The Search for Structure in Quantum Computation

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Outline

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What does *our* expertise buy us?

What challenges are open to us?

Some basic themes

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- How is behaviour described?
- How do behavioural descriptions *compose*?
- How do we *verify* systems?
- How do we design systems?

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Soon after it was realized that concurrent programs could not be modelled by functions *or even by relations*.

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But quantum computation poses entirely new challenges!

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- Depending on the type of particle the answer could be 1/3 (bosons) or 1 (fermions).

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There are complex numbers and interference which blatantly violate basic commonsense rules of logic.

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I would like to add a third: using the *non-local* nature of quantum computation to achieve distributed computing tasks.

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(b) if we don't physics will take all the money and run!

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More precisely if the filter makes an angle θ with the photon polarization then the photon gets through with probability $\cos^2\theta$.

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Quantum systems are not "just" probabilistic.

Non-locality and Entanglement

A key idea due to Einstein, Podolsky and Rosen (1935) which was intended as an attack on quantum mechanics. Turned out to be revolutionary and led to the notion of non-locality and entanglement.

EPR - Bohm's version

Two-state quantum particle: $|\uparrow\rangle$ for spin up and $|\downarrow\rangle$ for spin down.

Two-particle basis states written: $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$.

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Consider the state: $\frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$. This state can be *prepared in a laboratory*. Measuring the spin of one particle "makes" the other one have the opposite spin.

Information is *nonlocal*, a quantum mechanical state is nonlocal. We can substitute entanglement for communication.

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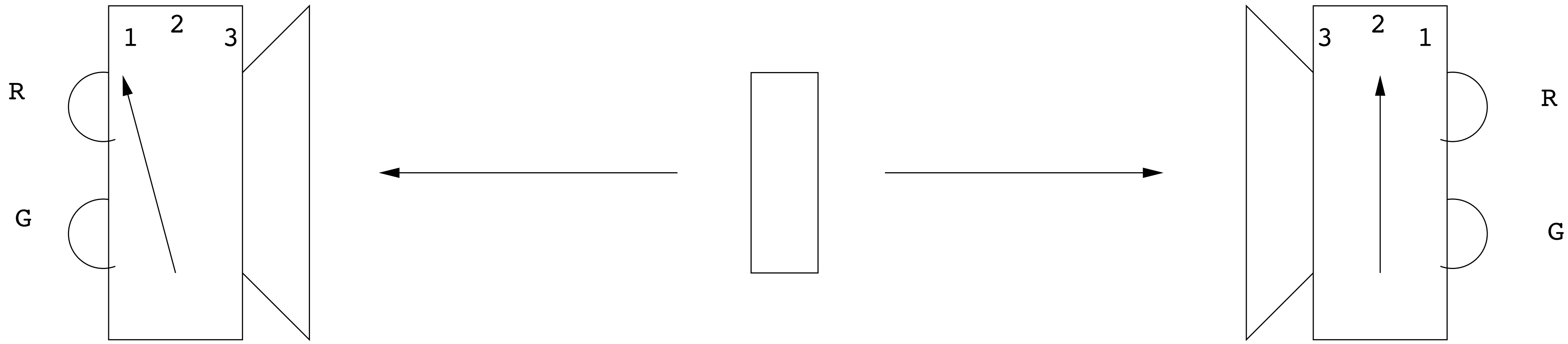
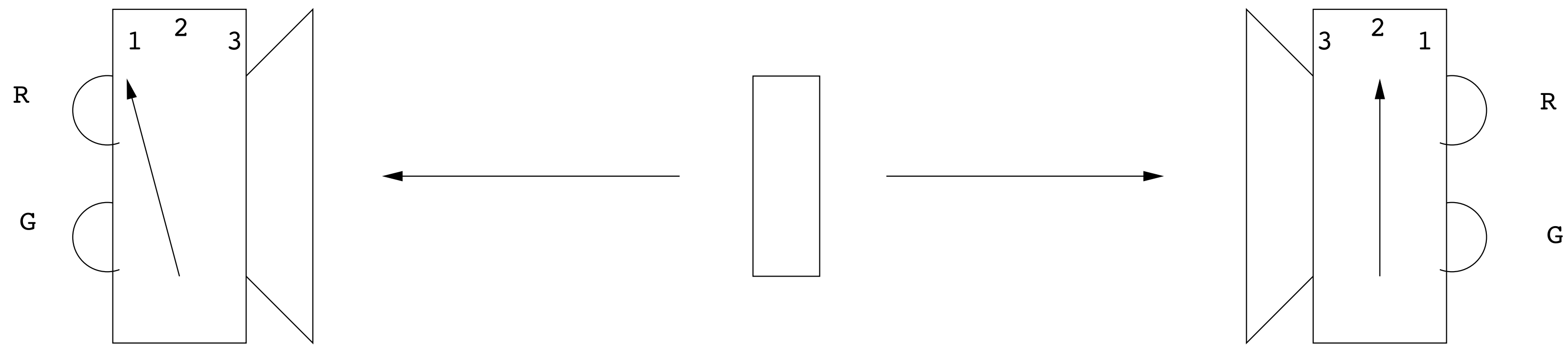
A simple version of Bell's inequality that can be understood easily.



