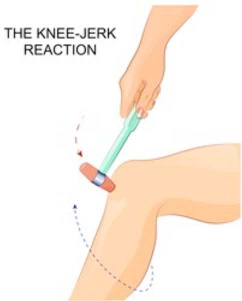
RL: Policy Gradient --
Deterministic Policy Gradient
PPO

How do we decide what to do?

- Emotions/Intuition  $V_t(s)$ $Q_t(s, a)$

- Thinking  $S_{t+1} = M(S_t, A_t, \theta)$

- Reflexes/Habits 

$A_t = \pi(S_t, \theta)$

Policy Approximation

We want to learn this directly!

$$\pi(a|s, \boldsymbol{\theta})$$

Actor-Critic Algorithms

- ACTOR: policy π
- CRITIC: value fct V (or Q)

Policy Gradient Theorem:

$$\nabla_{\boldsymbol{\theta}} J_{\boldsymbol{\theta}}(\pi) = \mathbb{E}_\pi[\sum_{t=0}^{T} (\gamma^t G_t)\, \nabla_{\boldsymbol{\theta}} \log \pi(A_t|S_t)]$$

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Policy Gradient Theorem:

REINFORCE Estimates G with Monte-Carlo

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Actor-Critic Algorithms

- ACTOR: policy π
- CRITIC: value fct V (or Q)

Policy Gradient Theorem:

Actor-Critic: use V and/or Q to estimate G, e.g. TD(0)

$$\nabla_{\boldsymbol{\theta}} J_{\boldsymbol{\theta}}(\pi) = \mathbb{E}_{\pi}[\sum_{t=0}^{T} \left(\gamma^{t} G_{t}\right) \nabla_{\boldsymbol{\theta}} \log \pi(A_t | S_t)]$$

Actor-Critic 1-step TD / TD(0) estimate:

Policy Gradient Theorem:

$$\nabla_\theta J_\theta(\pi) = \mathbb{E}\left[\sum_{t=0}^{T} \gamma^t \left(q_\pi(S_t, A_t) - v_\pi(S_t)\right) \nabla_\theta \log(\pi)\right]$$

$$\underbrace{\qquad\qquad\qquad}_{\text{Advantage}}$$

What about if we want a Deterministic Policy?

We can't use the Policy Gradient Theorem :

$$\nabla_{\boldsymbol{\theta}} J_{\boldsymbol{\theta}}(\pi) = \mathbb{E}_{\pi}[\sum_{t=0}^{T} (\gamma^t G_t) \nabla_{\boldsymbol{\theta}} \log \pi(A_t | S_t)]$$

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$$= \nabla_A Q_\pi \left(A = \pi(S, \theta), S\right) \nabla_\theta \pi(S, \theta)$$

Deterministic Policy Gradient:

How can we estimate $\boldsymbol{\nabla_\theta J_\theta(\pi)}$? When $A = \pi(S, \theta)$

$A = (a_1, \ldots, a_m),\ \pi = (\pi_1, \ldots, \pi_m)$

$$\nabla_\theta J_\theta(\pi \,|\, S_0 = S) \approx \sum_i^m \frac{\partial Q_\pi(A = \pi(S, \theta), S)}{\partial a_i} \nabla_\theta \pi_i(S, \theta)$$

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http://proceedings.mlr.press/v32/silver14.pdf

Deterministic Policy Gradient (on Continuous Control Tasks):

