

# Excursions in Computing Science: Table of Contents

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| i     |      | 01   |    | 001  | <b>Rules and sums.</b>                                      |
| i     |      | 02   |    | 003  | <b>Some visualizations.</b>                                 |
| i     |      | 03   |    | 005  | <b>Rules and differences</b>                                |
| i     |      | 04   |    | 005  | <b>Rules and programming.</b>                               |
| i     |      | 05   |    | 009  | <b>Reasoning with rules.</b>                                |
| i     |      | 06   |    | 010  | <b>Square roots and cube roots</b>                          |
| i     |      | 07   |    | 010  | <b>Primes.</b>  |
| i     |      | 08   |    | 012  | <b>Multiplication: rectangles.</b>                          |
| i     |      | 09   |    | 012  | <b>Division: slopes.</b>                                    |
| i     |      | 10   |    | 013  | <b>Negative numbers.</b>                                    |
| i     |      | 11   |    | 013  | <b>Pictures of rules.</b>                                   |
| i     |      | 12   |    | 015  | <b>Nonlinear plots.</b>                                     |
| i     |      |      | 01 | 016  | <b>Mathematical truth</b>                                   |
| i     |      |      | 02 | 017  | <b>Counting in tongues.</b>                                 |
| i     |      |      | 06 | 017  | <b>Squares.</b>   |
| i     |      |      | 07 | 017  | <b>Cubes.</b>   |
| i     |      |      | 08 | 017  | <b>Simplex numbers.</b>                                     |
| i     |      |      | 09 | 018  | <b>Hypercube numbers.</b>                                   |
| i     |      |      | 23 | 020  | <b>Simplexes: higher-dimensional triangles.</b>             |
| i     |      |      | 24 | 020  | <b>Hypercubes: higher-dimensional squares.</b>              |
| i     |      |      | 25 | 021  | <b>Hyperdipyramids: higher-dimensional squares, part 2.</b> |
| i     |      |      | 40 | 024  | <i>Sieve of Eratosthenes</i>                                |
| i     |      |      | 41 | 025  | <b>Rectangular numbers: multiplication</b>                  |
| i     |      |      | 44 | 025  | <b>Al jabr</b>  |
| i     |      |      | 44 | 025  | <b>The mystery of cancellation.</b>                         |
| i     |      |      | 45 | 025  | <b>Slopes: division</b>                                     |
| i     |      |      | 45 | 026  | <b>Approximate arithmetic</b>                               |
| i     |      |      | 46 | 027  | Brocot fractional approximation                             |
| i     |      |      | 47 | 027  | Better than Brocot - <b>continued fractions.</b>            |
| i     |      |      | 48 | 029  | <b>Making fractions clear.</b>                              |
| i     |      |      | 49 | 030  | <b>Coprimes and greatest common divisor.</b>                |
| i     |      |      | 50 | 030  | <b>Modular arithmetic and Euclid's algorithm.</b>           |
| i     |      |      | 51 | 031  | <b>Euclid's algorithm.</b>                                  |
| i     |      |      | 52 | 032  | <b>How many intervals?</b>                                  |
| i     |      |      | 53 | 035  | <b>Fermat and Euler on modular powers.</b>                  |
| i     |      |      | 54 | 036  | <b>Secret codes and encryption.</b>                         |
| i     |      |      | 55 | 038  | <b>Cracking RSA.</b>  |
| i     |      |      | 56 | 039  | <b>Fermat's little theorem again.</b>                       |

| Wk Bk | Part | Note | Ex | Page | Topic                                 |
|-------|------|------|----|------|---------------------------------------|
| ii    |      | 01   |    | 001  | <b>Powers.</b>                        |
| ii    |      | 02   |    | 001  | <b>Trees</b>                          |
| ii    |      | 03   |    | 003  | <b>Inverting trees.</b>               |
| ii    |      | 04   |    | 004  | <b>Properties of powers.</b>          |
| ii    |      | 05   |    | 005  | <b>Interest.</b>                      |
| ii    |      | 06   |    | 006  | <b>Binomial coefficients.</b>         |
| ii    |      | 07   |    | 007  | <b>Factorials.</b>                    |
| ii    |      | 08   |    | 008  | <b>Really big numbers.</b>            |
| ii    |      | 09   |    | 014  | <b>Really small numbers.</b>          |
| ii    |      |      | 01 | 017  | Perfect numbers                       |
| ii    |      |      | 15 | 019  | <b>Golden ratio.</b>                  |
| ii    |      |      | 34 | 023  | <b>Monotonic paths.</b>               |
| ii    |      |      | 38 | 024  | (Zeno's dichotomy paradox.)           |
| ii    |      |      | 40 | 024  | <b>Factorial trees.</b>               |
| ii    |      |      | 41 | 024  | <b>Choose trees.</b>                  |
| ii    |      |      | 42 | 025  | <b>Ways trees.</b>                    |
| ii    |      |      | 47 | 026  | Nine constellations.                  |
| ii    |      |      | 60 | 031  | <b>Hypercubes.</b>                    |
| iii   |      | 01   |    | 001  | <b>Base 10.</b>                       |
| iii   |      | 02   |    | 003  | <b>Base 2.</b>                        |
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| iii   |      | 06   |    | 007  | <b>The Genetic Code.</b>              |
| iii   |      | 07   |    | 008  | <b>Polynomials.</b>                   |
| iii   |      | 08   |    | 009  | <b>Multiplying polynomials.</b>       |
| iii   |      | 09   |    | 010  | <b>Polynomial division.</b>           |
| iii   |      | 10   |    | 012  | <b>Bases and polynomials</b>          |
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| iii   |      |      | 27 | 018  | <b>Legendre polynomials.</b>          |
| iii   |      |      | 27 | 018  | <b>Hermite polynomials.</b>           |
| iii   |      |      | 29 | 018  | <b>Modular arithmetic.</b>            |
| iv    |      | 01   |    | 001  | <b>Matrix multiplication.</b>         |
| iv    |      | 02   |    | 002  | <b>Vectors.</b>                       |
| iv    |      | 03   |    | 003  | <b>Identity matrix.</b>               |
| iv    |      | 04   |    | 003  | <b>Matrix inverse.</b>                |
| iv    |      | 05   |    | 004  | <b>Vectors in space.</b>              |
| iv    |      | 06   |    | 006  | <b>Positions and intervals.</b>       |
| iv    |      | 07   |    | 006  | <b>Transforming space.</b>            |
| iv    |      | 08   |    | 007  | <b>Rotations.</b>                     |
| iv    |      | 09   |    | 008  | <b>Shear.</b>                         |