Two detectors each with 3 settings and 2 indicators (Red and Green). The detectors are set independently and uniformly at random.

The detectors are not connected to each other or to the source.

Source of correlated particles in the middle.



Whatever the setting on a detector, the red or the green lights flash with equal probability, but never both at the same time.
When the settings are the same the two detectors **always** agree.

When the settings are different the detectors agree $\frac{1}{4}$ of the time!

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We write GGR for a typical particle type: it means that for the detector settings 1,2 and 3 respectively the light flashes green, green and red respectively.

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This applies for any of the combinations: $RRG, RGR, GRR, GGR, GRG, RGG$.

For particles of type RRR and GGG the colours **always** match whatever the settings.

Thus *whatever the distribution of particle types* the probability that the lights match when the settings are different is at least $\frac{1}{3}$!.

This experiment can be realized in the lab.

The data do not support the reasoning above: when the settings are different the lights match only a $\frac{1}{4}$ of the time!

What is wrong with the reasoning?

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What kinds of transition systems are these?
We have studied all kinds of transition systems,
we should analyze *from the perspective of transition system theory* the kinds of systems that arise in quantum mechanics.

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- The **record** or **result** of a measurement is an objective property.

The Stern-Gerlach Experiment

- Send neutral atoms through a varying magnetic field.
- Observe two peaks - the beam is split into an "up" and a "down."
- Rotate the apparatus and still observe the beam split in two.

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Note that $\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3 = 0$. So the sum of the measured components must add up to zero.

But 3 number chosen from the set $\{+1, -1\}$ cannot add up to zero!

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This is called **non-contextuality**.

Kochen-Specker Theorem

Quantum mechanics is not non-contextual
or, quantum mechanics is contextual.

There is a compatible family M of quantum measurements such that the following statements contradict each other:

If A, B, C are in M and $C = A + B$
then $val(C) = val(A) + val(B)$
and similarly for products,

all the members of M have definite values.

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Let $\mathcal{S}$ be a family of subsets of a finite set X. We say that $\mathcal{S}$ is a *KS family* if $\mathrm{card}(\mathcal{S})$ is odd, and for each $x \in X$, $c(x)$ is even, where:

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$$v = \sum_{x \in X} c(x)\phi(x)$$ which is clearly even!

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But people have constructed KS sets of triples of orthogonal directions!

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The outcome is created by the measurement process, there is no “value” that can be assigned before the system is measured!

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To describe the state of n qubits I have to specify 2^n complex numbers.

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There are *provably impossible* classical distributed computing tasks that can be done with suitable entangled states.

Categorical Quantum Mechanics

Abramsky, Coecke, Selinger, Landsman, Jacobs and others...

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basic algebraic aspects of tensor product and reversibility and duality.

Captures "abstract" quantum mechanics, one can explore "toy" quantum mechanics and ask what is the minimal structure needed to reveal key aspects of quantum mechanics.

