General Value Functions, General Policy Evaluation and General Policy Improvement

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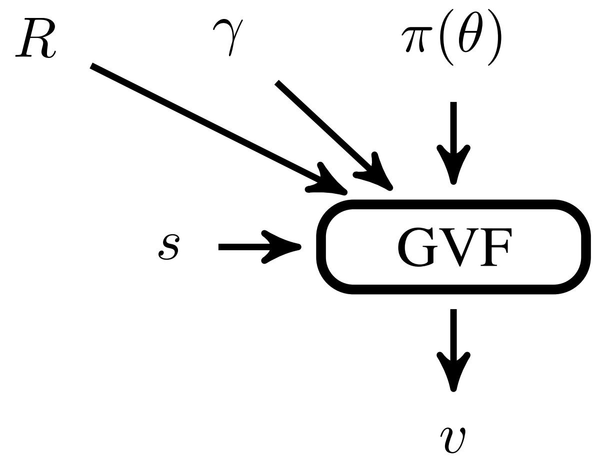
Recall: General Value Functions (GVFs)

- Given a cumulant function c, state-dependent continuation function γ and policy π, the General Value Function $v_{\pi,\gamma,c}$ is defined as:

$$v_{\pi,c,\gamma}(s) = \mathbf{E}\left[\sum_{k=t}^{\infty} c(S_k, A_k, S_{k+1}) \prod_{i=t+1}^{k} \gamma(S_i) | S_t = s, A_{t:\infty} \sim \pi\right]$$

- *Cumulant* c can output a vector (even a matrix)
- *Continuation function* γ maps states to [0,1] (further generalizations are possible)
- Cf. Horde architecture (Sutton et al, 2011); Adam White's thesis; inspiration from Pandemonium architecture
- Special case: policy is optimal wrt c, γ, $v^*_{c,\gamma}$ - Universal Value Function approximation (UVFA) (Schaul et al, 2015)
- No single task is required, just a multitude of cumulants and time scales!

GVFs as building blocks of knowledge



- Note that one can take the output of a GVF and make it an input to another GVF
- Or, the output of a GVF could become part of the "state" for another GVF

Option models are GVFs

- The reward model for an option ω is defined as:

$$r_\omega(s) = \mathbb{E}_\omega[r(S_t, A_t) + \gamma(1 - \beta_\omega(S_{t+1}))r_\omega(S_{t+1})|S_t = s]$$

- This means the option reward model is a GVF:
  - policy is π_ω
  - *cumulant* is the environment reward r
  - *continuation function* is $\gamma(1 - \beta_\omega)$
- Option transition model can be similarly written as a GVF

Many other approaches that can be expressed as GVFs

- Option-value functions (Precup, 2000; Sutton, Precup & Singh, 1999)
- Feudal networks (Dayan, 1994; Vezhnevets et al, 2017)
- Value transport (Hung et al, 2018)
- Auxilliary tasks (Jaderberg et al, 2016)
- *Are GVFs just an interesting insight or can they be useful?*