Deterministic Policy Gradient Algorithms

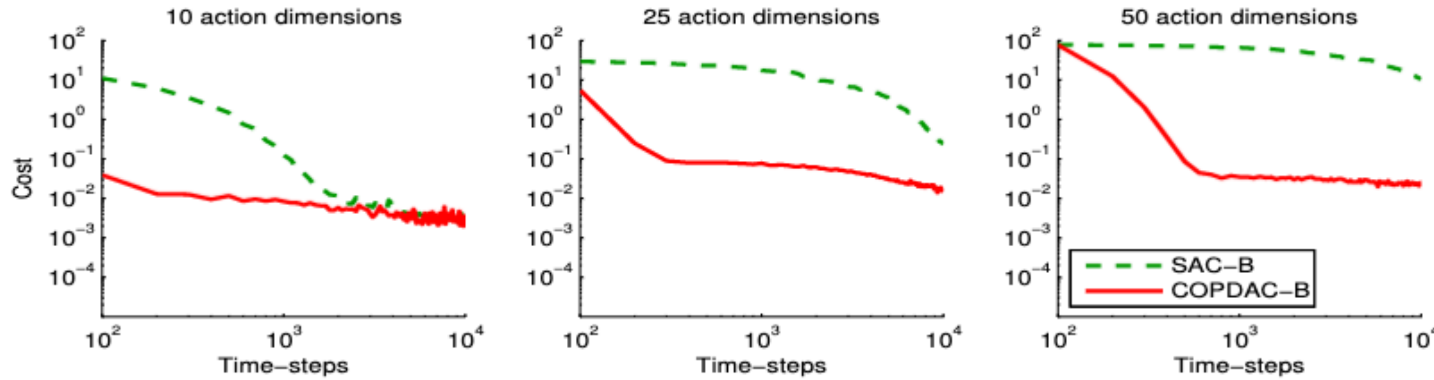
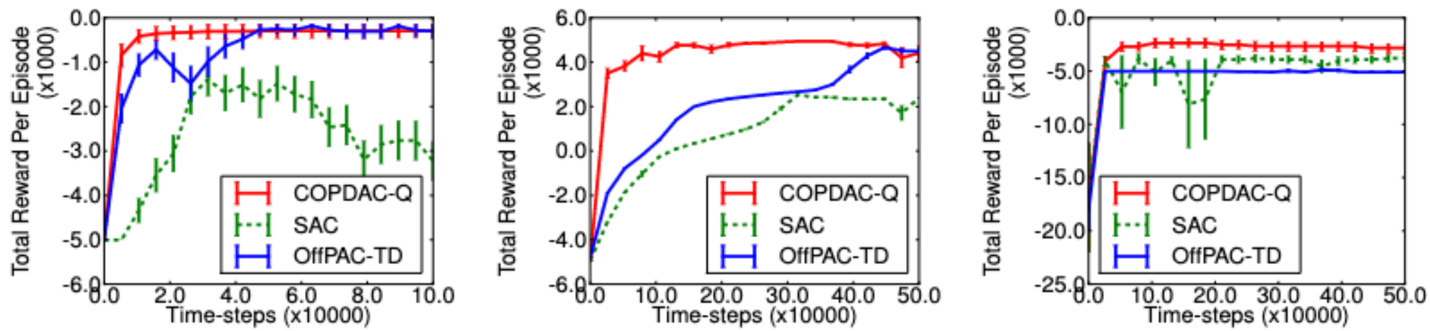


Figure 1. Comparison of stochastic actor-critic (SAC-B) and deterministic actor-critic (COPDAC-B) on the continuous bandit task.



(a) Mountain Car (b) Pendulum (c) 2D Puddle World

Figure 2. Comparison of stochastic on-policy actor-critic (SAC), stochastic off-policy actor-critic (OffPAC), and deterministic off-policy actor-critic (COPDAC) on continuous-action reinforcement learning. Each point is the average test performance of the mean policy.