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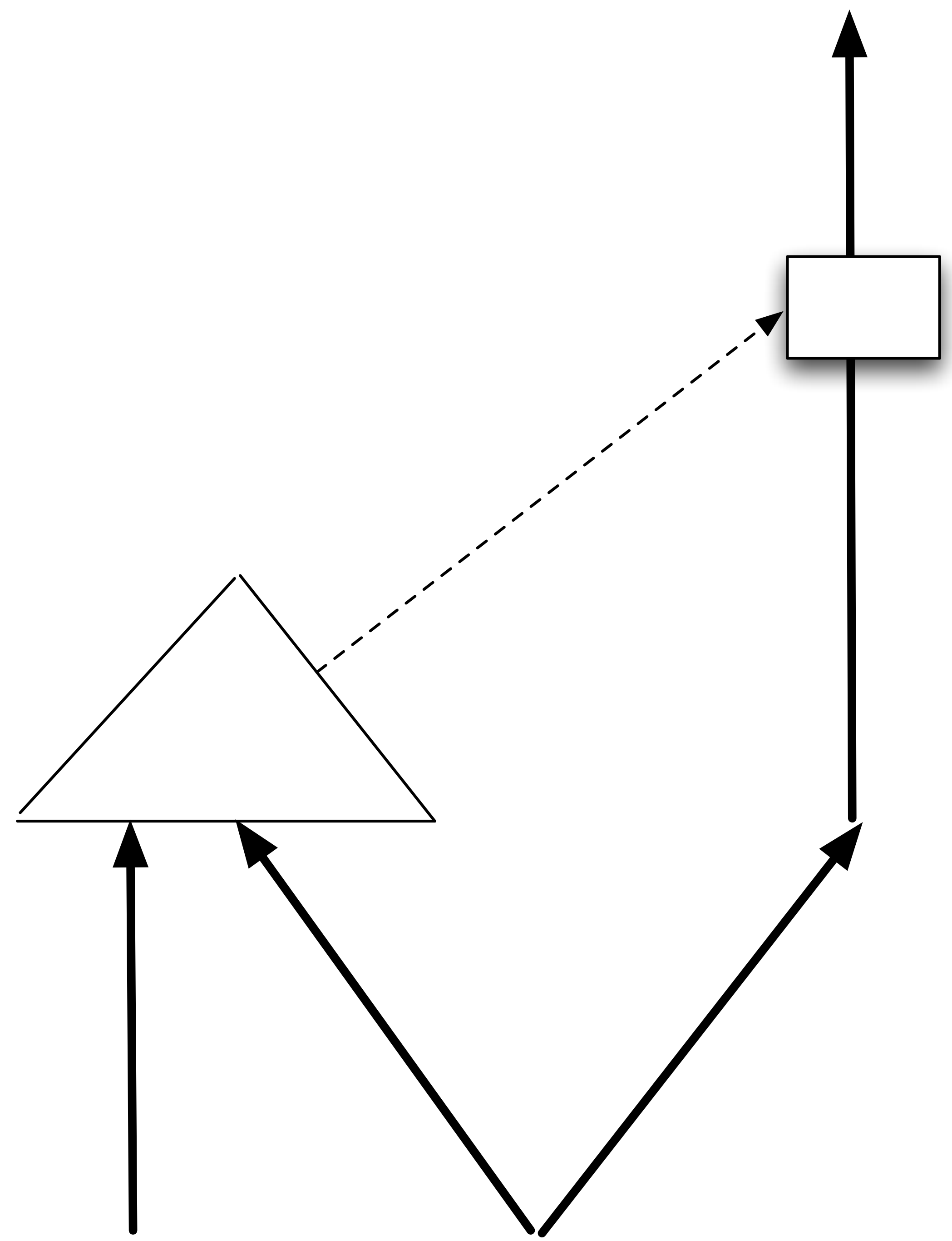
