McGill University COMP360 Winter 2011 **Instructor: Phuong Nguyen**

Assignment 2

Due January 24 at the beginning of lecture

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offense, and will be dealt with accordingly.

Question 1 Consider the reduction from SAT to 3CNF-SAT. Give the 3CNF formula that results from transforming the following formula:

$$(\neg x_1 \vee x_2) \wedge \big((x_1 \wedge x_2) \vee \neg(\neg x_1 \wedge x_2)\big)$$

Clearly list the new variables and the clauses. Give a satisfying truth assignment to the resulted 3CNF formula.

Question 2 Consider the following problem. The input consists of

- an $m \times n$ integer matrix A,

$$A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \dots & & & \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{pmatrix}$$

  where all A_{ij} $(1 \le i \le m, 1 \le j \le n)$ are integers, and

- a column vector $\vec{b}$ of m coordinates, $\vec{b} = (b_1, b_2, \dots, b_m)$, where all $b_1, b_2, \dots, b_m$ are integers.

The problem is to decide whether there is a column vector $\vec{x}$ of n coordinates, $\vec{x} = (x_1, x_2, \dots, x_n)$ where each x_i $(1 \le i \le n)$ can take value either 0 or 1, such that $A\vec{x} \le \vec{b}$, that is, whether there exists $\vec{x} = (x_1, x_2, \dots, x_n) \in \{0,1\}^n$ such that

$$\begin{aligned} A_{11}x_1 + A_{12}x_2 + \ldots + A_{1n}x_n &\le b_1 \\ A_{21}x_1 + A_{22}x_2 + \ldots + A_{2n}x_n &\le b_2 \\ &\dots \\ A_{m1}x_1 + A_{m2}x_2 + \ldots + A_{mn}x_n &\le b_m \end{aligned}$$

Show that the problem is NP-complete by giving a nondeterministic polytime algorithm for it, and show that 3CNF-SAT is polytime reducible to it.