| iv    |      | 10   |    | 009  | <b>Diagonalizing matrices.</b>        |
| iv    |      |      | 09 | 014  | <b>Cosine and sine.</b>               |
| iv    |      |      | 10 | 017  | <b>2-dimensional numbers</b>          |
| iv    |      |      | 11 | 017  | <b>Three-dimensional rotations.</b>   |
| iv    |      |      | 16 | 019  | Velocities in timespace.              |
| iv    |      |      | 17 | 021  | <b>Reflections, etc.</b>              |
| iv    |      |      | 21 | 023  | <b>Projection and reflection.</b>     |
| iv    |      |      | 22 | 024  | <b>Diagonalizing matrices.</b>        |
| iv    |      |      | 23 | 027  | <b>Array addressing.</b>              |
| iv    |      |      | 24 | 028  | <b>Simplex arrays.</b>                |

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|-------|------|------|----|------|---|
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| v     | 1    | 02   |    | 001  | Picture of $x^2 - x - 1 = 0$  |
| v     | 1    | 03   |    | 002  | <b>Cartesian Plane.</b>   |
| v     | 1    | 04   |    | 004  | <b>Symmetry.</b>  |
| v     | 1    | 05   |    | 004  | <b>Functions.</b>   |
| v     | 1    | 06   |    | 006  | <b>Zeros of a function.</b>   |
| v     | 1    | 07   |    | 006  | <b>Symmetry, again.</b>   |
| v     | 1    | 08   |    | 007  | <b>Slopes.</b>  |
| v     | 1    | 09   |    | 010  | <b>Slopes of curves.</b>  |
| v     | 1    | 10   |    | 012  | <b>Centering.</b>   |
| v     | 1    | 11   |    | 013  | <b>Limits.</b>  |
| v     | 1    | 12   |    | 015  | <b>Back to the zeros of <math>\frac{5}{2}x^2 - \frac{1}{2}x - 2</math>.</b> |
| v     | 1    | 13   |    | 016  | <b>Square roots.</b>  |
| v     | 1    | 14   |    | 018  | <b>Self-slope.</b>  |
| v     | 1    | 15   |    | 020  | <b>Infinite series.</b>   |
| v     | 1    | 16   |    | 021  | <b>Programming the infinite series.</b>                                     |
| v     | 1    | 17   |    | 023  | <b>Slope equations.</b>   |
| v     | 1    | 18   |    | 024  | <b>Ninety-degree rotations.</b>   |
| v     | 1    | 19   |    | 026  | <b>Two-dimensional numbers.</b>   |
| v     | 1    | 20   |    | 027  | <b>Slope of <math>c</math> and <math>s</math>.</b>                          |
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| v     | 2    | 02   |    | 002  | <b>Slope.</b>   |
| v     | 2    | 03   |    | 002  | <b>Approximations.</b>  |
| v     | 2    | 04   |    | 003  | <b>Slope of cubic.</b>  |
| v     | 2    | 05   |    | 003  | <b>The root.</b>  |
| v     | 2    | 06   |    | 004  | <b>Square roots.</b>  |
| v     | 2    | 07   |    | 004  | <b>Antislopes.</b>  |
| v     | 2    | 08   |    | 005  | <b>Areas.</b>   |
| v     | 2    | 09   |    | 008  | <b>Volumes.</b>   |
| v     | 2    | 10   |    | 009  | <b>Antislopes and areas.</b>  |
| v     | 2    | 11   |    | 010  | <b>The Fundamental Theorem of Calculus.</b>                                 |
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| v     | 2    |      | 04 | 013  | <b>Pythagoras.</b>  |
| v     | 2    |      | 11 | 016  | <b>Gini and IGE.</b>  |
| v     | 2    |      | 11 | 018  | <b>Least squares fit.</b>   |
| v     | 2    |      | 11 | 019  | <b>Minimizing.</b>  |
| 01    |      | 01   |    | 001  | One polarizing filter   |
| 01    |      | 02   |    | 002  | Two polarizing filters  |
| 01    |      | 03   |    | 003  | Theory  |
| 01    |      | 04   |    | 004  | Cosine  |
| 01    |      | 05   |    | 005  | A third polarizing filter   |
| 01    |      | 06   |    | 005  | Theory predicts   |
| 01    |      | 07   |    | 007  | Theory predicts wrong?  |
| 01    |      | 08   |    | 007  | Two polarizing filters again  |
| 01    |      | 09   |    | 008  | Testing theory  |
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| 01    |      |      | 01 | 010  | <b>Quantitative thinking is important.</b>            |
| 01    |      |      | 03 | 010  | <b>Measuring angles.</b>                              |
| 01    |      |      | 06 | 011  | (Trig. and triangles)                                 |
| 01    |      |      | 07 | 011  | Eugene's Clock  |
| 01    |      |      | 08 | 012  | <b>Celtic knots and other fun with cos and sin.</b>   |
| 01    |      |      | 14 | 016  | <b>Useful cos and sin.</b>                            |
| 01    |      |      | 15 | 017  | Trigonometry can help us locate the stars.            |
| 01    |      |      | 16 | 021  | <b>Hyperspheres.</b>                                  |
| 01    |      |      | 19 | 022  | <b>Can quantum physics be made more complete?</b>     |
| 01    |      |      | 20 | 024  | <b>Quantum key distribution.</b>                      |
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| 02    |      | 02   |    | 002  | Light beam as vector                                  |
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| 02    |      | 04   |    | 002  | Component at angle                                    |
| 02    |      | 05   |    | 003  | Vector multiplication                                 |
| 02    |      | 06   |    | 005  | Polarizing filter as projection operator              |
| 02    |      | 07   |    | 005  | Three projections                                     |
| 02    |      | 08   |    | 006  | $c^2$ , $s^2$ , $cs$                                  |
| 02    |      | 09   |    | 007  | two rotations, Pythagoras                             |
| 02    |      | 10   |    | 008  | product of functions, sum of arguments                |
| 02    |      |      | 02 | 009  | values for cos, sin                                   |
| 02    |      |      | 16 | 011  | linear operator                                       |
| 02    |      |      | 17 | 012  | ruler and compass                                     |
| 03    |      | 01   |    | 001  | time and distance                                     |
| 03    |      | 02   |    | 001  | speed   |
| 03    |      | 03   |    | 001  | timespace, Galileo, Stan and Trav                     |
| 03    |      | 04   |    | 003  | maximum speed   |
| 03    |      | 05   |    | 004  | modifying Galileo, eigenvalues                        |
| 03    |      | 06   |    | 005  | matrix from fixed directions, eigenvalues             |
| 03    |      | 07   |    | 005  | unit determinant                                      |
| 03    |      | 08   |    | 006  | <b>Digression on determinants.</b>                    |
| 03    |      | 09   |    | 007  | Lorentz transformation                                |
| 03    |      | 10   |    | 009  | <b>Using the Lorentz transformation.</b>              |
| 03    |      | 11   |    | 010  | <b>Physical principles behind special relativity.</b> |