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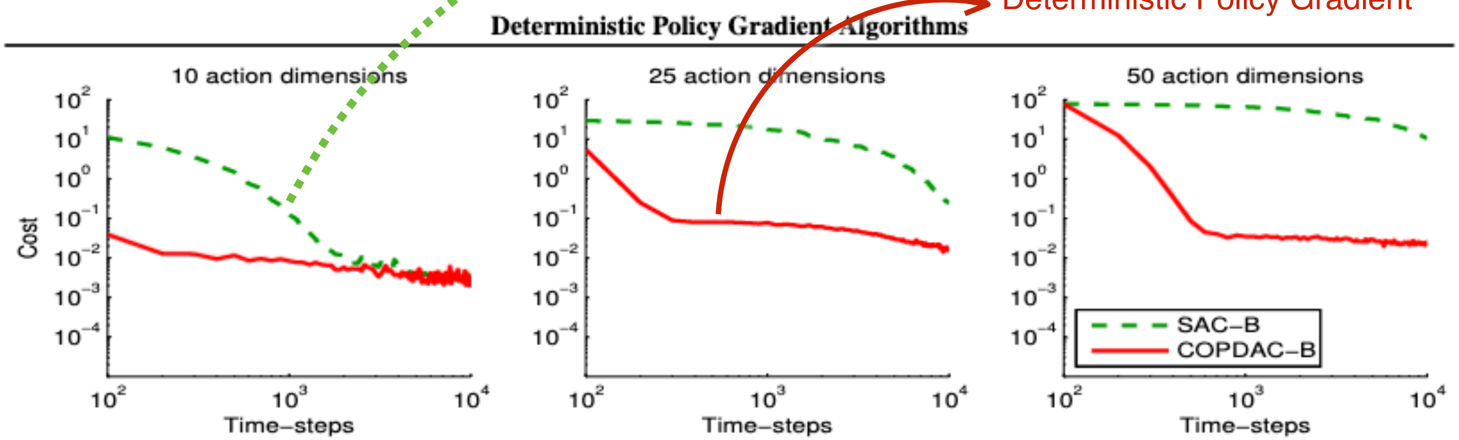
Deterministic Policy Gradient (on Continuous Control Tasks):

Actor Critic with stochastic policy

Deterministic Policy Gradient



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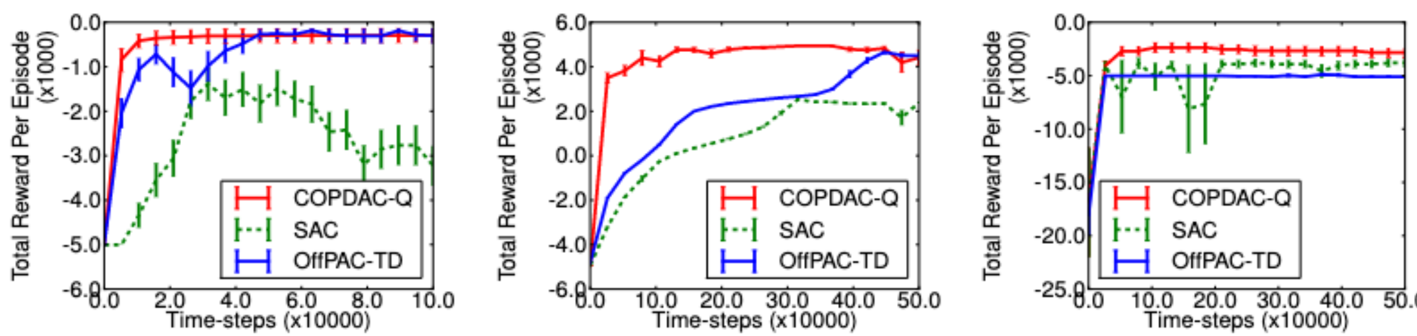

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Deep Deterministic Policy Gradient (DDPG):

