radiation” incoming due to extra CO_2 in the atmosphere.

First, a small change f in the flux F induces a small change t in the temperature T :

$$F + f = \sigma(T + t)^4 = \sigma(T^4 + 4T^3t + 6T^2t^2 + 4Tt^3 + t^4) \approx \sigma(T^4 + 4T^3t) = F + 4\sigma T^3t$$

The approximation holds since t is much less than T .

That is,

$$f \approx 4\sigma T^3t$$

or

$$t \approx \frac{f}{4\sigma T^3}$$

T is the effective temperature, 255°K .

We need to know f , the additional effective flux due to extra CO_2 . The Intergovernmental Panel on Climate Change has captured the measurements of this as $1.4 \times 10^{-5} \text{ W/m}^2/\text{ppb}$ [FRA⁺07, Table 2.14 on pp.212–13] so we can calculate the effect of an additional 280 ppm as

$$f = 1.4 \times 10^{-2} \times 280 = 3.92 \text{ W/m}^2$$

This gives our first estimate of *climate sensitivity*, the temperature increase due to an additional 280 ppm of CO_2 on top of the 280 ppm of CO_2 that was the norm before industrialization in 1750.

$$t \approx \frac{f}{4\sigma T^3} = \frac{3.92}{4 \times 5.67 \times 10^{-8} \times 255^3} = 1 \text{ }^\circ\text{K}$$

There's that 1 degree again. But we've already quoted another IPCC report which says that the 1-degree increase was reached in 2017, when the U.N. reported CO_2 at 405.5 ppm, an increase of only 125 ppm since 1750, not 280.

What we've left out is feedback.

Feedback brings us into territory much harder to quantify and hence predict. There are many feedbacks, some negative (slowing down the warming) but most positive (speeding it up).

The first obvious feedback is the reduction of the Earth's albedo by melting ice. We can see this in the decline of glaciers. Here is the Pedersen glacier in Alaska, viewed from the same point in 1917 and in 2005.



And here is the Careser glacier in Italy in 1933 and 2012.



We can see that the ground exposed after the ice has gone is darker and so will absorb heat that much more readily.

The significant ice-albedo feedback reinforcing climate warming is the loss of sea-ice in the Arctic, especially in summer when the Sun can shine nonstop. The water exposed by melted ice has a lower albedo (0.06 compared with 0.5 to 0.7) and so warms even faster.

Ice melting has a further positive reinforcement. The Greenland ice sheet, especially, is melting internally, with the water running down sinkholes in the ice to flow out to sea underneath. This lubricates the land the ice is sitting on and allows it to slide more readily towards the ocean, where it breaks into icebergs that float away and melt. Ice shelves that are already floating do not raise the sealevel, but ice floating away from land support does, even though it loses 10% of its volume by melting.

When the ice is underground as permafrost near the poles, it contains plant matter and especially methane. I've mentioned that methane is a greenhouse gas significantly more absorbent than CO₂, and releasing it provides a tremendous reinforcement to climate warming.

In another positive feedback loop, warming dries out forests which become more susceptible to wildfire. California's fall 2020 fires released 1/10 GtCO₂. This is only a third of a percent of our global CO₂ emissions, but it is also only one fire, big though it was. Fire soot, landing on ice, reduces the albedo, also accelerating warming. However, fire aerosols may increase the albedo of