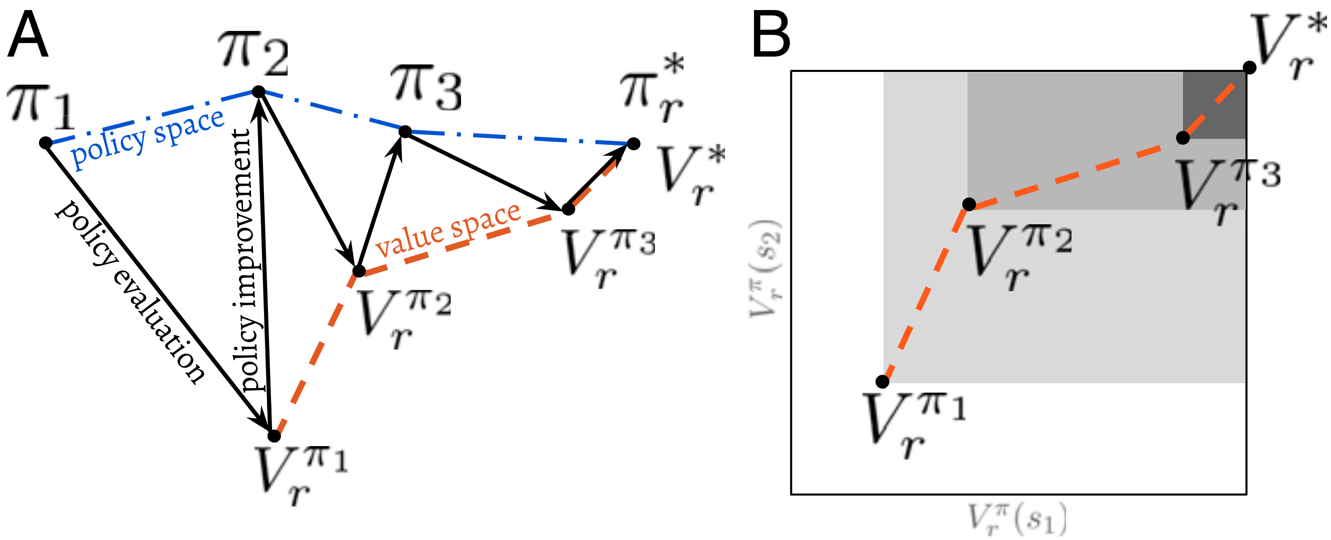
Policy Evaluation and Policy Improvement

- Consider a Markov Decision Process $\langle \mathcal{S}, \mathcal{A}, P, r \rangle$ and a policy $\pi : \mathcal{S} \to Dist(\mathcal{A})$
- Classic dynamic programming relies on two basic operations:
  - *Policy evaluation*: given policy π, compute the value function V_r^π and/or Q_r^π
  - *Policy improvement*: given value function Q_r^π, compute an improved policy: $\pi'(s) = \arg\max_{a' \in \mathcal{A}} Q_r^\pi(s, a')$
- Policy improvement guarantee:

$$Q_r^{\pi'}(s, a) \geq Q_r^\pi(s, a), \, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}$$

- Dynamic programming: interleave these steps (executed exactly)
- Reinforcement learning: carry out these steps approximately

Visualizing Policy Evaluation and Policy Improvement



- Generalize this process to *multiple reward functions (ie tasks)* $r \in \mathcal{R}$ and *multiple policies* $\pi \in \Pi$