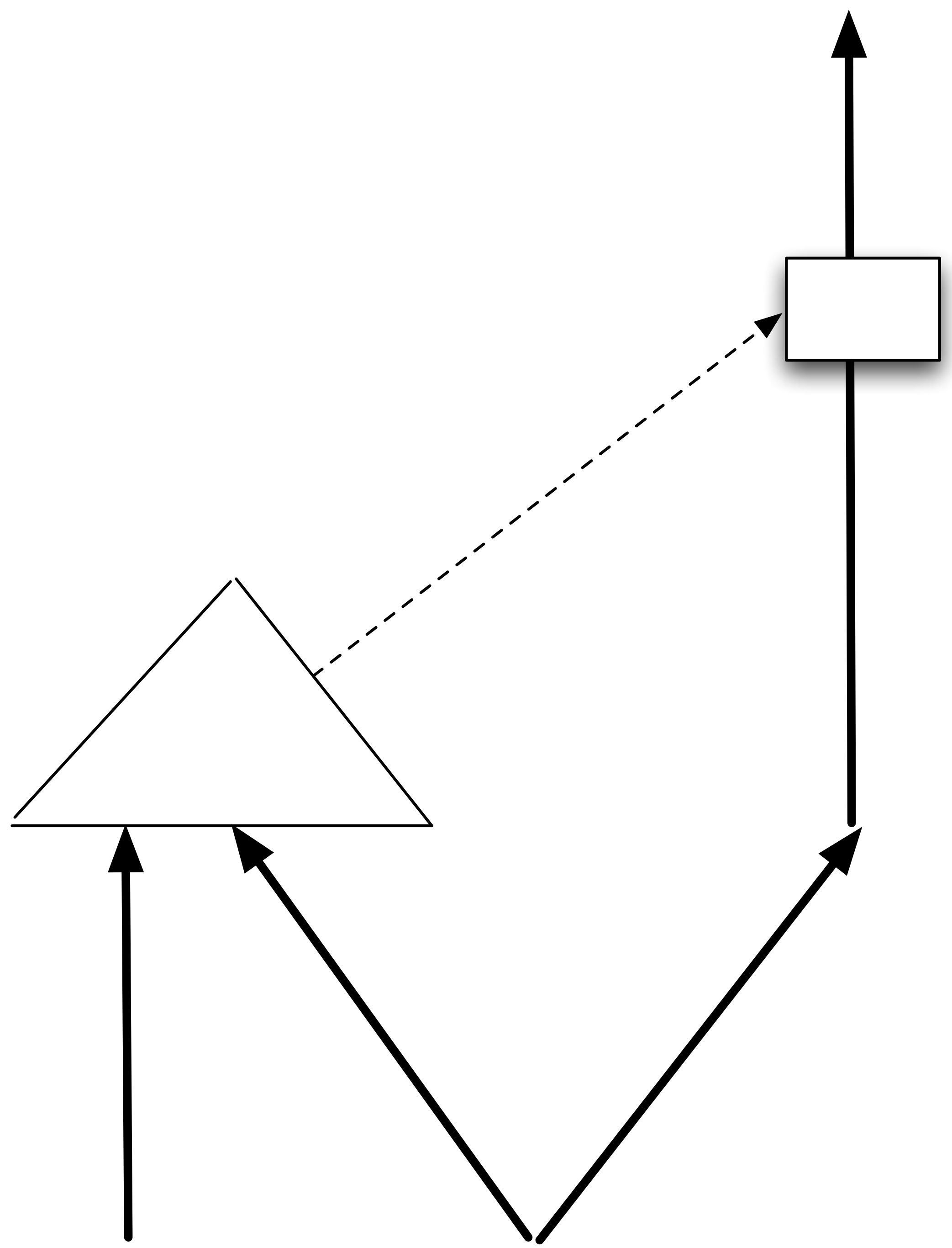
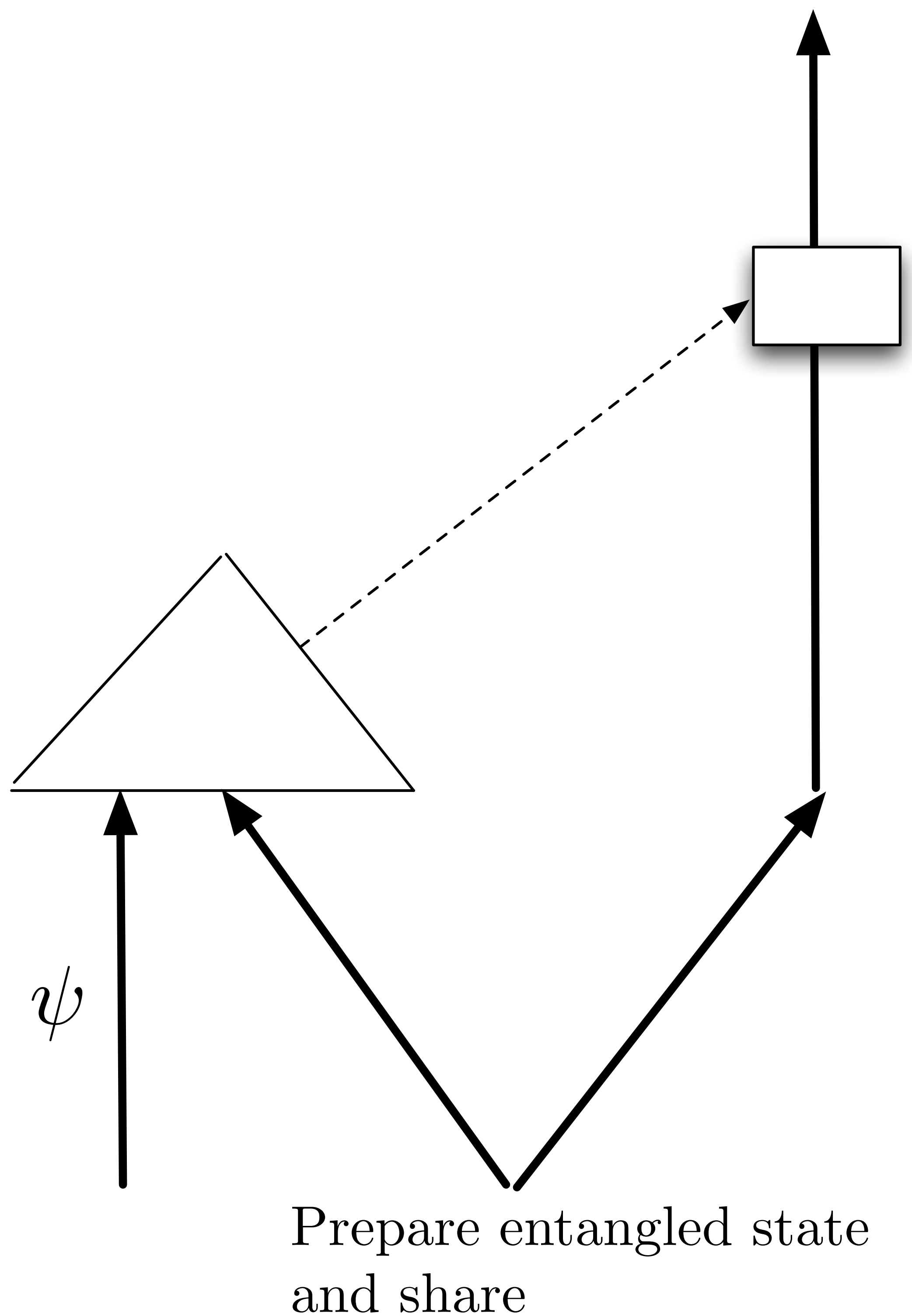
and more!