| 03    |      |      | 18 | 012  | time dilation   |
| 03    |      |      | 19 | 013  | <b>Lorentz (<math>c \neq 1</math>).</b>               |
| 03    |      |      | 20 | 013  | <b>The weird fizzle matrices.</b>                     |
| 04    |      | 01   |    | 001  | Matrices as numbers?                                  |
| 04    |      | 02   |    | 001  | One-dimensional numbers and 90-degree rotation        |
| 04    |      | 03   |    | 001  | 90-degree rotation as $\sqrt{-1}$                     |
| 04    |      | 04   |    | 002  | $\cos + \sqrt{-1}\sin$ as rotation                    |
| 04    |      | 05   |    | 002  | $x + \sqrt{-1}y$ as vector                            |
| 04    |      | 06   |    | 002  | $\sqrt{-1}$ as "imagine that!", 2-dimensional numbers |
| 04    |      | 07   |    | 003  | formal properties of numbers                          |
| 04    |      | 08   |    | 003  | Adding 2-numbers                                      |
| 04    |      | 09   |    | 003  | Multiplying 2-numbers                                 |
| 04    |      | 10   |    | 004  | <b>Turtle graphics.</b> Total Turtle Turning          |

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| 04    |      |      | 01 | 006  | De Moivre   |
| 04    |      |      | 03 | 007  | Logo programming language   |
| 04    |      |      | 04 | 007  | Army ant Eciton burchellie bivouacs                                 |
| 04    |      |      | 05 | 007  | Treasure Island   |
| 04    |      |      | 06 | 008  | Interpreting numbers  |
| 04    |      |      | 07 | 008  | Moving parabolas and their shadows                                  |
| 04    |      |      | 09 | 010  | <b>Roots of unity.</b>  |
| 04    |      |      | 10 | 010  | <b>Conjugates.</b>  |
| 04    |      |      | 11 | 010  | shear and 2-numbers   |
| 04    |      |      | 12 | 010  | eigenvalues of rotation   |
| 04    |      |      | 13 | 011  | anticommuting matrices  |
| 04    |      |      | 14 | 011  | Fourier transform inverse   |
| 04    |      |      | 15 | 011  | field axioms  |
| 04    |      |      | 16 | 011  | ruler and compass cannot trisect angles                             |
| 04    |      |      | 18 | 011  | <b>Field trip</b> modulo arithmetic, solving by radicals            |
| 04    |      |      | 21 | 014  | ring axioms   |
| 04    |      |      | 22 | 014  | <b>“Symmetric polynomials”.</b>                                     |
| 05    |      | 01   |    | 001  | Amplitudes as 2-numbers   |
| 05    |      | 02   |    | 001  | <b>Photons</b>  |
| 05    |      | 03   |    | 002  | Feynman’s rules   |
| 05    |      | 04   |    | 002  | Probabilities for Young’s double slit                               |
| 05    |      | 05   |    | 003  | Wavelength  |
| 05    |      | 06   |    | 004  | Trigonometry for double slit  |
| 05    |      | 07   |    | 005  | Light travels in straight lines                                     |
| 05    |      | 08   |    | 007  | Two ways to collide alpha-particles                                 |
| 05    |      | 09   |    | 008  | Two ways to collide electrons - new Feynman rule - bosons, fermions |
| 06    |      | 01   |    | 001  | <b>States.</b>  |
| 06    |      | 01   |    | 001  | Two polarization states of light                                    |
| 06    |      | 02   |    | 001  | Two polarization states of electrons                                |
| 06    |      | 03   |    | 002  | Euler angles, transformation matrix                                 |
| 06    |      | 04   |    | 003  | Half angles   |
| 06    |      | 05   |    | 003  | Pauli matrices  |
| 06    |      | 06   |    | 003  | Special Euler angles  |
| 06    |      | 07   |    | 004  | Transforming amplitudes   |
| 06    |      | 08   |    | 004  | Two q-bits  |
| 06    |      | 09   |    | 004  | Symmetric and antisymmetric states                                  |
| 06    |      | 10   |    | 005  | Spin-1 and polarized light  |
| 06    |      | 11   |    | 005  | Circular to linear polarization                                     |
| 06    |      | 12   |    | 006  | Linear polarization rotates in two dimensions                       |
| 06    |      | 13   |    | 006  | Lightspeed eliminates a third dimension                             |
| 06    |      | 14   |    | 007  | <b>Fermions and Bosons.</b>   |
| 06    |      |      | 11 | 008  | <b>Spin 3/2.</b>  |
| 06    |      |      | 12 | 010  | <b>Spin 2.</b>  |
| 06    |      |      | 15 | 010  | The periodic table  |

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|-------|------|------|----|------|---|
| 07a   |      | 01   |    | 001  | <b>Overview.</b>  |
| 07a   |      | 02   |    | 001  | Phase of particle $e^{-i\omega t}$                                    |
| 07a   |      | 03   |    | 002  | Phase is independent of observer                                      |
| 07a   |      | 04   |    | 002  | Energy, momentum and frequency, wavenumber - Planck's constant        |
| 07a   |      | 05   |    | 003  | <b>Anti-Pythagoras</b>  |
| 07a   |      | 06   |    | 003  | Interval is invariant   |
| 07a   |      | 07   |    | 004  | <b>Dimensional analysis and pre-timespace limits.</b>                 |
| 07a   |      | 08   |    | 004  | <b>The pre-timespace limit for freqnum.</b>                           |
| 07a   |      | 09   |    | 005  | Gives the transformation of energentum, $E^2 - c^2 p^2 = m^2 c^4$     |
| 07a   |      | 10   |    | 005  | Energentum is conserved.  |
| 07a   |      | 11   |    | 006  | <b>Inelastic collisions</b>   |
| 07a   |      | 12   |    | 006  | Centre-of-mass reference frame  |
| 07a   |      | 13   |    | 007  | <b>Fusion</b>   |
| 07a   |      | 14   |    | 007  | <b>Fission</b>  |
| 07a   |      | 15   |    | 008  | <b>Elastic collision</b>  |
| 07a   |      | 16   |    | 009  | <b>Equations of relativistic quantum mechanics</b>                    |
| 07a   |      |      | 01 | 010  | <b>Waves</b>  |
| 07a   |      |      | 02 | 010  | <b>Visualizing waves</b>  |
| 07a   |      |      | 11 | 012  | <b>Spin statistics, etc.</b>  |
| 07a   |      |      | 16 | 013  | Energy bands in crystals  |
| 07b   |      | 01   |    | 001  | Strong Church-Turing thesis   |
| 07b   |      | 02   |    | 001  | Switch-light question needs 1 function call                           |
| 07b   |      | 03   |    | 001  | Logic circuits and gates, not, Hadamard                               |
| 07b   |      | 04   |    | 003  | Classical circuit for switch-light                                    |
| 07b   |      | 05   |    | 003  | Quantum circuit for switch-light                                      |
