The One Way to Quantum Computation

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- hence the name “One-way quantum computer.”

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- Conclusions.

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  - The type structure of computations
- Ideas from the semantics/concurrency/type theory community will prove useful.

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  - local,
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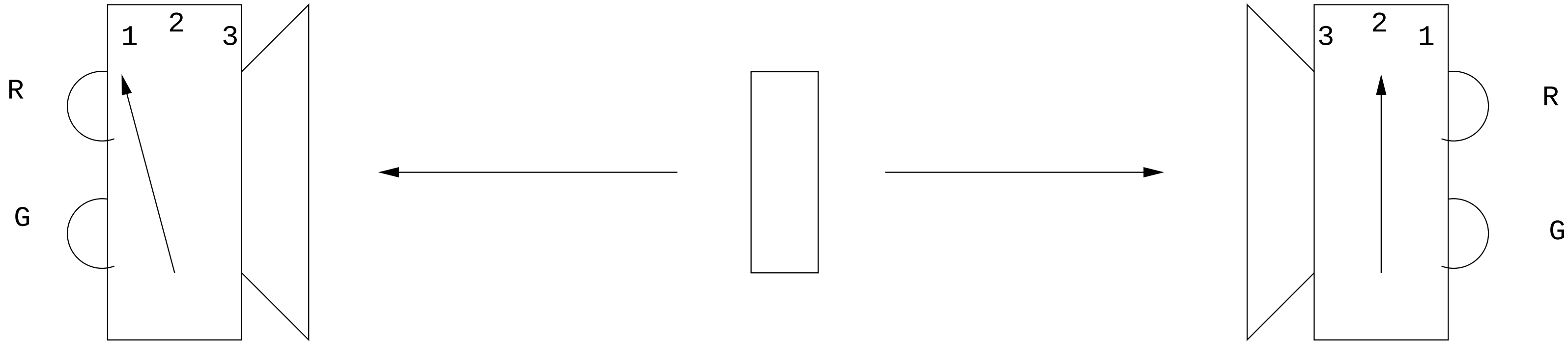
  - based on a deterministic state.

Mermin's example I

A simple version of Bell's inequality that can be understood easily.

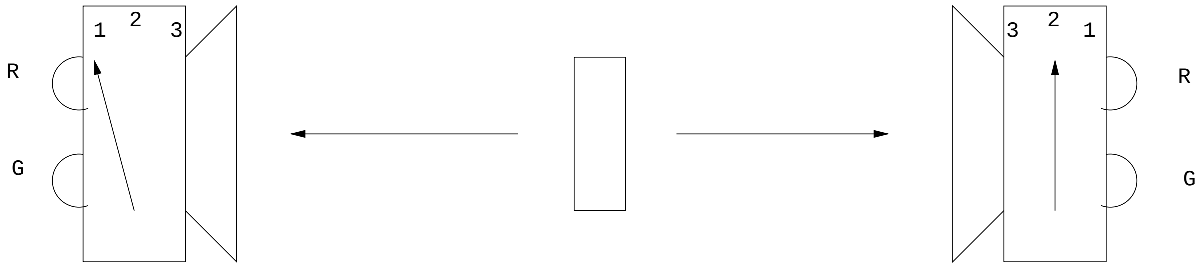


Two detectors each with 3 settings and 2 indicators (Red and Green). The detectors are set independently and uniformly at random.

The detectors are not connected to each other or to the source.

Source of correlated particles in the middle.

Mermin's Example II



Whatever the setting on a detector, the red or the green lights flash with equal probability, but never both at the same time.

When the settings are the same the two detectors **always** agree.

When the settings are different the detectors agree $\frac{1}{4}$ of the time!

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- There must be some “hidden” property of the particles that determines which colour is chosen for each setting; the two correlated particles must be identical with respect to this property, *whether or not the switches are set the same way.*
- Let us write GGR mean that for the three settings, $1, 2, 3$, the detectors flash green, green and red respectively for a type GGR particle.

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- This applies for any of the combinations: $RRG, RGR, GRR, GGR, GRG, RGG$.
- For particles of type RRR and GGG the colours **always** match whatever the settings.

The inescapable conclusion

Thus *whatever the distribution of particle types* the probability that the lights match when the settings are different is at least $\frac{1}{3}$!.

This just ain't what we see in nature!

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- a detector cannot predict the future sequence of particles and alter its behaviour: causal.
- no ordinary probabilistic automaton or MDP can reproduce the observed behaviour without breaking locality or causality.

Bell's inequality

The inequality,

$$Prob(\text{lights agree}|\text{settings different}) \geq \frac{1}{3},$$

is a simple special case of Bell's inequality.

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- Bell's inequality has been **experimentally tested** and it is plainly violated

The Point of this discussion

The probabilistic nature of quantum mechanics does not arise as an abstraction of things that could be known. State is not enough to predict the outcomes of measurements; **state is enough to predict evolution to new states.**

If you want to know where the $\frac{1}{4}$ comes from and a description of the "real" experiment, see me later.