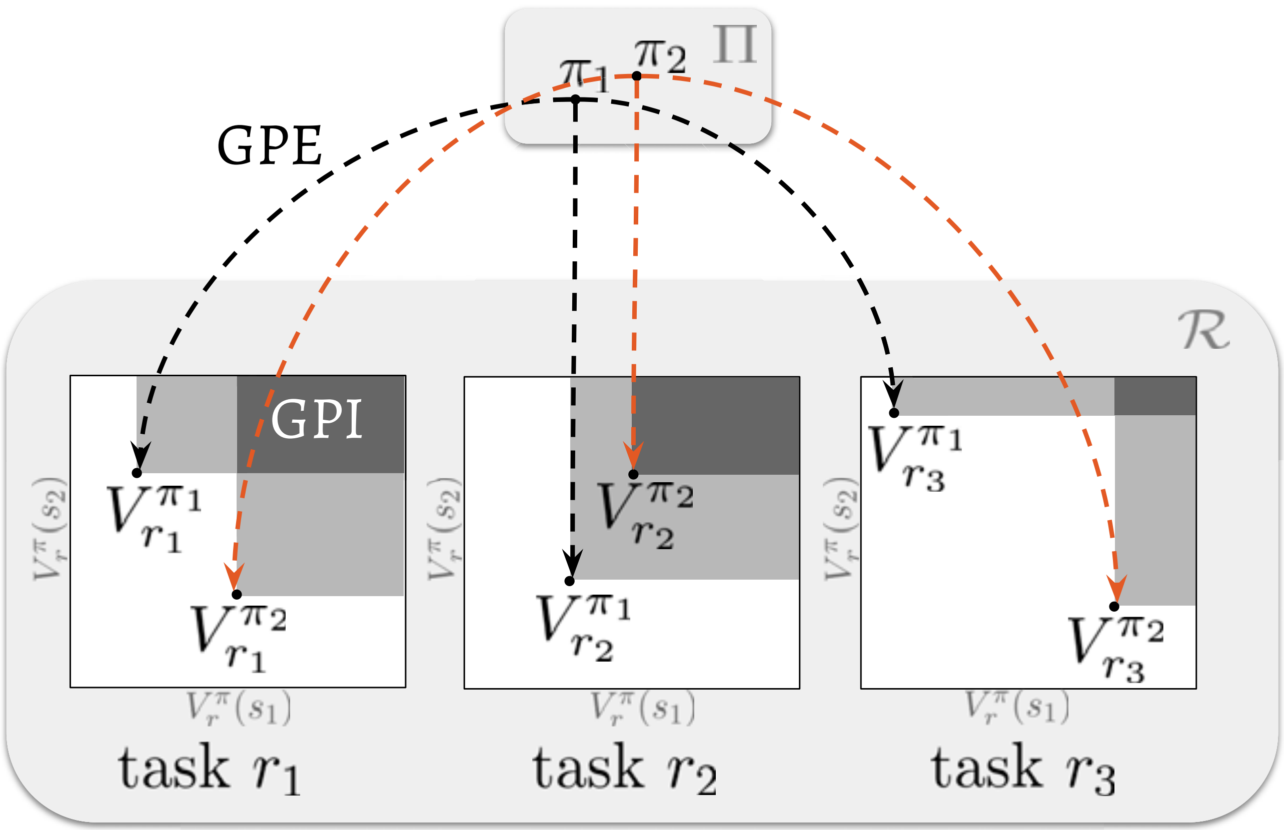
Generalized Policy Updates

- *Generalized policy evaluation (GPE)*: compute the value of a policy π on a set of reward functions $\mathcal{R}$
- *Generalized policy improvement (GPI)*: given a set of policies Π and a reward function r, compute a new policy such that:

$$Q_r^{\pi'}(s,a) \geq \sup_{\pi \in \Pi} Q_r^{\pi}(s,a),\ \forall s \in \mathcal{S} \forall a \in \mathcal{A}$$

- If we have only one r and one π, we recover usual policy evaluation and policy improvement

Visualizing Generalized Policy Updates



Fast Generalized Policy Evaluation

- If we had a nice map from r to Q_r^π, GPE could be efficient
- Consider the class of reward functions that are linear in some feature space $\phi(s,a)$:

$$r_{\mathbf{w}}(s,a) = \mathbf{w}^T \phi(s,a) \text{ and } \mathcal{R}_\phi = \{r_{\mathbf{w}} | \mathbf{w} \in \mathbb{R}^d\}$$

  Note that ϕ can be learned and non-linear
- *Successor features*: $\psi^\pi(s,a) = \mathbf{E}_\pi[\sum_{t=1}^\infty \gamma^t \phi(s_t,a_t) | s_0 = s, a_0 = a]$
- Then the value function for a specified reward function can be easily computed as a function of the successor features:

$$Q_{\mathbf{w}}^\pi(s,a) = \mathbf{w}^T \psi^\pi(s,a)$$

- *Successor features can be pre-computed for π once and re-used thereafter (a form of model!)*
- Connections to hippocampus representations