| 07b   |      | 06   |    | 004  | State space and entanglement  |
| 07b   |      | 07   |    | 005  | <b>Developments after Deutsch's seminal 1985 paper</b>                |
| 07b   |      | 08   |    | 005  | <b>Physical construction of quantum computers</b>                     |
| 07c   |      | 01   |    | 001  | <b>Vectors are real.</b>  |
| 07c   |      | 02   |    | 002  | Some pairs are not vectors  |
| 07c   |      | 03   |    | 003  | Height, width as eigenvalues not coordinates                          |
| 07c   |      | 04   |    | 004  | Is twirl a tensor?  |
| 07c   |      | 05   |    | 005  | Twirl and area are "pseudovectors" or "axial vectors"                 |
| 07c   |      | 06   |    | 006  | <b>Vectors and Areas and .. All Together</b>                          |
| 07c   |      | 07   |    | 007  | <b>Rotation</b>   |
| 07c   |      | 08   |    | 008  | <b>Reflection</b>   |
| 07c   |      | 09   |    | 009  | <b>3D rotations</b>   |
| 07c   |      | 10   |    | 010  | <b>Intervals plus locations.</b>                                      |
| 07c   |      | 11   |    | 010  | <b>Interval algebra in 3D.</b>  |
| 07c   |      | 13   |    | 013  | <b>Appendix; Summary of vector and matrix operations</b>              |
| 07c   |      |      | 01 | 014  | <b>Dot product.</b>   |
| 07c   |      |      | 16 | 015  | <b>Tetrahedron.</b>   |
| 07c   |      |      | 20 | 017  | Five Platonic solids  |
| 07c   |      |      | 24 | 017  | Euler's formula, edges, faces, vertices                               |
| 07c   |      |      | 26 | 018  | Relational composition  |
| 07c   |      |      | 30 | 019  | <b>Direction cosines.</b>   |
| 07c   |      |      | 31 | 020  | Gibbs cross product   |
| 07c   |      |      | 32 | 020  | <b>Nonorthogonal axes and tensor notation.</b>                        |
| 07c   |      |      | 33 | 024  | <b>Tensor Calculator I. Matrix representation.</b>                    |
| 07c   |      |      | 34 | 026  | <b>Tensor Calculator I. Relational representation.</b>                |
| 07c   |      |      | 37 | 031  | <b>Factoring sums and differences of squares by Clifford algebra.</b> |
| 07c   |      |      | 38 | 031  | <b>Matrix representations of Clifford "numbers".</b>                  |
| 07c   |      |      | 39 | 033  | <b>Matrix Mechanics.</b>  |
| 07c   |      |      | 40 | 038  | <b>Spin and Clifford Algebra.</b>                                     |
| 07c   |      |      | 41 | 045  | <b>Creation and Annihilation.</b>                                     |

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|-------|------|------|----|------|---|
| 08    |      | 01   |    | 001  | Importance of a web page - cycles   |
| 08    |      | 02   |    | 002  | Importance of a web page - no cycles  |
| 08    |      | 03   |    | 003  | Add equal columns of small numbers as source vector                             |
| 08    |      | 04   |    | 004  | Linear equations - <b>Elementary row operations</b>                             |
| 08    |      | 02   |    | 005  | <b>Gauss elimination</b>  |
| 08    |      | 06   |    | 006  | <b>3D for 2D graphics</b> - tetrahedron to triangle to collinear                |
| 08    |      | 07   |    | 007  | Sketchpad example 1 (linear constraint)   |
| 08    |      | 08   |    | 008  | Sketchpad example 2 (nonlinear constraint)                                      |
| 08    |      | 09   |    | 010  | <b>Iterate!</b>   |
| 08    |      | 10   |    | 011  | <b>Underdetermined equations</b> - Sketchpad example 3 (point collinear with li |
| 08    |      | 11   |    | 012  | Minimize subject to constraint  |
| 08    |      | 12   |    | 013  | <b>Overdetermined equations</b>   |
| 08    |      | 13   |    | 014  | <b>Overdetermined equations</b>   |
| 08    |      |      | 10 | 017  | Factor analysis/Principal components analysis in the measurement of personal    |
| 08    |      |      | 14 | 021  | <b>Null spaces and stoichiometry.</b>   |
| 08    |      |      | 15 | 024  | <b>Quadratic forms and stability.</b>   |
| 08    |      |      | 16 | 025  | Binomial coefficients   |
| 08    |      |      | 28 | 027  | <b>Slopes of cos and sin.</b>   |
| 08    |      |      | 38 | 032  | Surveying   |
| 08c   | 1    | 01   |    | 001  | Triangle symmetry   |
| 08c   | 1    | 02   |    | 002  | Generating reflections  |
| 08c   | 1    | 03   |    | 002  | Permutation notation  |
| 08c   | 1    | 04   |    | 003  | Group properties  |
| 08c   | 1    | 05   |    | 003  | Reflecting rotations, rotating reflections                                      |
| 08c   | 1    | 06   |    | 004  | Invariant sets  |
| 08c   | 1    | 07   |    | 005  | Invariant (normal) subgroups  |
| 08c   | 1    | 08   |    | 005  | Matrix invariant sets share eigenvalues, determinant, trace                     |
| 08c   | 1    | 09   |    | 006  | Other representations - trivial, direct, permutation                            |
| 08c   | 1    | 10   |    | 007  | Using traces to block-diagonalize a whole matrix representation                 |
| 08c   | 1    | 11   |    | 008  | Regular representation  |
| 08c   | 1    | 12   |    | 010  | <b>Molecules 1D</b>   |
| 08c   | 1    | 13   |    | 012  | Triangle molecule   |
| 08c   | 1    | 14   |    | 017  | <b>Greenhouse gases CO<sub>2</sub></b>  |
| 08c   | 1    | 15   |    | 018  | <b>Greenhouse gases H<sub>2</sub>O</b>  |
| 08c   | 1    | 16   |    | 019  | <b>Tetrahedron.</b>   |
| 08c   | 1    | 17   |    | 024  | <b>Hexa/Octahedra.</b>  |
| 08c   | 1    | 18   |    | 025  | <b>Dodeca/Icosahedra.</b>   |
| 08c   | 1    |      | 06 | 028  | Car wheels  |
| 08c   | 1    |      | 37 | 031  | <b>Spring constants.</b>  |
| 08c   | 1    |      | 53 | 034  | <b>Symmetric field trip - Galois and the quintic</b>                            |
| 08c   | 2    | 19   |    | 002  | <b>Infinite groups</b>  |
| 08c   | 2    | 20   |    | 003  | <b>1D crystals, gratings, CDs</b>   |
| 08c   | 2    | 21   |    | 006  | <b>2D crystals</b>  |
| 08c   | 2    | 22   |    | 011  | <b>2D waves</b>   |
| 08c   | 2    | 22   |    | 012  | <b>Periodic boundary conditions</b>   |
| 08c   | 2    | 23   |    | 015  | <b>Brillouin zone.</b>  |
| 08c   | 2    | 24   |    | 016  | <b>Non-translational crystal symmetries.</b>                                    |
| 08c   | 2    | 25   |    | 016  | <b>Wallpaper groups.</b>  |
| 08c   | 2    |      | 03 | 022  | poetic symmetry   |
| 08c   | 2    |      | 04 | 023  | CDs and DVDs  |
| 08c   | 2    |      | 13 | 023  | <b>Dot product.</b>   |
| 08c   | 2    |      | 23 | 026  | <b>Wallpaper groups.</b>  |
