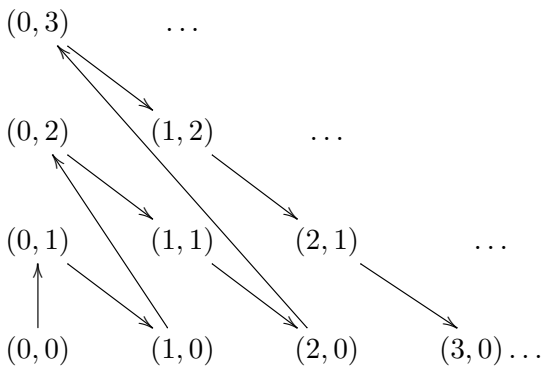
First we note that we can give a one-to-one and onto map between natural numbers and pairs of natural numbers. Think of writing the pairs in an infinite square table with the pairs of the form $(0, n)$ in the first row, the pairs of the form $(1, n)$ in the second row and so on. If we look at the diagonals from lower left rising towards the right we see that in every such row the numbers in each pair add up to the same value. We can thus enumerate the pairs by just going down these diagonals. We get

$$\tau(i, j) = \frac{1}{2}(i + j)(i + j + 1) + i.$$

We can see the pattern of the enumeration in the picture below



The function τ is clearly one-to-one and onto. We can invert τ by writing