

COMP 330 Winter 2021
Assignment 3
Due Date: 18th February 2021

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4th February 2021

There are **5** questions for credit and some extra questions at the end. There are three questions which are accessible to everyone but perhaps harder than the usual questions. There is one for your spiritual growth. All the regular questions are excellent practice for the mid-term. The extra questions will not help you prepare for the mid-term. Please submit the homework through myCourses by 5pm on the due date.

Question 1[20 points] Are the following statements true or false? Prove your answer in each case, but the proof need only be a simple example or a couple of lines of explanation. We have some fixed alphabet Σ with at least two letters. In the following A and B stand for languages, *i.e.* subsets of Σ^* .

- If A is regular and $A \subseteq B$ then B must be regular. [3]
- If A and AB are both regular then B must be regular. [7]
- If $\{A_i | i \in \mathbb{N}\}$ is an infinite family of regular sets then $\bigcup_{i=1}^{\infty} A_i$ is regular. [5]
- If A is not regular it cannot have a regular subset. [5]

Question 2[20 points]

Show that the following language is not regular using the pumping lemma.

$$\{a^n b^{2n} | n > 0\}$$

Question 3[20 points] Show that the language

$$F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

is not regular. Show, however, that it satisfies the statement of the pumping lemma as I proved it in class, i.e. there is a p such that all three conditions for the pumping lemma are met. Explain why this does not contradict the pumping lemma.

Question 4[20 points] Let D be the language of words w over the alphabet $\{a, b\}$, such that w has an even number of a 's and an odd number of b 's and does not contain the substring ab . By this last statement I mean that one can never have an a followed by a b .

1. Give a DFA with *only five* states, including any dead states, that recognizes D .
2. Give a regular expression for this language.

Question 5[20 points] Consider the language $L = \{a^n b^m \mid n \neq m\}$; as we have seen this is not regular. Recall the definition of the equivalence \equiv_L which we used in the proof of the Myhill-Nerode theorem; we used the notation R_L in the notes but it means the same thing as \equiv_L . Since this language is not regular \equiv_L cannot have finitely many equivalence classes. Exhibit explicitly, infinitely many distinct equivalence classes of \equiv_L .

Please turn over for the spiritual growth question.

Extra Question 1[0 points]

If L is a language over an alphabet with strictly more than one letter we define $CYC(L) = \{uv|u, v \in \Sigma^*, vu \in L\}$. Show that if L is regular then $CYC(L)$ is also regular; [12]. Give an example of a *non-regular* language such that $CYC(L)$ is regular.

Extra Question 2[0 points] In assignment 1 we had a question that asked you to prove that if a language is regular then the lefthalf of the language is also regular. Similarly, if I define the *middle thirds* of a regular language by

$$\text{mid}(L) = \{y \in \Sigma^* | \exists x, z \in \Sigma^* \text{ s.t. } xyz \in L \text{ and } |x| = |y| = |z|\}$$

then $\text{mid}(L)$ is also regular. *I am not asking you to prove this; it is too easy after you have done left-half.* What if I delete the “middle” and keep the outer portions? More precisely define,

$$\text{outer}(L) = \{xz | \exists y \in \Sigma^*, xyz \in L, \text{ and } |x| = |y| = |z|\}$$

then is it true that $\text{outer}(L)$ is regular if L is regular? Give a proof if your answer is “yes” and a counter-example, with a proof that it is not regular, if your answer is “no.”

Extra Question 3[0 points] Consider regular expressions as an algebraic structure with operations of \cdot and $+$ and constants of \emptyset and ε . Now consider equations of the form

$$X = A \cdot X + B$$

where A and B are regular expressions. Show that this always has a solution given by $A^* \cdot B$. Show, in addition, that if A does not contain the empty word this is the unique solution.

Spiritual Growth Question[0 points] Consider a probabilistic variant of a finite automaton. Come up with a formalization of what this might mean. Suppose that you have a reasonable definition and now you define acceptance to mean that your word causes the machine to reach an accept state with probability at least $\frac{2}{3}$. Show that such automata can recognize non-regular languages.