Prepare entangled state
and share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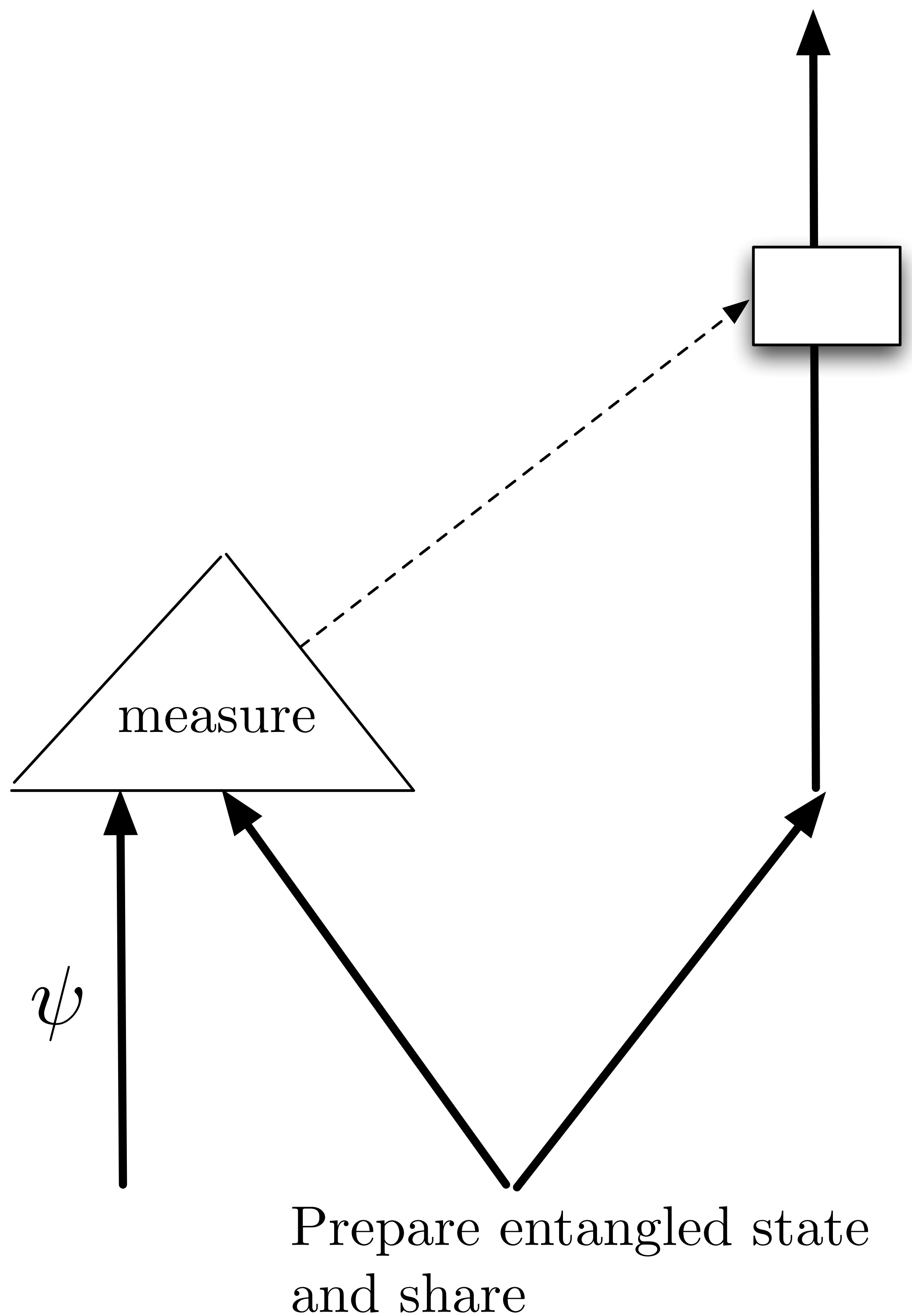
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