COMP 330 Winter 2021 Quiz 5 Solutions

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Question 1: Which is the only true statement in the following?

1. If $P \leq Q$ and $Q \leq R$ then $P \leq R$.
2. If $P \leq Q$ and Q is undecidable then P is undecidable.
3. If $P \leq Q$ and P is decidable then Q is decidable.
4. If $P \leq Q$ and $Q \leq P$ then P and Q are both decidable.
5. If $P \leq Q$ and $Q \leq P$ then P and Q are both undecidable.

Answer 1.

Question 2: What type of language is $\{\langle M, w\rangle | M(w) \uparrow\}$?

1. Computably enumerable (CE).
2. co-Computably enumerable (coCE)
3. Decidable.
4. Both CE and coCE.
5. Neither CE nor coCE.

Answer 2.

Question 3: What is the only false statement among the following?

1. VALCOMPS is used to establish undecidability of some questions concerning context-free grammars.
2. The Post correspondence problem (PCP) is used to establish undecidability of the validity problem for formulas of first-order logic.

3. To show undecidability one needs to use a *direct* reduction from the halting problem.
4. The PCP can be used to show that ambiguity of context-free grammars is undecidable.
5. In class we proved

$$\mathsf{HP}_{TM} := \{\langle M, w\rangle | M(w)\downarrow\} \leq_m \mathsf{VALCOMPS}(M, w) \neq \emptyset.$$

Answer 3.

Question 4: The language $\{\langle G_1, G_2\rangle | L(G_1) \cap L(G_2) = \emptyset\}$, where $G_{1,2}$ are context-free grammars, is

1. context-free.
2. decidable.
3. CE but not co-CE
4. co-CE but not CE.
5. regular.

Answer 4.

Question 5: Which is the only true statement among the following? The symbols M_1, M_2 refer to Turing machines.

1. $\{\langle M_1, M_2\rangle | L(M_1) = L(M_2)\}$ is both CE and co-CE.
2. $\{\langle M_1, M_2\rangle | L(M_1) = L(M_2)\}$ is neither CE nor co-CE.
3. $\{\langle M_1, M_2\rangle | L(M_1) = L(M_2)\}$ is CE but not co-CE.
4. $\{\langle M_1, M_2\rangle | L(M_1) = L(M_2)\}$ is not CE but it is co-CE.
5. $\{\langle M_1, M_2\rangle | L(M_1) = L(M_2)\}$ is context free.

Answer 2.