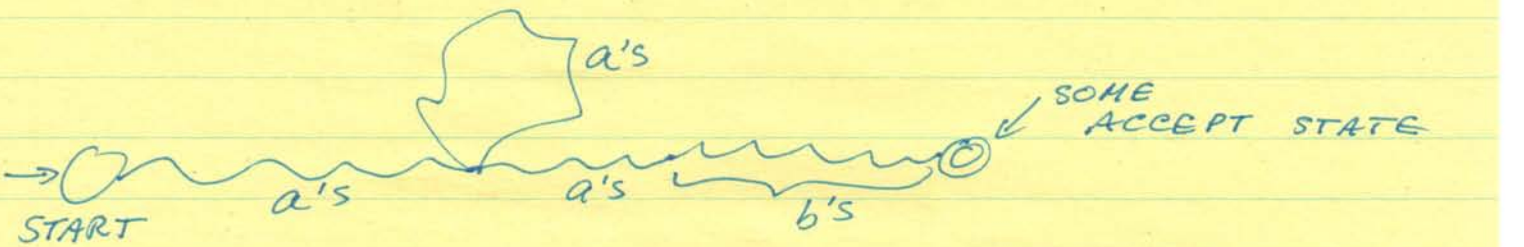
PUMPING LEMMA

No DFA can accept $L = \{a^n b^n \mid n \geq 0\}$ because no DFA can count how many a's there are.

<u>Idea</u> Suppose we have a putative recognizer for L & this is a DFA. Suppose it has n states. Choose a string of the form $a^m b^m$ where $m > n$.



Such a string must hit the same state twice as it reads the a's. So there is a loop of size, say k driven by a's. But now I should be able to recognize

$a^{m+k} b^m$: going around the loop twice
$a^{m+2k} b^m$: going around the loop 3 times
$\vdots$

also $a^{m-k} b^m$: skipping the loop

FORMAL STATEMENT: If L is a regular language
$\exists p \in \mathbb{N}$ $p > 0$ s.t. $\forall w \in L$ with $|w| \geq p$
$\exists x, y, z \in \Sigma^*$ s.t. $w = xyz$ & $|xy| \leq p$ & $|y| > 0$
$\forall i \in \mathbb{N}$ $xy^iz \in L$

L regular $\Rightarrow$ L can be pumped
$\Updownarrow$
L cannot be pumped $\Rightarrow$ L is not regular

NOTE
L can be pumped does <u>NOT</u> imply L is regular

CONTRAPOSITIVE : Suppose $L \subseteq \Sigma^*$ is a language
~~then~~ s.t. $\forall p > 0\ \exists \omega \in L$ with $|\omega| \geq p$ s.t.

$\forall x, y, z \in \Sigma^*$ s.t. $\omega = xyz$ & $|xy| \leq p$ & $|y| > 0$

$\exists i \in \mathbb{N}$ s.t. $xy^i z \notin L$

then L is not regular.

How to cope with all those quantifiers?
GAMES.

$\forall$: Demon $\qquad$ $\exists$: You

(1) Demon chooses p
(2) You choose ω with $|\omega| \geq p$
(3) Demon chooses x, y, z meeting the conditions above
(4) You choose i & show $xy^i z \notin L$

Demon's choices are (1) symbolic to cover all cases & (3) you have to analyze all demonic choices.

EXAMPLE $\quad L = \{a^n b^n \mid n \in \mathbb{N}\}$

(1) Demon chooses p
(2) I choose $a^p b^p$
(3) Demon has to choose x, y, z with $|xy| \leq p$
so x, y consist purely of a's & $y \neq \varepsilon$ since $|y| > 0$
(4) I choose $i = 2$ so the string $xy^i z$ is
$a^{p+i \cdot l} b^p$ where $l = |y| > 0$
$p + i \cdot l \neq p$ so this string is not in L.

Thus L is not regular