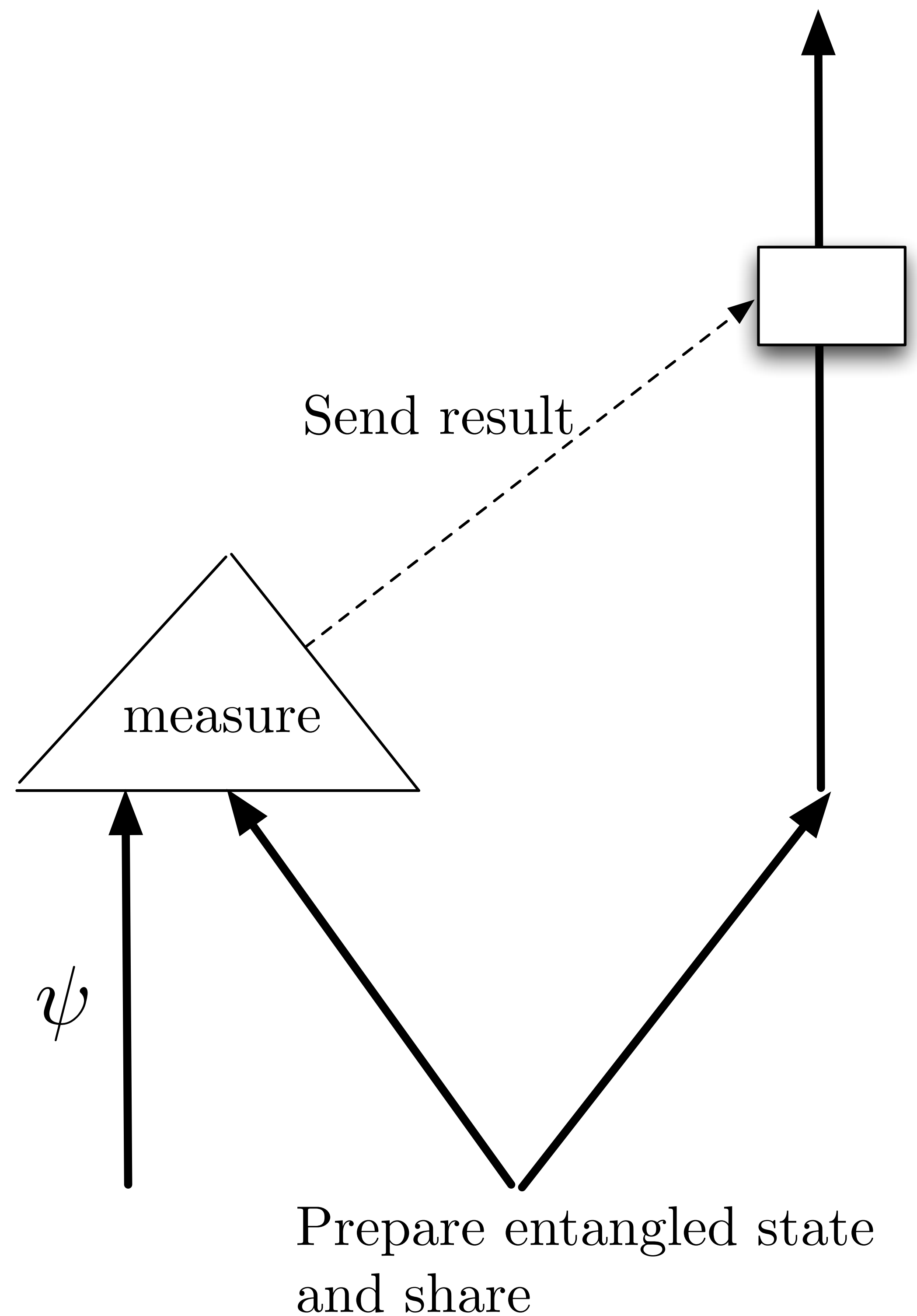
Send result
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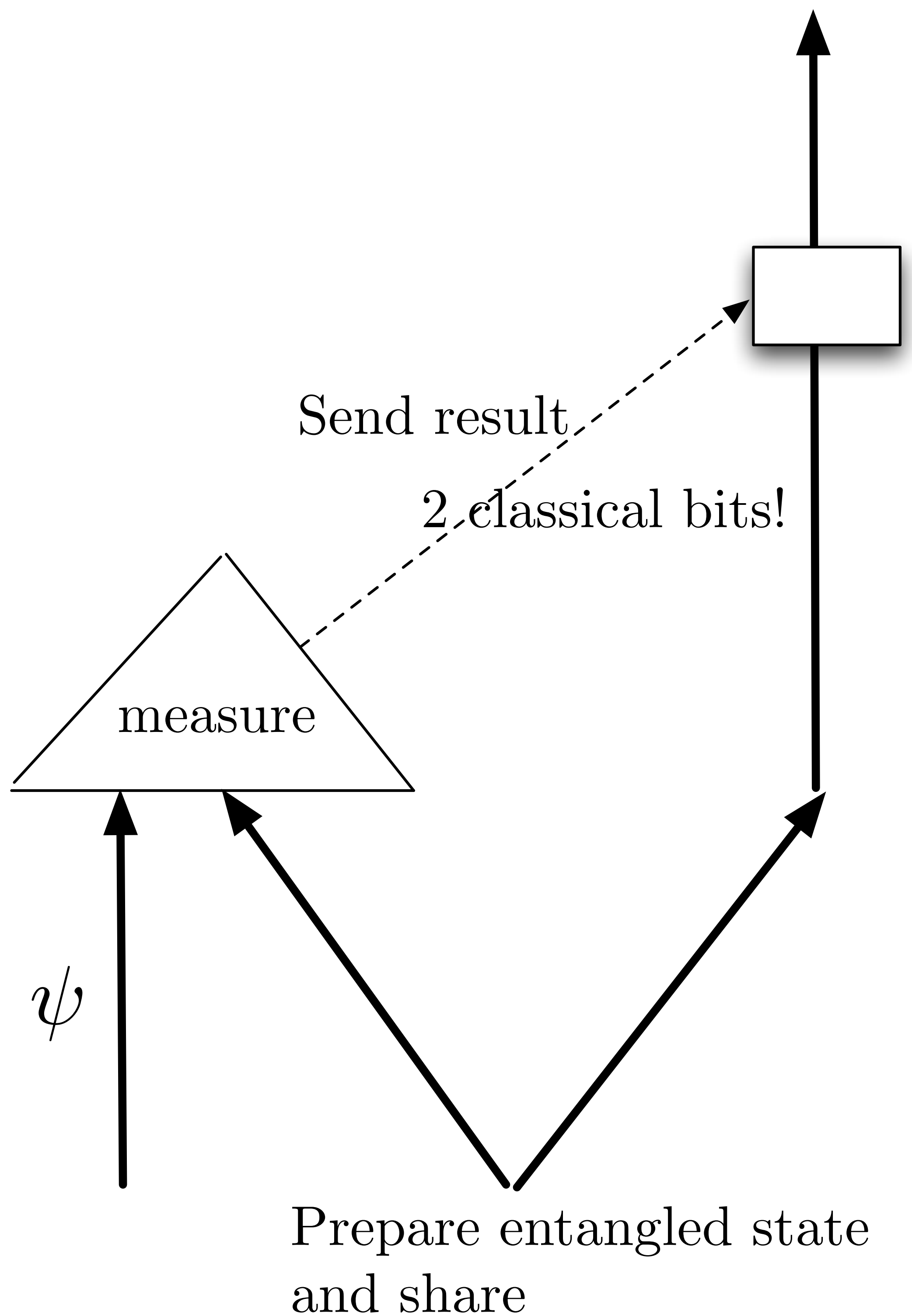
Send result
2 classical bits!