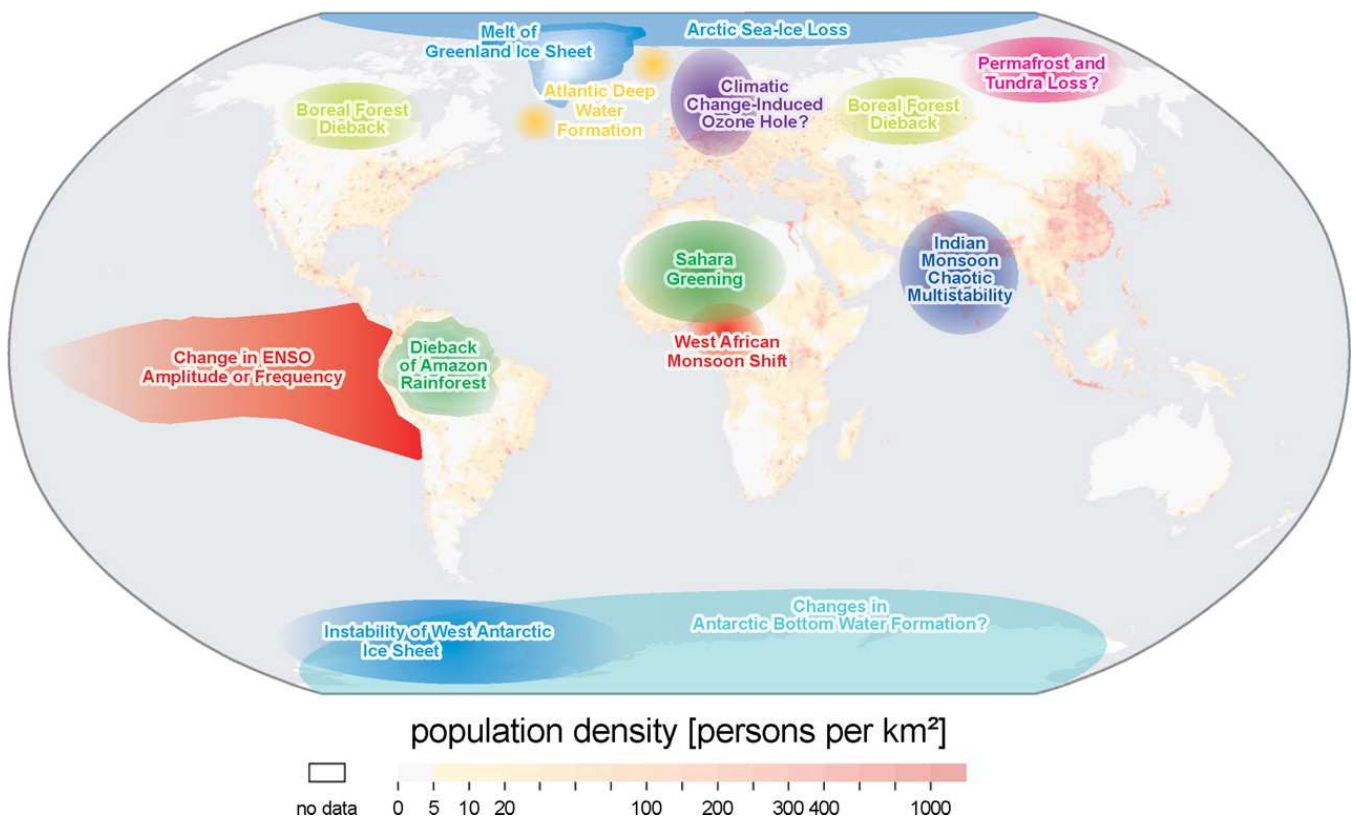
the atmosphere somewhat, slowing warming down.⁶

Positive feedback is what happens in a sound system when the microphone picks up little sounds from the loudspeakers, feeding them back into the amplifier where they grow, in principle without limit. This illustration may help you appreciate the idea of “tipping point” where the screech becomes unbearable and you must turn the system off.

Methane release may be an example of an abrupt, and after a certain point, irreversible, climate change. A more localized tipping point would be in the “thermohaline system” of ocean currents. Warm salty water is less dense than cold salty water, and so rises to the top of the ocean. The Gulf Stream is an example, bringing warm water from the eastern seaboard of North America across the north Atlantic to warm Iceland, Britain and all the way up the Baltic Sea to Scandinavia and Russia. The cold returning current is forced down and so returns south without affecting weather.

But cold fresh water, from Greenland ice melt, is less dense even than warm salty water, and so will interfere with this pattern. It runs the risk even of “turning the Gulf Stream off”. That would significantly affect northern Europe, with global warming paradoxically making a large population a lot colder.

Here are some more potential tipping points, from [LHK⁺08].



The upshot is that science has been unable to calculate climate sensitivity. The best estimate is that, instead of 1 degree increase after 280 more ppm of CO₂ have been added, the increase will

⁶Wildfires give a dramatic way to visualize a gigatonne of carbon dioxide. The U.S. West Coast fires in 2020 burned some million hectares, which is 10,000 square kilometers (100 hectares in a (Km)²). A gigatonne would result from ten times this. So make a line from you to a place 300 Km away. (That’s a 3-hour highway drive.) Square that. Pack the square with mature trees. Burn it. You’ve just generated a GtCO₂. Humanity is now doing that 36 times per year and increasing.

range from 1.5 to 4.5C°. This range has not successfully been narrowed in three decades of research. At our present annual rate of 3–4.6 ppm CO₂ we'll have doubled our pre-industrial emissions to 560 ppm by 2050–2060. And the rate has not started to decrease.