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

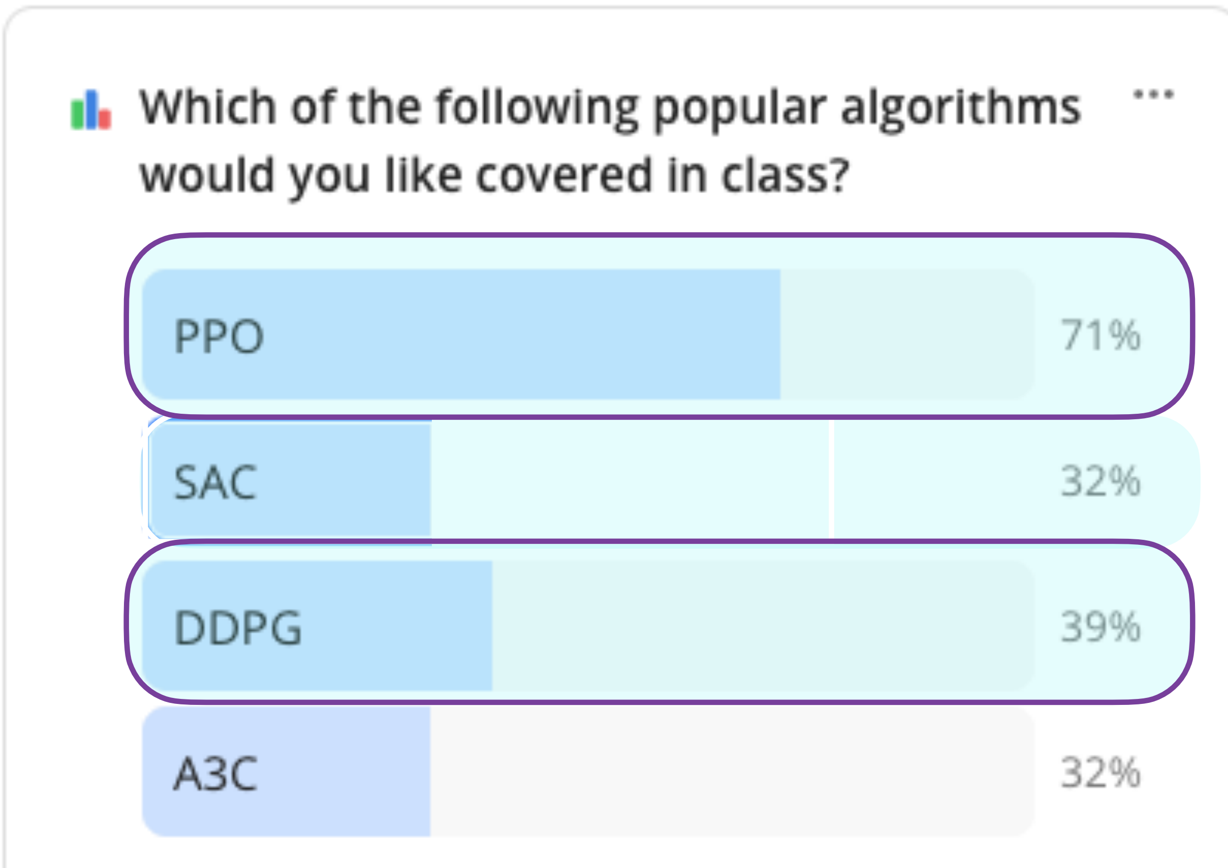
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

https://arxiv.org/pdf/1509.02971.pdf

Conclusion

- Policy Gradient Theorem: $\nabla_{\boldsymbol{\theta}} J_{\boldsymbol{\theta}}(\pi) = \mathbb{E}_{\pi}[\sum_{t=0}^{T} (\gamma^t G_t) \nabla_{\boldsymbol{\theta}} \log \pi(A_t | S_t)]$
- REINFORCE: PGT + MC for estimate of G
- Actor-Critic: PGT + V,Q for estimate of G
- Deterministic Policy Gradient: $\nabla_{\theta} J_{\theta}(\pi | S_0 = S) \approx \nabla_{\theta} Q_{\pi}(\pi(S, \theta), S)$


Which of the following popular algorithms would you like covered in class?
PPO
71%
SAC
32%
DDPG
39%
A3C
32%

TRPO -> PPO

- Trust Region Policy Optimization: https://arxiv.org/pdf/1502.05477.pdf
- Proximal Policy Optimization: https://arxiv.org/pdf/1707.06347.pdf

TRPO:

$$\eta(\pi) = \mathbb{E}_{s_0, a_0, \ldots} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t) \right], \text{ where}$$

$$s_0 \sim \rho_0(s_0),\ a_t \sim \pi(a_t | s_t),\ s_{t+1} \sim P(s_{t+1} | s_t, a_t).$$

$$\textit{Policy Gradient} : \nabla_\theta \eta(\pi)$$

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$$Policy\ Gradient: \nabla_\theta \eta(\pi)$$

$$L_\pi = \eta(\pi)$$

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$$Q_\pi(s_t, a_t) = \mathbb{E}_{s_{t+1}, a_{t+1}, \ldots}\left[\sum_{l=0}^{\infty} \gamma^l r(s_{t+l})\right],$$

$$V_\pi(s_t) = \mathbb{E}_{a_t, s_{t+1}, \ldots}\left[\sum_{l=0}^{\infty} \gamma^l r(s_{t+l})\right],$$

$$A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s), \text{ where}$$

$$a_t \sim \pi(a_t|s_t), s_{t+1} \sim P(s_{t+1}|s_t, a_t) \text{ for } t \geq 0.$$