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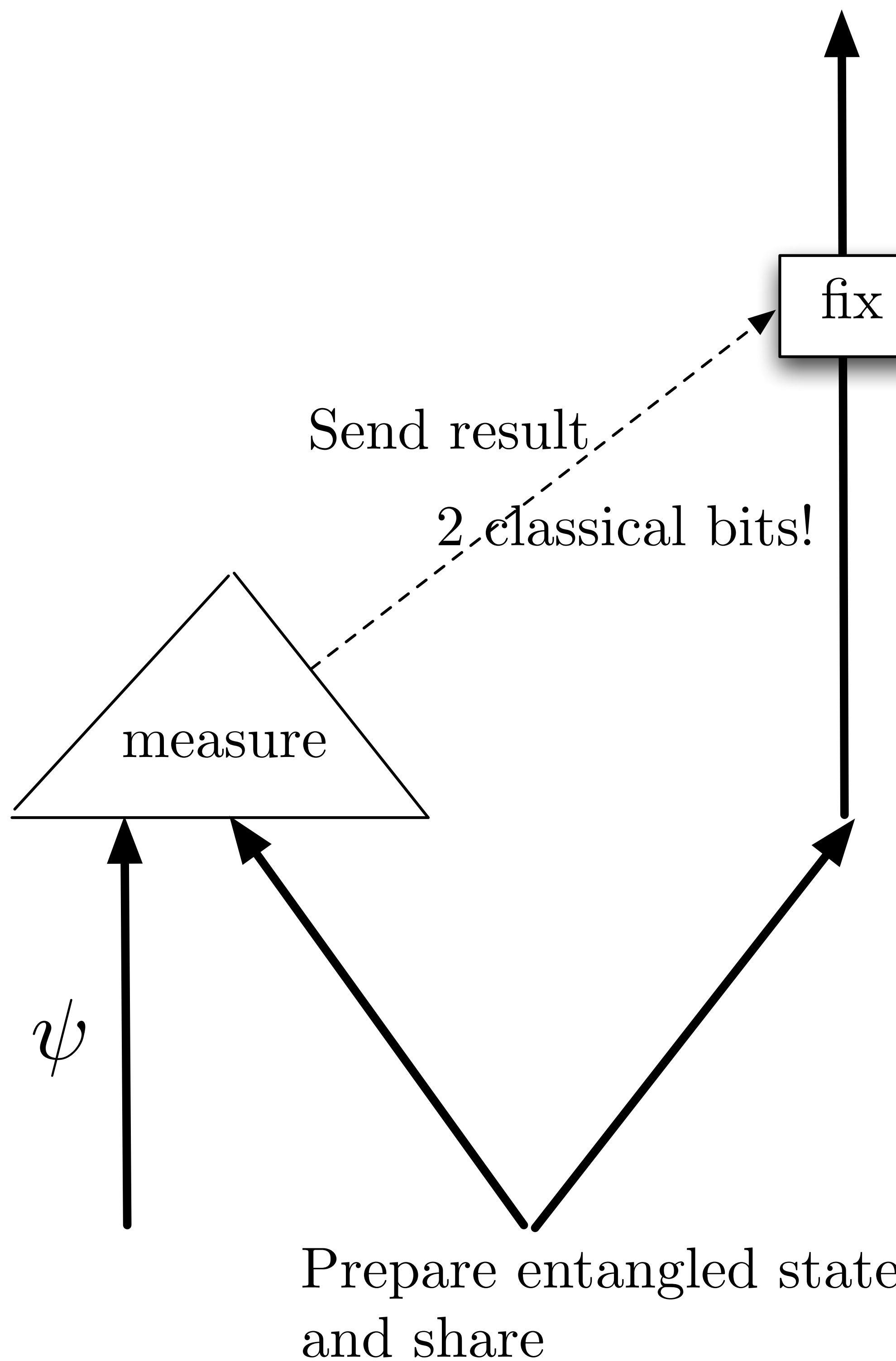
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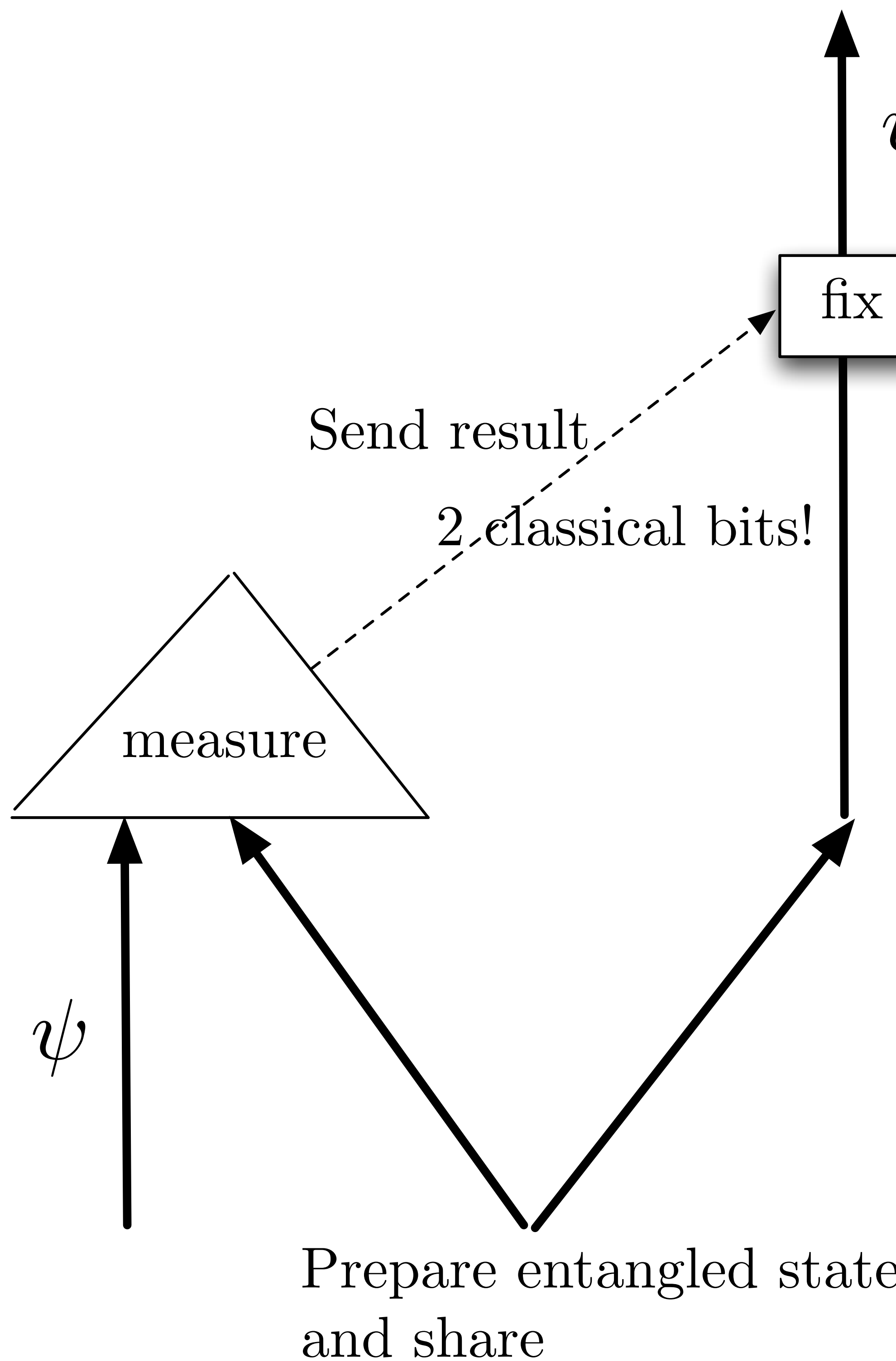