| 08c   | 2    |      | 38 | 028  | <b>Periodic interactions, vernier, moiré</b>                                    |



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| 08c   | 3    | 26   |    | 002  | <b>Continuous groups.</b>                                  |
| 08c   | 3    | 27   |    | 004  | <b>Spherical symmetry.</b>                                 |
| 08c   | 3    | 28   |    | 006  | <b>Commutator algebra.</b>                                 |
| 08c   | 3    | 29   |    | 007  | <b>Representations of the spherical group.</b>             |
| 08c   | 3    | 30   |    | 013  | <b>Spherical harmonics, slope rules</b>                    |
| 08c   | 3    | 31   |    | 020  | <b>Atomic physics.</b>                                     |
| 08c   | 3    |      | 06 | 027  | Angular momentum   |
| 08c   | 3    |      | 09 | 027  | <b>Kepler II.</b>  |
| 08c   | 3    |      | 12 | 028  | Half-Pauli matrices  |
| 08c   | 3    |      | 16 | 028  | Commutator algebra, Lie groups                             |
| 08c   | 3    |      | 22 | 029  | Slope rules, product rule, chain rule                      |
| 08c   | 3    |      | 35 | 030  | <b>Bohr atom.</b>  |
| 08c   | 3    |      | 36 | 031  | <b>Kepler III</b>  |
| 08c   | 3    |      | 37 | 032  | <b>Conic sections.</b>                                     |
| 08c   | 3    |      | 38 | 034  | <b>Kepler I.</b>   |
| 08c   | 4    | 32   |    | 002  | <b>SU(2) formal and informal.</b>                          |
| 08c   | 4    | 33   |    | 006  | <b>SU(3).</b>  |
| 08c   | 4    | 34   |    | 013  | <b>Isospin and quarks</b>                                  |
| 08c   | 4    | 35   |    | 016  | <b>Symmetry and Conservation; Complementary Quantities</b> |
| 08c   | 4    | 36   |    | 017  | <b>Symmetry and Conservation; Energy</b>                   |
| 08c   | 4    | 37   |    | 020  | <b>Principle of Stationary Action.</b>                     |
| 08c   | 4    | 38   |    | 028  | <b>Symmetry and Conservation; Noether's Theorem</b>        |
| 08c   | 4    | 39   |    | 030  | <b>The Hamiltonian and Schrödinger's Equation</b>          |
| 08c   | 4    |      | 19 | 037  | <b>Billiards.</b>  |
| 08c   | 4    |      | 20 | 040  | <b>Molecules</b>   |
| 08c   | 4    |      | 21 | 042  | <b>Forces and deformations.</b>                            |
| 08c   | 4    |      | 22 | 047  | <b>Spring oscillators.</b>                                 |
| 08c   | 4    |      | 25 | 049  | Potential energy   |
| 08c   | 4    |      | 37 | 050  | <b>Legendre transformation.</b>                            |
| 08c   | 4    |      | 41 | 052  | Dirac equation   |
| 09    |      | 01   |    | 001  | 2D numbers give 5D vector space                            |
| 09    |      | 02   |    | 002  | Fourier transform, FT                                      |
| 09    |      | 03   |    | 004  | Approximating function using important frequencies only    |
| 09    |      | 04   |    | 005  | Joint Photographic Experts Group, JPEG                     |
| 09    |      | 05   |    | 007  | <b>The fast Fourier transform, FFT</b>                     |
| 09    |      | 06   |    | 009  | <b>Divide and Conquer</b>                                  |
| 09    |      | 07   |    | 010  | <b>The Uncertainty Principle</b>                           |
| 09    |      | 08   |    | 012  | <b>Compression and Content</b>                             |
| 09    |      |      | 14 | 014  | <b>FFT again.</b>  |
| 09    |      |      | 17 | 015  | <b>Modified inverse FT.</b>                                |
| 09    |      |      | 18 | 017  | <b>FT with vector <math>k, l</math>.</b>                   |
| 09    |      |      | 19 | 019  | <b>Continuous FT.</b>                                      |
| 09    |      |      | 20 | 020  | <b>Artefacts of sampling.</b>                              |
| 09    |      |      | 23 | 022  | <b>Two-dimensional discrete Fourier transform</b>          |
| 09    |      |      | 24 | 022  | <b>Digital signal processor, DSP filters.</b>              |
| 09    |      |      | 25 | 033  | Tachyons faster than light                                 |

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| 09c   | 1    | 01   |    | 001  | <b>Histograms.</b>   |
| 09c   | 1    | 02   |    | 002  | <b>Histogram arithmetic.</b>   |
| 09c   | 1    | 03   |    | 005  | <b>Distributions and densities.</b>                                  |
| 09c   | 1    | 04   |    | 007  | <b>Aggregates; the moments of distributions.</b>                     |
| 09c   | 1    | 05   |    | 011  | <b>Quantum distributions: the density matrix.</b>                    |
| 09c   | 1    | 06   |    | 014  | <b>The normal distribution.</b>                                      |
| 09c   | 1    | 07   |    | 021  | <b>Expectation, surprise and ignorance.</b>                          |
| 09c   | 1    | 08   |    | 024  | <b>Does ignorance ever decrease?</b>                                 |
| 09c   | 1    | 09   |    | 028  | <b>Inside knowledge; the clients of Joe and Sue revisited.</b>       |
| 09c   | 1    | 10   |    | 031  | <b>Correlation and co-ignorance.</b>                                 |
| 09c   | 1    | 11   |    | 034  | <b>Conditional distributions and ignorance.</b>                      |
| 09c   | 1    |      | 07 | 042  | Program to add histograms  |
| 09c   | 1    |      | 13 | 043  | <b>Mean, median, mode.</b>   |
| 09c   | 1    |      | 14 | 044  | <b>Geometric and harmonic means.</b>                                 |
| 09c   | 1    |      | 22 | 045  | <b>Notation, integrals and antislopes</b>                            |
| 09c   | 1    |      | 24 | 046  | <b>Legendre and Hermite polynomials.</b>                             |
| 09c   | 1    |      | 34 | 051  | <b>Increasing ignorance.</b>   |
| 09c   | 1    |      | 38 | 053  | <b>Newton's method 1.</b>  |
| 09c   | 1    |      | 39 | 055  | <b>Newton's method 2.</b>  |
| 09c   | 1    |      | 40 | 057  | <b>Continuous distributions.</b>                                     |
| 09c   | 1    |      | 41 | 058  | <b>More on continuous distributions.</b>                             |
| 09c   | 1    |      | 42 | 059  | <b>Ignorance and continuous distributions.</b>                       |
| 09c   | 1    |      | 43 | 061  | <b>Continuous distributions by antislope: average distance.</b>      |
| 09c   | 1    |      | 44 | 064  | <b>Hyperspheres.</b>   |
| 09c   | 1    |      | 46 | 068  | <b>Geometrical interpretations of constant sums.</b>                 |
| 09c   | 1    |      | 47 | 069  | <b>Simplex coordinates.</b>  |
| 09c   | 1    |      | 48 | 069  | <b>When is a distribution the product of its marginals?</b>          |