Successor states and successor features are GVFs

- *Successor features* (Barreto et al, 2017, 2018) are a natural extension of successor states (Dayan, 1992)
- Successor states give the expected occupancy of future states
- If states are defined by a feature vector $\phi(s)$, successor features give the expected, discounted sum of future feature vectors from a state.
- In GVF terms, the *cumulant is $c = \phi$*, and there is a fixed policy and discount
- Interesting property highlighted in Barreto et al:

$$v_{\pi,\mathbf{w}^T c,\gamma}(s) = \mathbf{w}^T v_{\pi,c,\gamma}(s)$$

which leads to one-shot computation of new GVFs

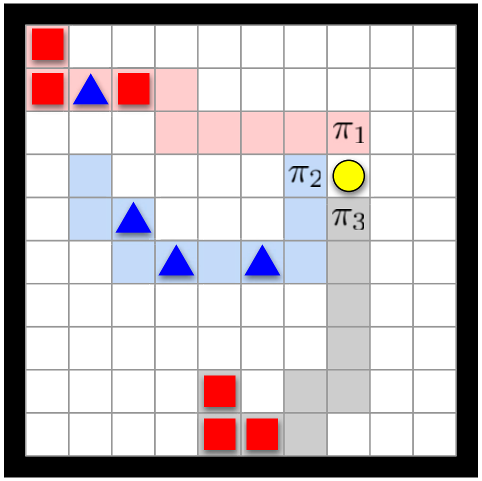
Fast Generalized Policy Improvement

- Compute the improved policy as:

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} \max_{\pi \in \Pi} Q_r^{\pi}(s,a)$$

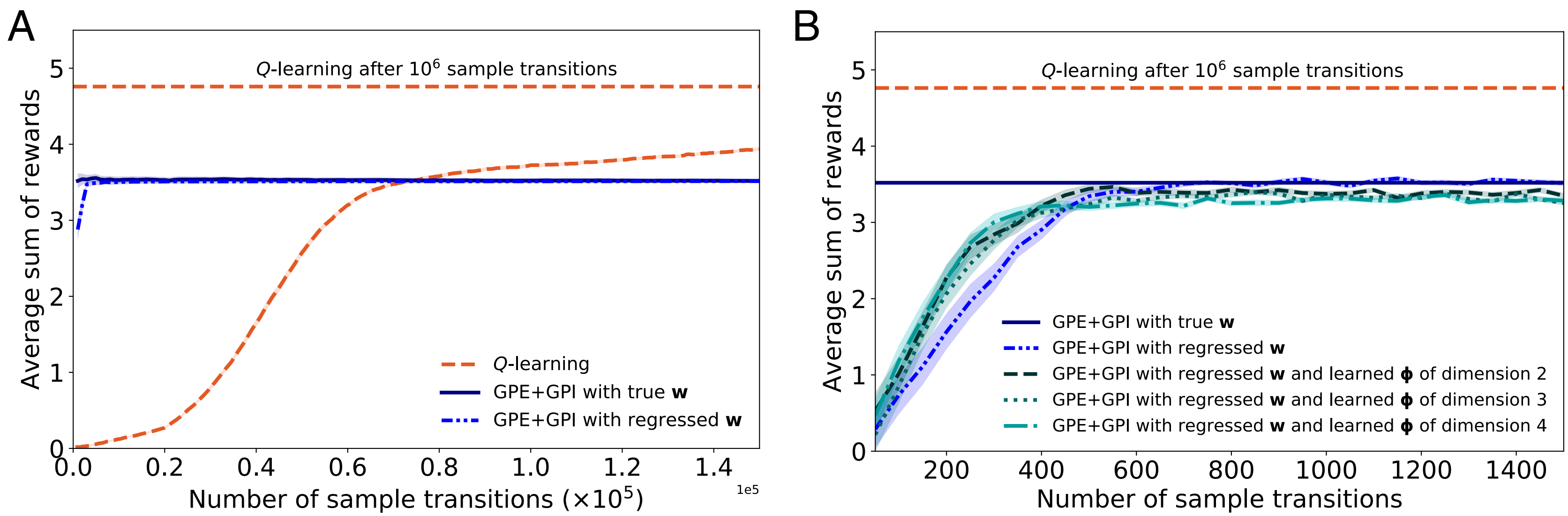
- Note that π' *could choose actions that are not chosen by any of the* π
- The process takes only *one iteration*, after which no further change to the policy π' would happen
- In contrast with iterative policy improvement...