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- The evolution of an *isolated* system is governed by a *unitary* transformation. This is determinate evolution.
- Measurements are described by Hermitian operators. Why operators?

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- If M is measured and λ_i is obtained the system gets knocked into a eigenstate of M with eigenvalue λ_i.
- If we measure M immediately again then we will certainly get the value λ_i again.

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- Typically quantum computation is presented in terms of circuits that implement various unitaries.

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- The “size” (dimensionality) of the combined state space grows *exponentially*.
- This is what gives quantum computation its power.

Notation for Quantum Computation

- The basic unit is a two-dimensional state space called a **qubit**. The basis states are typically written $|0\rangle = \binom{1}{0}$ and $|1\rangle = \binom{0}{1}$. Note that $|0\rangle$ is not the zero of the vector space!

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- Tensor product is denoted by juxtaposition: $|0\rangle \otimes |0\rangle = |00\rangle$.
- We can measure in the computational basis by using the Hermitian operator $|0\rangle\langle 0| + |1\rangle\langle 1|$.

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- A few basic gates are enough to approximate all the possible unitary operations: universality.
- The basic gates are certain one-qubit operations plus a particular two-qubit operation: CNOT.

Some example gates

The Pauli matrices:

$$\sigma_x \text{ or just } X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y \text{ or just } Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z \text{ or just } Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Hadamard matrix

The Hadamard matrix:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

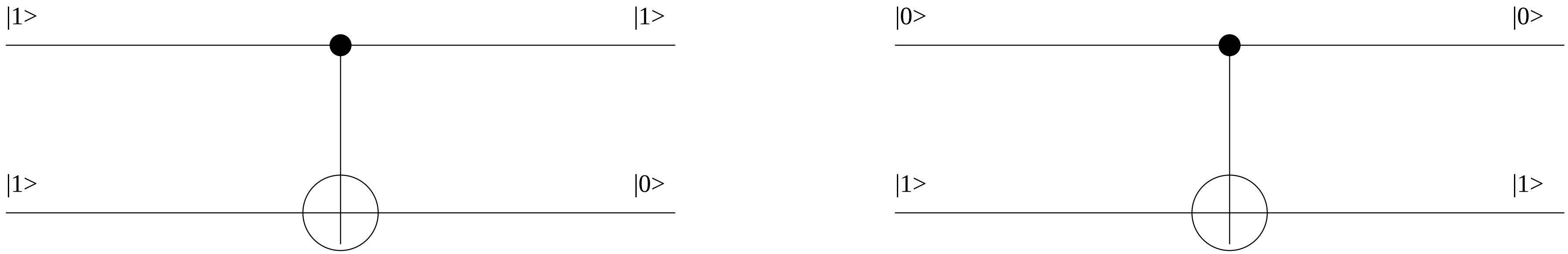
We have

$$H|0\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle$$

and

$$H|1\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle$$

The CNOT gate



It acts as follows

$$|xy\rangle \mapsto |x(x \oplus y)\rangle.$$

The x bit controls whether a **not** is applied to the second bit.

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- There are many notions of entanglement and proposed measures of how entangled two states are. For two qubits the state $|01\rangle + |10\rangle$ is maximally entangled, as is, e.g. $|00\rangle + |11\rangle$. They are called Bell states or Bell pairs.

CNOT entangles

When $CNOT$ is applied to $H|0\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle$ we get $\frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$.

Controlled versions of other one-qubit gates are possible.

Thus CZ stands for controlled Z. *This is also an entangler*.

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- Suppose that one of them performs a measurement to determine the state. He will get the outcome $|0\rangle$ or $|1\rangle$ with equal probability.
- The other observer will detect the same outcomes and by themselves these outcomes will seem random. However, the two sets of outcomes will be perfectly correlated.

Teleportation

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- She sends to B the outcome of the measurement (2 bits).
- B applies a unitary transformation to his state and it ends up being $|\psi\rangle$.

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- After combining $|\psi\rangle$ with her state the combined system is in

$$\frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)],$$

The algebra behind teleportation cont.*

which equals

$$\frac{1}{2\sqrt{2}}[(|00\rangle + |11\rangle)(\alpha|0\rangle + \beta|1\rangle) + (|00\rangle - |11\rangle)(\alpha|0\rangle - \beta|1\rangle)$$

$$+(|10\rangle + |01\rangle)(\beta|0\rangle + \alpha|1\rangle) + (|10\rangle - |01\rangle)(\beta|0\rangle - \alpha|1\rangle)$$

If A gets the first basis state she tells this to B and he knows that he now has $|\psi\rangle$. If A gets the second result, B has to fix up his state to get $|\psi\rangle$, and so on. This "fixing up" amounts to applying Pauli operations.

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- Can we compute more interesting functions?

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- One can modify the teleportation protocol to implement an arbitrary one-qubit unitary and some selected two-qubit unitaries: this is enough to get universality [Gottesman and Chuang 1999].
- Teleportation looks like it involves two-qubit measurements but it can be reduced to one-qubit measurements.

The Measurement-based Slogan

Measurements, followed by unitary corrections which may depend on the measurement outcomes can implement determinate quantum computations.