| 09c   | 1    |      | 49 | 070  | <b>Mutual information from Chesapeake clams.</b>                     |
| 09c   | 1    |      | 51 | 072  | <b>Ascendency.</b>   |
| 09c   | 1    |      | 52 | 074  | <b>A playpenful of baby ecologies.</b>                               |
| 09c   | 1    |      | 53 | 077  | <b>Maximizing ascendency.</b>  |
| 09c   | 1    |      | 59 | 078  | <b>Black boxes.</b>  |
| 09c   | 1    |      | 61 | 080  | <b>Thongs.</b>   |
| 09c   | 1    |      | 62 | 081  | <b>Order of a symbol system.</b>                                     |
| 09c   | 1    |      | 63 | 082  | <b>Marilyn vos Savant on the Monty Hall problem.</b>                 |
| 09c   | 1    |      | 64 | 083  | <b>Decision trees.</b>   |
| 09c   | 2    | 12   |    | 001  | <b>A gas simulation 1; the collisions</b>                            |
| 09c   | 2    | 13   |    | 011  | <b>A gas simulation 2; statistics.</b>                               |
| 09c   | 2    | 14   |    | 016  | <b>The Boltzmann and Maxwell distributions.</b>                      |
| 09c   | 2    | 15   |    | 021  | <b>Fluctuations, variations and samples.</b>                         |
| 09c   | 2    | 16   |    | 026  | <b>Entropy.</b>  |
| 09c   | 2    | 17   |    | 029  | <b>Temperature.</b>  |
| 09c   | 2    | 18   |    | 035  | <b>Pressure.</b>   |
| 09c   | 2    | 19   |    | 039  | <b>State function for monatomic gases.</b>                           |
| 09c   | 2    | 20   |    | 044  | <b>Thermostatic equations of state.</b>                              |
| 09c   | 2    | 21   |    | 048  | <b>More on multivariate slopes.</b>                                  |
| 09c   | 2    | 22   |    | 052  | <b>Work and heat.</b>  |
| 09c   | 2    | 22   |    | 057  | <b>Carnot's reversible heat engine, second law of thermodynamics</b> |
| 09c   | 2    | 22   |    | 061  | <b>Emergence.</b>  |

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| 09c   | 2    |      | 10 | 064  | <b>First simulation.</b>  |
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| 09c   | 2    |      | 12 | 066  | <b>Dominance.</b> (A kinetic theory of conflict, 2.)                              |
| 09c   | 2    |      | 13 | 069  | <b>Hierarchies.</b>   |
| 09c   | 2    |      | 14 | 071  | <b>Cycles.</b>  |
| 09c   | 2    |      | 19 | 074  | <b>Maxwell distribution.</b>  |
| 09c   | 2    |      | 29 | 075  | <b>Sackur-Tetrode equation.</b>   |
| 09c   | 2    |      | 32 | 076  | <b>Local inverses of functions.</b>   |
| 09c   | 2    |      | 33 | 077  | <b>Complementary variables.</b>   |
| 09c   | 2    |      | 45 | 078  | <b>Causality.</b>   |
| 09c   | 2    |      | 46 | 079  | Emergence   |
| 09c   | 3    | 23   |    | 002  | <b>Correlation.</b>   |
| 09c   | 3    | 24   |    | 012  | <b>Collision theory.</b>  |
| 09c   | 3    | 25   |    | 016  | <b>Mobility, diffusivity and Brownian motion.</b>                                 |
| 09c   | 3    | 26   |    | 019  | <b>Three potentials and dissipation.</b>  |
| 09c   | 3    | 27   |    | 022  | <b>Active transport and biochemistry.</b>   |
| 09c   | 3    | 28   |    | 024  | <b>Combined transport.</b> Onsager relations.                                     |
| 09c   | 3    |      | 05 | 027  | <b>Least squares filter.</b>  |
| 09c   | 3    |      | 19 | 029  | <b>Kirchhoff circuits.</b>  |
| 09c   | 3    |      | 20 | 030  | <b>Resistors in series and parallel.</b>  |
| 09c   | 3    |      | 21 | 031  | <b>Kirchhoff lattices; relations and operators.</b>                               |
| 09c   | 3    |      | 24 | 037  | <b>Viscosity and heat conduction.</b>   |
| 09c   | 3    |      | 25 | 041  | <b>“Ohm’s law” for viscosity.</b>   |
| 09c   | 3    |      | 26 | 041  | <b>Elasticity and viscosity: stress, strain and flow tensors.</b> Poisson’s ratio |
| 09c   | 3    |      | 26 | 044  | <b>Hookean solid.</b>   |
| 09c   | 3    |      | 26 | 046  | <b>Newtonian liquid.</b>  |
| 09c   | 3    |      | 26 | 047  | <b>Principal axes.</b>  |
| 09c   | 3    |      | 27 | 048  | <b>Lattice statics and dynamics.</b>  |
| 09c   | 3    |      | 30 | 056  | <b>Chemical kinetics and differential equations.</b>                              |
| 09c   | 3    |      | 30 | 062  | <b>Coupled linear first-order differential equations.</b>                         |
| 09c   | 3    |      | 31 | 075  | <b>Chemistry in the language of physics.</b>                                      |
| 09c   | 4    | 29   |    | 002  | <b>Phase transitions.</b>   |
| 09c   | 4    | 30   |    | 003  | <b>Phase transitions in random graphs.</b>  |
| 09c   | 4    | 31   |    | 009  | <b>Point-to-point resistance in a network.</b>                                    |
| 09c   | 4    | 32   |    | 012  | <b>Van der Waals.</b>   |
| 09c   | 4    | 33   |    | 016  | <b>Sublimation.</b>   |
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| 09c   | 4    | 35   |    | 020  | <b>Particle individuality and Bose-Einstein condensation.</b>                     |
| 09c   | 4    |      | 06 | 027  | <b>Percolation.</b>   |
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| 09c   | 4    |      | 22 | 029  | <b>Gibbs’ paradox.</b>  |
| 09c   | 4    |      | 23 | 030  | <b>Ovations.</b>  |
| 09c   | 4    |      | 24 | 033  | social phase transitions  |
| 09c   | 4    |      | 25 | 033  | <b>Power laws.</b>  |
| 09c   | 4    |      | 26 | 033  | <b>Self-Organized Criticality.</b>  |
| 09c   | 4    |      | 27 | 035  | Ising model.  |
| 10    |      | 01   |    | 001  | Axioms of an algebra  |
| 10    |      | 02   |    | 002  | Boolean algebra of two elements   |
| 10    |      | 03   |    | 003  | Implication operator  |
| 10    |      | 04   |    | 004  | Sixteen binary operators  |
| 10    |      | 05   |    | 005  | Switching circuits; halfadder, fulladder  |
| 10    |      | 06   |    | 007  | <b>Reversibility</b> controlled-not, controlled-exchange                          |
| 10    |      | 07   |    | 010  | Operators as matrices   |
| 10    |      | 09   |    | 011  | <b>Appendix Binary Arithmetic</b>   |

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| 10    |      |      | 26 | 017  | Analogic reasoning   |
| 10    |      |      | 34 | 019  | Sets and probability   |
| 10    |      |      | 36 | 036  | Birthday paradox   |
| 10    |      |      | 37 | 020  | Genetic overlap  |
| 10    |      |      | 52 | 021  | <b>Matrix logic.</b>   |
| 10    |      |      | 53 | 024  | <b>xor</b> and <b>and</b> .  |
| 10    |      |      | 54 | 026  | <b>C<sup>k</sup>N circuits.</b>                                      |
| 10    |      |      | 55 | 026  | <b>Basic matrices for logic.</b>                                     |
