An example of exponential blow-up in determinization

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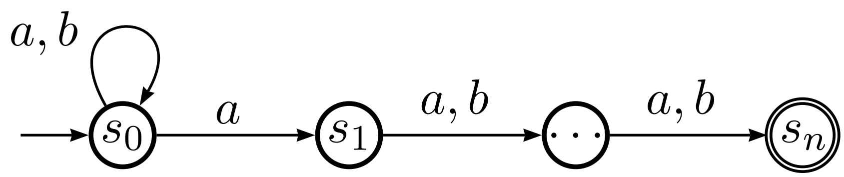
We fix the alphabet $\Sigma = \{a, b\}$. Consider the problem of recognizing strings where the nth character from the *end* is an a. The following NFA does the job.



Any deterministic automaton will take at least 2^n states. Suppose that a DFA M has fewer than 2^n states. Consider all possible strings of length n: there are 2^n such strings. Thus if we see where the machine ends up after processing each of these strings *there must be two strings that end up with the machine in the same state*; let these two strings be called x and y. Suppose that they are the same up to position k and in position k, x has an a whereas y has a b. Consider the new strings $x' = xa^{k-1}, y' = ya^{k-1}$. Now clearly x' should be accepted whereas y' should be rejected. But at the end of processing either x or y the machine is in the same state and then both strings x' and y' are identical there after. Since it is a DFA the transitions must be identical so it has to treat the two strings x' and y' the same way. This is a contradiction.