One degree of warming, or even 4.5 degrees, does not immediately sound too dire. It even sounds comforting in northern countries. But, quite apart from this being but the thin end of the wedge, it brings with it unpredictable threats. Such as insect-borne disease vectors (Lyme's disease, Zika), increasingly energetic hurricanes and cyclones due to ocean warming (wind, flooding) complemented by increasing drought in mid-latitudes as the climatic Hadley cells are driven polewards (wildfires), and the threat to coastal cities of sealevel rise.

Climate models are probably not ready to make full predictions of all of this, so we are left to look at past occurrences.

A seventh of the way through the geologically short period of time between the extinction of the dinosaurs and the present, paleontology reveals the Paleocene-Eocene Thermal Maximum (PETM). That was the last time CO₂ concentrations were as high as our present 400+ ppm, and they went to over 1000 ppm. Temperatures increased 5–8C°. There was no ice or snow. The cause is still being debated, but carbon was released into the atmosphere over some 20,000 to 50,000 years, the warm period (no ice) lasted 200,000 years and the recovery back to temperatures humans have hitherto known took another 300,000 years.

Further estimates are that 12Tt (tera tonnes: exagrams) of carbon (equivalent to 44 Tt CO₂) were released over that, say, 20Kyr period. That averages to 2.2GtCO₂ equivalent per year, and probably less. For comparison, in 2018 we put out 36Gt, 16 times that rate. During the PETM, species adapted and evolved. (Indeed our primate ancestors appeared then, as did the ancestors of horses.) This may not have time to happen in the anthropocene.

How do we measure temperatures and CO₂ concentrations going back 55 million years? By isotopes in seafloor sediments.

These sediments contain the calcium carbonate, CaCO₃, from shells and bones of creatures growing in seawater. Both carbon and oxygen have various stable *isotopes*—atoms with the same number of protons in their nuclei, and so having the same chemistry, but with different numbers of neutrons. Thus the usual ¹²C₆ and ¹⁶O₈ are accompanied by heavier ¹³C₆ and ¹⁸O₈. Heavier molecules of H₂O and CO₂ do not evaporate quite as readily as the lighter versions, and so build up a concentration in the sea: glaciers, say, and seawater have different mixes of the oxygen isotopes, which can be detected. So a sudden melting of ice will show up as sedimentary layers with a sudden relative increase in the lighter isotopes. Conversely, we can also measure how much ice there was out of the oceans and on land.

(A similar analysis detects whether the CO₂ in ice core bubbles came from burning former plants—fossil fuels—with their slight surfeit of lighter carbon isotopes. It did.)

Before we can do anything about fouling our collective nest this way we should know *where* we are generating the most carbon. My Google search on “sources CO₂” produced, first, a simple answer from a European 3-year consortium of unidentified authors:

CO ₂ Human Emissions, World 2017 (https://www.che-project.eu/)	
Fossil fuels	87%
Land clearing & use	9%
Industry, e.g., cement	4%

Further answers were conflicting, owing no doubt to different criteria and regional focus (e.g., U.S.A. and global). There is some agreement that agriculture is responsible for about 10% of human carbon emissions. The other economic sectors (transportation, industry, residential and commercial) divide the remaining 90% fairly equally, with transportation emitting somewhat more

than the rest and commercial, and maybe residential, somewhat less.

Some of the political confusion about what to do may be due to lack of clarity in the data that might tell us where to begin. But it seems clear—without need for further studies—that we must start with fossil fuels.

Petroleum molecules, and natural gas and coal to a lesser extent, have wonderful potential, so burning them into CO₂ is woefully short-sighted. (Of course, we must also stop dumping the plastics they produce into the oceans.)

Solar power—and its derivative, wind power—can easily supply our needs for energy. The world consumed 0.4ZJ (zeta = 10²¹ joules) in 2017, which translates to 12.7TW (tera = 10¹² watts = joules/sec). A “Kardashev Type I” technical civilization would use the entire solar energy incident on Earth, which is 1KW per square meter (let’s accept the albedo and reduce the 1366W/m² to 70%) times $\pi r^2 = (20M\text{ m})^2/\pi = 127T\text{ m}^2$ (remember the definition of a meter) or 127PW (peta = 10¹⁵). That is 10,000 times what we are now using: there is no need to whine about running out of power. Nor about jobs: we have lots of work ahead.

To make this abundant energy portable, we can use it to dissociate water into hydrogen and oxygen, carry the hydrogen in fuel cells to power electric vehicles or burn it directly in modified internal combustion engines. The product is water in both cases—whose vapour is also a greenhouse gas, but a transient one. Hydrogen becomes an “energy currency”, a clean form of energy storage.