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The Point of Teleportation

The result of a measurement tells you what fix to apply
in order to get a determinate result.

It did not matter that the measurement outcome is indeterminate,
the whole procedure is determinate.

This is a computation - of the identity function (!) - that
is guided by measurement outcomes.

Can we compute more interesting functions?

Measurements, followed by corrections, which may depend on the measurement outcomes, can implement *all possible* determinate quantum computations.

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Extremely hard to prove general results based on example patterns. The physicists intuitions are so good that they (almost) never make mistakes.

But their proofs tend to be example demonstrations.

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A success story for the ideas from this community.

We give a precise textual syntax for patterns. We do not worry about the geometrical layout but refer to qubits by name.

The language comes with a natural compositional structure: inductive definition of possible patterns.

We give a precise operational semantics and denotational semantics for the patterns.

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A lot of work has been done since 2007 by Elham Kashefi and her many collaborators using this framework.

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We have no clue what is the right modal logic for quantum systems in the spirit of van Benthem-Hennessy-Milner.

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Is this the right way to proceed?

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She cannot "write it down"; that would violate the no-cloning theorem.

Topological Quantum Computing

Quantum systems are extremely unstable, how do we manipulate them with the exquisite precision needed while maintaining entanglement?

Many ideas in the physics community (each more expensive than the last).

One brilliant idea (due to Kitaev): use topological configurations like knots and braids that do not come apart easily.

Beautiful and interesting mathemtics and many opportunities for us to formalize the appropriate methods for reasoning about such systems. See tutorial slides on my web page.

Nobody can drive us out of the paradise that Heisenberg has created for us!

with apologies to David Hilbert.


RBS
TAYLORS
31 ST GILES

OSC
OXFORD
SUPERCOMPUTING
CENTRE
MFPS XXV

Thanks!

 

and thanks again,

and a final thank you!