Illustration



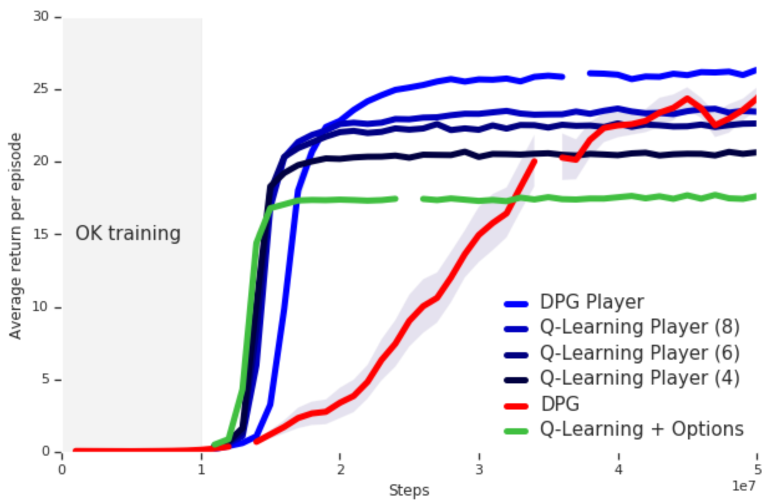
- The three policies correspond to three weight vectors: like red ($\mathbf{w}_1 = [1, 0]^T$), like blue ($\mathbf{w}_2 = [0, 1]^T$) and like red not blue ($\mathbf{w}_3 = [1, -1]^T$)
- *Note that* $\mathbf{w}$ *can be viewed as a preference function over features!*
- We can pre-train the policies that optimize for each preference, and train their successor features as well
- Then just do GPE/GPI!

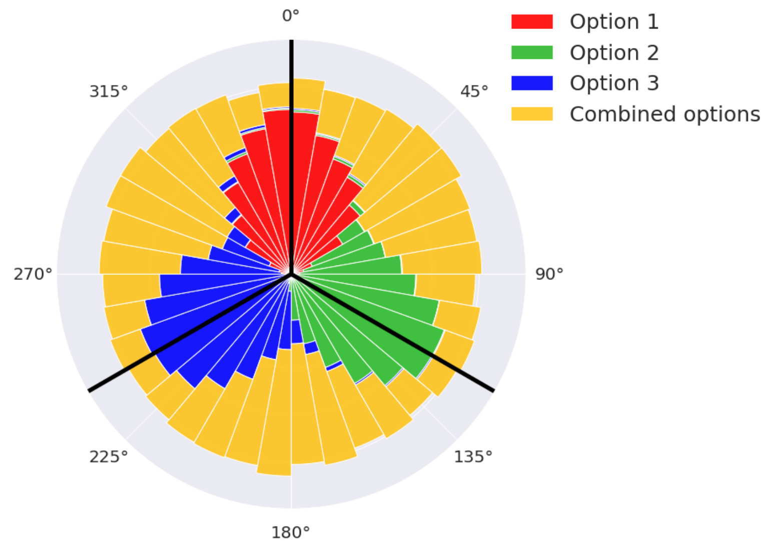
Illustration: Results



- Training the successor features for $\mathbf{w}_1$, $\mathbf{w}_2$ over 5×10^5 samples then GPE/GPI for $\mathbf{w}_3$
- GPE/GPI with successor features achieves *75x improvement in sample size* compared to Q-learning
- Obtaining $\mathbf{w}$, ϕ by learning almost as good as knowing these in advance

Synthesizing new behavior: Moving Target Arena





General way to synthesize quickly new behavior for combinations of reward functions!