We can conclude this Note, and this chapter, optimistically. Although the news of global warming has been met with massive herdthink—understandably, because the evidence is subtle and nuanced—we have two examples of successful political cooperation over similar issues, despite temporary economic downside and change of direction for various stakeholders.

The first was the 1970 U.S. Clean Air Act. Passed unanimously in the U.S. Senate and with one dissenting vote in the House of Representatives, it has made U.S. air 77% cleaner today (and car tailpipe pollution 99% cleaner)⁷ with benefits forty times the costs. The issue of National Geographic that contains a brief summary [Gar21] of the Act and its effects also highlights the downside of dirty air throughout the world, and notes the counterproductive effect of wildfires.

The second successful agreement was the Montreal Protocol to phase out ozone depleting substances (ODS) which were opening a vast hole in the ozone layer, which protects Earth from ultraviolet and other carcinogenic radiation. “Adopted on 15 September 1987, the Protocol is to date the only UN treaty ever that has been ratified by every country on Earth - all 198 UN Member States” (<https://www.unep.org/ozonaction/who-we-are/about-montreal-protocol> last accessed 2021 May 12). An Encyclopedia Britannica article by John Rafferty says that by 2016 “scientists got sufficient data to confidently reveal proof that the ozone layer was indeed on a path to recovery .. [and] expected the ozone layer to fully heal sometime between 2040 and 2070.” Those gases (chlorofluorocarbons CFCs, hydrofluorocarbons HFCs, hydrochlorofluorocarbons HCFCs and others) are also exceedingly powerful greenhouse gases, although very sparse, so the Montreal Protocol is a direct start on halting climate warming.

Appendix. Trigonometry and calculus.

43. Trigonometry.

44. Integral calculus.

45. Differential calculus.

II. The Excursions

You’ve seen lots of ideas. Now *do* something with them!

⁷I imported a car into North America in 1970. The British manufacturer had had to build in elaborate emission control equipment to meet California standards. A friend at the British Motors special tuning division, who adjusted the system before I put the car on the boat, said that the car ran better because waste was reduced.

1. The data for the graph in Note 39 come from [DR93]. Before quoting the results you should check out that paper and any subsequent papers that examine it critically. I am not an astrophysicist; don't take my word for it.
2. Extinction data in Note 40 come from [RS84] (black ticks on the plot) and [Smi10] (red asterisks). Note that one asterisk coincides with a tick. The other two should also coincide with ticks. That they do not shows the uncertainty of paleontological dating. These two publications are separated by 35 years, so improvements may account for the discrepancies. Other publications show yet other dates. Getting to the bottom of it all would take more time than I have. I am not a paleontologist either; don't take my word for it.
3. Elizabeth Kolbert [Kol14] elaborates on the “sixth extinction”. Elizabeth Royte [Roy21] spells out what a few extra degrees of heat can do.
4. A reference for stars approaching the Solar System is [Ben21]. It shows 5 stars approaching to some 3 light years within the next 50,000 years, and a number of stars close enough to pass through the Oort cloud in ± 5 megayears, especially some 2 Myr ago.
5. I have no background in ethology, the study of animal behaviour introduced in Note 41, so am particularly susceptible to being wrong, especially in pushing results past their realm of applicability, which is animals. (For most of the other topics in this Book I have been able to detect my own errors—but not guarantee correctness—by doing a calculation, mentally, pen-and-paper, or by writing a computer program. That approach is less available here—although zoology increasingly does modelling.) This Excursion invites you to make a career in ethology and correct me. A “mini-revue” which is still being cited links dominance with leadership but adds other considerations such as kinships and social bonds: [KC09]. It may give you a start.
6. An enlightening example, of vaccine hesitation reduced by making new friends with experienced people, was aired on the Canadian Broadcasting Corporation's *The Current* a year after the Covid pandemic started: [Gal16].
7. The history of vaccines in [Phi13] gives a perspective on the tremendous amount of work that has gone into vaccines and their enormous success in containing ravaging diseases such as smallpox (Edward Jenner, 1796; eradicated in 1980), rabies (Louis Pasteur, 1884), cholera (Jaime Ferrán, 1885, Waldemar Haffkine, 1911), diphtheria (William Park, 1914), yellow fever (Max Theiler, 1936), whooping cough (Pearl Kendrick and Grace Elderding, 1939), Japanese encephalitis (Maurice Hilleman, 1944), influenza (Thomas Francis Jr. and Jonas Salk, 1945), polio (Jonas Salk, 1952), measles (John Enders et al., 1963, Maurice Hilleman, 1968), mumps (Maurice Hilleman, 1967), influenza A (Maurice Hilleman, 1968) and rubella (Maurice Hilleman, 1969).