| 10    |      |      | 60 | 028  | <b>Euclid's Elements.</b>  |
| 11    |      | 01   |    | 001  | <b>Memory</b> flipflop, feedback automata                            |
| 11    |      | 02   |    | 005  | <b>Control</b> thermostat  |
| 11    |      | 03   |    | 006  | Electric blanket control   |
| 11    |      | 04   |    | 008  | <b>Biology</b> the lac operon of E.coli, gene expression, attractors |
| 11    |      | 05   |    | 009  | <b>Language</b> parsing by finite automaton                          |
| 11    |      | 06   |    | 011  | evaluation by actions  |
| 11    |      | 07   |    | 013  | nested parentheses and pushdown automaton                            |
| 11    |      | 08   |    | 014  | evaluation by reverse Polish; shunting                               |
| 11    |      | 09   |    | 016  | automata hierarchy   |
| 11    |      |      | 22 | 018  | attractors   |
| 11    |      |      | 26 | 019  | attractors   |
| 11    |      |      | 29 | 019  | tipping point  |
| 11    |      |      | 30 | 019  | gene expression, microarrays   |
| 11    |      |      | 35 | 021  | rich and poor  |
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| 11c   | 1    | 01   |    | 001  | <b>Fields and slopes.</b>  |
| 11c   | 1    | 02   |    | 002  | <b>"Reality" and coordinates.</b>                                    |
| 11c   | 1    | 03   |    | 005  | <b>Index notation and tensors.</b>                                   |
| 11c   | 1    | 04   |    | 008  | <b>Protors.</b>  |
| 11c   | 1    | 05   |    | 011  | <b>The protor calculator 1.</b>                                      |
| 11c   | 1    | 06   |    | 013  | <b>Divergence and Curl.</b>  |
| 11c   | 1    | 07   |    | 018  | <b>Classical gravity.</b>  |
| 11c   | 1    | 08   |    | 023  | <b>Gypsum coordinates.</b>   |
| 11c   | 1    | 09   |    | 029  | <b>The metric.</b>   |
| 11c   | 1    | 10   |    | 031  | <b>Fields in gypsum.</b>   |
| 11c   | 1    | 11   |    | 035  | <b>Polar coordinates.</b>  |
| 11c   | 1    | 12   |    | 037  | <b>The affine connection.</b>  |
| 11c   | 1    | 13   |    | 041  | <b>Parallel transport and geodesics.</b>                             |
| 11c   | 1    | 14   |    | 043  | <b>Absolute slopes.</b>  |
| 11c   | 1    | 15   |    | 046  | <b>Gradient, divergence and curl with absolute slope.</b>            |
| 11c   | 1    | 16   |    | 049  | <b>Spherical polar coordinates.</b>                                  |
| 11c   | 1    | 17   |    | 053  | <b>Curvature.</b>  |
| 11c   | 1    | 18   |    | 061  | <b>Negative curvature.</b>   |
| 11c   | 1    | 19   |    | 065  | <b>The Ricci tensor.</b>   |
| 11c   | 1    | 20   |    | 067  | <b>More protor calculator.</b>                                       |

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| 11c   | 1    |      | 01 | 075  | <b>Slopes commute?</b>   |
| 11c   | 1    |      | 13 | 077  | <b>Newton's constant.</b>  |
| 11c   | 1    |      | 14 | 077  | <b>Gauss' law for volume and surface integrals</b>                               |
| 11c   | 1    |      | 15 | 077  | <b>Message through the Earth.</b>  |
| 11c   | 1    |      | 16 | 079  | <b>Falling traffic lights</b>  |
| 11c   | 1    |      | 23 | 081  | <b>Levi-Civita symbol, alternating tensors.</b>                                  |
| 11c   | 1    |      | 24 | 083  | <b>Physical tensors.</b>   |
| 11c   | 1    |      | 25 | 084  | <b>Orthonormal polars.</b>   |
| 11c   | 1    |      | 36 | 087  | <b>Hotplate geometry.</b>  |
| 11c   | 1    |      | 37 | 090  | <b>Absolute slope and curvature.</b>   |
| 11c   | 1    |      | 38 | 090  | <b>Bianchi identities.</b>   |
| 11c   | 1    |      | 39 | 091  | <b>Divergence-free curvature.</b>  |
| 11c   | 1    |      | 40 | 092  | <b>Number of components of the curvature tensor.</b>                             |
| 11c   | 1    |      | 41 | 092  | <b>Angular excess at fixed latitude.</b>   |
| 11c   | 1    |      | 42 | 096  | <b>Local flatness.</b>   |
| 11c   | 1    |      | 43 | 099  | <b>The boundary of boundary is zero.</b>   |
| 11c   | 1    |      | 46 | 100  | <b>Curvature of lines.</b>   |
| 11c   | 1    |      | 49 | 104  | <b>What functions for ch, sh?</b>  |
| 11c   | 1    |      | 51 | 106  | <b>Hyperboloid.</b>  |
| 11c   | 1    |      | 52 | 107  | <b>Paraboloid.</b>   |
| 11c   | 2    | 22   |    | 002  | <b>Curved timespace.</b>   |
| 11c   | 2    | 23   |    | 005  | <b>Gravitational redshift.</b>   |
| 11c   | 2    | 24   |    | 009  | <b>Spherically symmetric gravity.</b>  |
| 11c   | 2    | 25   |    | 011  | <b>Schwarzschild orbits and black holes.</b>                                     |
| 11c   | 2    | 26   |    | 016  | <b>Rotationally symmetric gravity.</b>   |
| 11c   | 2    | 27   |    | 019  | <b>Kerr orbits and black holes.</b>  |
| 11c   | 2    | 28   |    | 026  | <b>Tides.</b>  |
| 11c   | 2    | 29   |    | 028  | <b>Light orbits.</b>   |
| 11c   | 2    | 30   |    | 030  | <b>The source of gravity.</b>  |
| 11c   | 2    | 31   |    | 033  | <b>Cosmology.</b>  |
| 11c   | 2    | 32   |    | 037  | <b>Negative pressure and dark energy.</b>  |
| 11c   | 2    | 33   |    | 039  | <b>Gravitational irreversibility.</b>  |
| 11c   | 2    | 34   |    | 043  | <b>Alternatives to geometry.</b>   |
| 11c   | 2    |      | 02 | 048  | <b>Schwarzschild metric.</b>   |
| 11c   | 2    |      | 09 | 049  | <b>The ergosphere.</b>   |
| 11c   | 2    |      | 24 | 052  | <b>Newtonian limit.</b>  |
| 11c   | 2    |      | 25 | 052  | <b>Inflation.</b>  |
| 11c   | 2    |      | 28 | 053  | <b>Negative energy; warp drives.</b>   |
| 11c   | 2    |      | 29 | 056  | <b>Negative mass.</b>  |
| 11c   | 2    |      | 30 | 056  | <b>Blandford-Znajek mechanism.</b>   |
| 11c   | 2    |      | 32 | 056  | <b>Gravitational waves.</b>  |
| 11d   | 1    | 01   |    | 001  | <b>Central Forces.</b>   |
| 11d   | 1    | 02   |    | 004  | <b>Gravity vs. Electricity.</b>  |
| 11d   | 1    | 03   |    | 004  | <b>Energy and momentum scales.</b>   |
| 11d   | 1    | 04   |    | 008  | <b>Divergence, gradient and <math>\vec{\text{div}} \vec{\text{grad}}</math>.</b> |
| 11d   | 1    | 05   |    | 009  | <b>Electrodynamics departs from gravitation.</b>                                 |
| 11d   | 1    | 06   |    | 013  | <b>Invariants, cross-products and convention.</b>                                |
| 11d   | 1    | 07   |    | 019  | <b>Electromagnetic waves.</b>  |
| 11d   | 1    |      | 04 | 022  | <b>Gauss' law for fields from charge.</b>  |
| 11d   | 1    |      | 09 | 022  | <b>Visualizing magnetic fields.</b>  |

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| 11d   | 2    | 08   |    | 001  | <b>Partial Slope Equations; Laplace's Equation.</b>           |