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- Use measurements to guide the computation; use Pauli corrections to eliminate the indeterminacy.
- One-qubit measurements and one-qubit corrections suffice [Raussendorf and Briegel 2001]
- provided one has the “right” entanglement to start with.
- One standard type of entangled state suffices and further entanglement is not necessary.

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- The angle along which a measurement is made may depend on the result of a previous measurement.
- Once a qubit is measured it is never used again.

The CNOT Pattern

control	X	Y	Y	Y	Y	Y	O
				Y			
target	X	X	X	Y	X	X	O

Each square is a qubit; X means measure the observable σ_x. Later it was realized that one can do this with fewer qubits.

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- Understanding the previous pattern is painful if you are not a physicist: too low level.
- Very hard to prove general results based on example patterns. The physicists' intuitions are so good that they rarely make wrong statements but their "proofs" tend to be demonstrations by example.
- No *systematic* understanding of how patterns can be composed.

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- The language comes with a natural compositional structure: inductive definition of possible patterns.
- We give a precise operational semantics and denotational semantics for the patterns.
- We develop a *calculus* of patterns and using rewriting theory arguments show that all patterns can be put in a normal form.

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- M_i^α measure qubit i in a basis rotated by angle α.

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- Programs are just sequences of commands read from right to left.

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- We write ${}^t[M_i^\alpha]^s$ for measurement $[M_i^\alpha]$ modified by signals s and t.
- X_i^s, Z_i^s: dependent corrections.

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(D3) a qubit i is measured if and only if i is not an output.

The Execution of Patterns

Entangle according to the E commands, then measure the qubits as indicated by the M commands and finally apply the corrections.

An Example Pattern

$$\mathcal{H} = (\{1,2\},\{1\},\{2\},X_2^{s_1}M_1^0E_{12}).$$

If the input is $\alpha|0\rangle + \beta|1\rangle$ then after E we get

$$\frac{1}{\sqrt{2}}[\alpha(|00\rangle + |01\rangle) + \beta(|10\rangle - |11\rangle).$$

After the measurement there are two possible outcomes:

$$\frac{1}{2}[(\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle] \text{ or } \frac{1}{2}[(\alpha-\beta)|0\rangle + (\alpha+\beta)|1\rangle].$$

The correction only kicks in for the second branch and it makes the two outcomes identical.

This pattern implements Hadamard.

Semantics

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- The denotational semantics associates to each pattern a certain type of operator (a completely positive map, for those who know and care).
- The proof that the two semantics agree is easy.

Composing Patterns 1

Two patterns $\mathcal{P}_1$ and $\mathcal{P}_2$ may be composed if $V_1 \cap V_2 = O_1 = I_2$.

The composite pattern $\mathcal{P}_2\mathcal{P}_1$ is defined as:

— $V := V_1 \cup V_2$, $I = I_1$, $O = O_2$,

— commands are concatenated.

Composing Patterns 2

Two patterns $\mathcal{P}_1$ and $\mathcal{P}_2$ may be tensored if $V_1 \cap V_2 = \emptyset$. The tensor pattern $\mathcal{P}_1 \otimes \mathcal{P}_2$ is defined as:

— $V = V_1 \cup V_2$, $I = I_1 \cup I_2$, and $O = O_1 \cup O_2$,

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- We use the semantics to show that our pattern language is universal.
- We came up with a striking new implementation of controlled unitary; only 14 qubits instead of $40+$ known before.
- We came up with a new - very simple - set of generators for unitaries as part of our proof of universality. [DKP Phys. Rev. A. Dec 2005]

A Huge Problem

Composing patterns ruins the nice EMC form of the patterns.

This form is very important if we want to avoid generating new entanglements on the fly.

Rewriting Rules

Using the algebra of the Pauli operators and qubits we showed how to define a rewrite system for patterns. These rules allow one to flip the order of certain commands. Example

$$E_{ij} X_i^s = X_i^s Z_j^s E_{ij}$$

which we orient as a rewrite rule

$$E_{ij} X_i^s \Rightarrow X_i^s Z_j^s E_{ij}.$$

Operators on disjoint qubits commute.

Standardization

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- Thus we can freely combine patterns and run it through the standardization engine to ensure that it is in EMC form.

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- One can analyze classical distributed computation problems in the quantum setting, e.g. leader election [DP 06]
- One can discuss epistemic concepts in a quantum setting and analyze knowledge flow in e.g. teleportation [DP05]

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- Suppose that two agents want to choose one of themselves as a leader and they share a Bell pair or, if there are three agents, they share $W_3 = |001\rangle + |010\rangle + |100\rangle$.

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- They can each measure in the basis $|0\rangle, |1\rangle$; the one who gets $|1\rangle$ is the leader.
- Each agent has the same chance of getting elected, the process is guaranteed to terminate in one step. Exactly what is classically impossible!

Conclusions

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- We have benefited from the formalization of measurement-based computation: EMC form, composing patterns, succinct representations.
- In fact there are several measurement calculi and translations between them which show how the models are related.
- The one-way quantum computer plays a central role in relating these models.

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