Before Jenner, smallpox immunization was achieved by “variolation”, the risky direct exposure to the smallpox pathogen itself (*variola* = smallpox). Jenner's innovation was to use cowpox, a less virulent (to humans) relative of smallpox. The procedure became known as vaccination (*vacca* = cow). Modern vaccines use a dead or weakened form of the pathogen, or parts of it. Two of the vaccines for SARS-CoV-2, the Covid-19 coronavirus, use messenger ribonucleic acid, mRNA, to induce the patient's cells to produce a protein specific to the virus for the immune system to recognize and target.

Coronaviruses, as well as influenza, measles and some cold viruses, use RNA rather than DNA for their genomes. They mutate often and the advantage of mRNA-based vaccines is that they work from generic “platforms” which can be repurposed to deal with mutations or indeed any new mRNA. Chinese researchers decoded the SARS-CoV-2 genome, in Jan. 2020, ten days after the first cases were reported, and shared it. So Moderna, for instance, had a vaccine in Phase I clinical trials two months later. This very speed, unprecedented as it is, has led to mistrust of SARS-CoV-2 vaccines, as has government bungling in some

jurisdictions. [Yon21]

The timeline also lists setbacks, which would have given rise to misgivings about vaccination. In 1928 and 1929 vaccination disasters in Queensland, Australia, and Lübeck, Germany, for diphtheria and tuberculosis, respectively, were caused by mistakes in the production processes. Similar oversights led to polio vaccination being temporarily suspended by the U.S. Surgeon General in 1955.

And, of course, in 1997 a subsequently discredited and retracted article in *The Lancet* by Andrew Wakefield claimed that the measles, mumps and rubella (MMR) vaccine increased autism in children.

But misinformation about vaccination predates even these excuses. The Anti-Vaccination League of America held its first meeting in New York in 1882, claiming that smallpox was not contagious but caused by dirt. It would be interesting to know what motivated that antipathy to successful science, and what (contagion) spread those ideas.

8. Attribute the following quotes.

- “It’s hard to get someone to understand an idea whose job depends on not understanding it.”
- “It’s hard to get someone to accept a fact who desperately needs to believe something else.”
- “If you think education is expensive then try ignorance.” But “learning is impossible without ignorance.”
- “It’s not what you don’t know that’s dangerous, but what you know that ain’t so.”
- “If you have power, you don’t need intelligence.”
- “It doesn’t take much cleverness to camouflage the obvious.”

9. If all the ice in the Antarctic and Greenland ice sheets were to melt (as in the PETM of Note 42) how high would the seas rise?

	Area (M(Km) ²)	Max. thick (Km)	Volume (M(Km) ³)
Antarctica	14	4.5	30
Greenland	1.7	3	2.6

(You’ll discover that I’ve applied a “fudge factor” of 1/2 to reduce maximum thickness to average thickness.)

The world’s oceans occupy 71% of the whole surface area, which is $4\pi r^2 = (2\pi r)^2/\pi$ where the circumference $2\pi r = 40\text{Mm}$ by the Napoleonic definition of a meter (1/10,000,000 of the distance from pole to equator). This works out to be 362M(Km) so the oceans will rise by $(30 + 2.6)\text{M(Km)}^3/362\text{M(Km)}^2 = 90\text{m}$.

Taking into account that water expands 10% on freezing (so that ice floats), this is reduced to 80m. I’ve found another estimate which says 65m. How many cities are not entirely higher than 65 meters?

Calculating the annual rise is trickier because it depends on the feedbacks and tipping points of Note 42. So far, the sea seems to have risen 0.2 m—some of which is due, not to ice melt, but to expansion of the water as it warms.

10. A review of the science of the Paleocene-Eocene Thermal Maximum (PETM: Note 42) is in [MW11]. (Foraminifera are single-celled animals: the planktic kind float in the ocean; the benthic kind live on the seafloor.) They give different estimates of onset, duration and recovery times, and of CO₂ levels: how do these rates of carbon emission compare with anthropocene levels? (They also use numbers which imply that 1 ppm CO₂ in the atmosphere is equivalent to 12GtCO₂. Their CO₂ estimate in the PETM of 16 times the 280 ppm preindustrial levels

gives 4500 ppm, which at 12GtCO_2 would require $(4500 - 280)/(36/12) = 1400$ years for us to reach at our present rate of $36/12 = 3$ ppm per year. I wouldn't get too relaxed: using 7.8GtCO_2 per ppm gets us to 1000ppm by year 2150 at $36 \text{ GtCO}_2/\text{year}$.

11. Any part of the Preliminary Notes that needs working through.

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