| 11d   | 2    | 09   |    | 007  | <b>The Wave Equation.</b>                                     |
| 11d   | 2    | 10   |    | 010  | <b>The Schrödinger Equation I; Physics.</b>                   |
| 11d   | 2    | 11   |    | 013  | <b>The Schrödinger Equation II; Animating in 1D.</b>          |
| 11d   | 2    | 12   |    | 019  | <b>The Schrödinger Equation III; Animating in 2D.</b>         |
| 11d   | 2    |      | 04 | 024  | <b>Stability</b> of numerical solutions of the wave equation. |
| 11d   | 2    |      | 05 | 025  | <b>The Hamiltonian.</b>                                       |
| 11d   | 2    |      | 07 | 026  | <b>Dispersion relations.</b>                                  |
| 11d   | 3    | 13   |    | 001  | <b>The electromagnetic Schrödinger equation.</b>              |
| 11d   | 3    | 14   |    | 003  | <b>Simulating a charged wavepacket moving near a current.</b> |
| 11d   | 3    | 15   |    | 006  | <b>Links with geometry.</b>                                   |
| 11d   | 3    | 16   |    | 008  | <b>Local action versus action-at-a-distance.</b>              |
| 11d   | 3    | 17   |    | 009  | <b>Other symmetries, other forces.</b>                        |
| 11d   | 3    |      | 01 | 012  | <b>Aharonov-Bohm effect.</b>                                  |
| 11d   | 3    |      | 03 | 013  | <b>Maxwell's tensor and equations.</b>                        |
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| 11d   | 3    |      | 06 | 014  | <b>Abelian and non-Abelian symmetries.</b>                    |
| 11d   | 3    |      | 07 | 016  | <b>Goldstone and Higgs mechanisms.</b>                        |
| 11d   | 3    |      | 08 | 020  | <b>Quarks and vacuum.</b>                                     |
| 11d   | 4    | 18   |    | 001  | <b>Introduction to Quantum Fields.</b>                        |
| 11d   | 4    | 19   |    | 008  | <b>Small matrices.</b>  |
| 11d   | 4    | 20   |    | 011  | <b>Tensor products.</b>                                       |
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| 11d   | 4    | 22   |    | 018  | <b>Vectors and spinors.</b>                                   |
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| 11d   | 4    | 24   |    | 024  | <b>A simple field.</b>  |
| 11d   | 4    | 25   |    | 028  | <b>The Yukawa potential.</b>                                  |
| 11d   | 4    | 26   |    | 031  | <b>Perturbation approximations.</b>                           |
| 11d   | 4    | 27   |    | 033  | <b>Fermions.</b>  |
| 11d   | 4    | 28   |    | 035  | <b>Slopes and antislopes of 2D numbers, etc.</b>              |
| 11d   | 4    | 29   |    | 042  | <b>Charge conservation and antimatter.</b>                    |
| 11d   | 4    | 30   |    | 045  | <b>Relativistic quantum field theory redux, so far.</b>       |
| 11d   | 4    |      | 02 | 046  | <b>Heisenberg uncertainty.</b>                                |
| 11d   | 4    |      | 03 | 046  | <b>More uncertainty.</b>                                      |
| 11d   | 4    |      | 06 | 048  | <b>Sphere of reflections.</b>                                 |
| 11d   | 4    |      | 07 | 049  | <b>Density matrices and inverse tensor product.</b>           |
| 11d   | 4    |      | 08 | 049  | <b>Certainty and entanglement.</b>                            |
| 11d   | 4    |      | 09 | 050  | <b>Can quantum physics be made more complete? (Part II).</b>  |
| 11d   | 4    |      | 26 | 054  | <b>Tensor product transposes.</b>                             |
| 11d   | 4    |      | 31 | 054  | <b>Hermitian transposes of reflections, etc.</b>              |
| 11d   | 4    |      | 33 | 055  | <b>Slopes of Clifford functions.</b>                          |
| 11d   | 5    | 31   |    | 002  | <b>Fermions.</b>  |
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| 11d   | 5    | 33   |    | 008  | <b>Gaussian integrals.</b>                                    |
| 11d   | 5    | 34   |    | 013  | <b>Diagrams and QED.</b>                                      |
| 11d   | 5    | 35   |    | 016  | <b>Chirality and electroweak.</b>                             |
| 11d   | 5    | 36   |    | 023  | <b>Green's functions.</b>                                     |
| 11d   | 5    | 37   |    | 030  | <b>Propagators.</b>   |
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| 11d   | 5    |      | 06 | 035  | <b>Matrix Gaussians.</b>                                      |
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| 11d   | 5    |      | 14 | 037  | <b>Current from faze invariance.</b>                          |
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| 11d   | 6    | 38   |    | 003  | <b>Quantum Computing.</b>                                 |
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| 11d   | 6    | 41   |    | 015  | <b>Finding periods.</b>                                   |
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| 11d   | 6    | 45   |    | 024  | <b>Detecting and correcting errors.</b>                   |
| 11d   | 6    | 46   |    | 028  | <b>Nonlocality; Einstein-Podolsky-Rosen.</b>              |
| 11d   | 6    | 47   |    | 029  | <b>Building a quantum computer.</b>                       |
| 11d   | 6    |      | 07 | 033  | <b>Implement swap gate.</b>                               |
| 11d   | 6    |      | 10 | 034  | <b>Representing functions.</b>                            |
| 12    |      | 01   |    | 001  | <b>Recursion.</b> Define “ancestor” in terms of “parent”. |
| 12    |      | 02   |    | 001  | <b>Precedence without parentheses</b>                     |
| 12    |      | 03   |    | 002  | <b>Fractals</b>   |
| 12    |      | 04   |    | 003  | <b>Mathematical induction.</b>                            |
| 12    |      | 05   |    | 004  | <b>Recursion.</b> Programming languages.                  |
| 12    |      | 06   |    | 004  | <b>L-systems</b>  |
| 12    |      | 07   |    | 006  | <b>Instantiation.</b>                                     |
| 12    |      | 08   |    | 007  | Copying state.  |
| 12    |      |      | 07 | 009  | Peano curve   |
| 12    |      |      | 13 | 010  | Euler’s formula, edges, faces, vertices                   |
| 12    |      |      | 14 | 010  | <i>Hexagonal numbers</i>                                  |
| 12    |      |      | 18 | 012  | <b>Calculus.</b>  |
| 12    |      |      | 19 | 017  | <b>Continuity.</b>  |
| 12    |      |      | 20 | 019  | <b>Series.</b> Binomial coefficients.                     |
| 12    |      |      | 28 | 021  | Fractional dimension                                      |