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The following useful identity expresses the expected return of another policy $\tilde{\pi}$ in terms of the advantage over π, accumulated over timesteps (see Kakade & Langford (2002) or Appendix A for proof):

$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{s_0, a_0, \cdots \sim \tilde{\pi}} \left[\sum_{t=0}^{\infty} \gamma^t A_\pi(s_t, a_t) \right] \qquad (1)$$

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$$L_\pi(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a). \quad (3)$$

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This is an approximation!

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Should be $\rho_{\tilde{\pi}}$ instead of ρ_π

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TRPO says: Approximately valid only if $\tilde{\pi} \sim \pi$

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$$\sum_a \pi_\theta(a|s_n) A_{\theta_{\text{old}}}(s_n, a) = \mathbb{E}_{a \sim q}\left[\frac{\pi_\theta(a|s_n)}{q(a|s_n)} A_{\theta_{\text{old}}}(s_n, a)\right]$$

Our optimization problem in Equation (13) is exactly equivalent to the following one, written in terms of expectations:

$$\underset{\theta}{\text{maximize}}\, \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}, a \sim q}\left[\frac{\pi_\theta(a|s)}{q(a|s)} Q_{\theta_{\text{old}}}(s, a)\right] \tag{14}$$

$$\text{subject to } \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}}\left[D_{\text{KL}}(\pi_{\theta_{\text{old}}}(\cdot|s) \,\|\, \pi_\theta(\cdot|s))\right] \leq \delta.$$

PPO:

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t \left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[r_t(\theta) \hat{A}_t \right].$$

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$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t\right] = \hat{\mathbb{E}}_t\left[r_t(\theta)\hat{A}_t\right].$$

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right]$$

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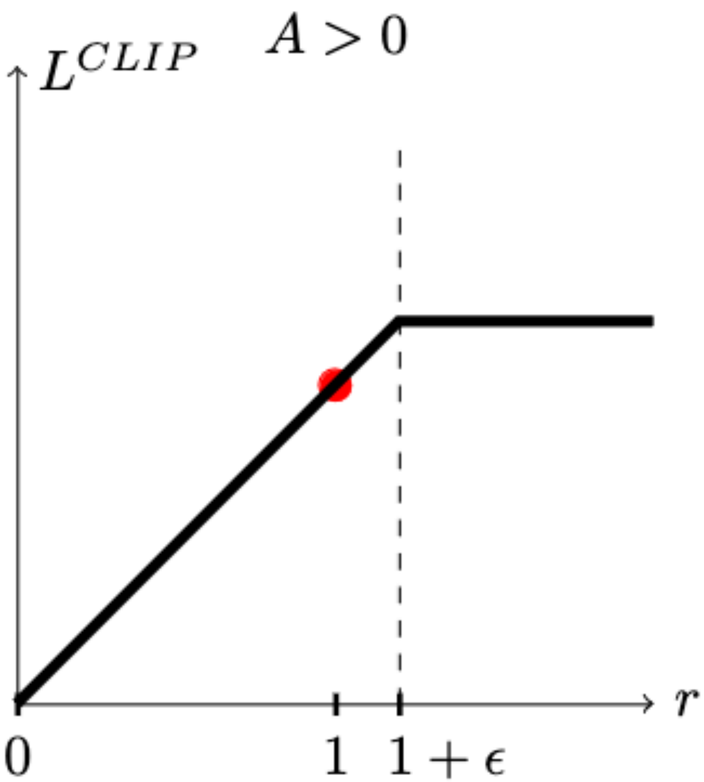

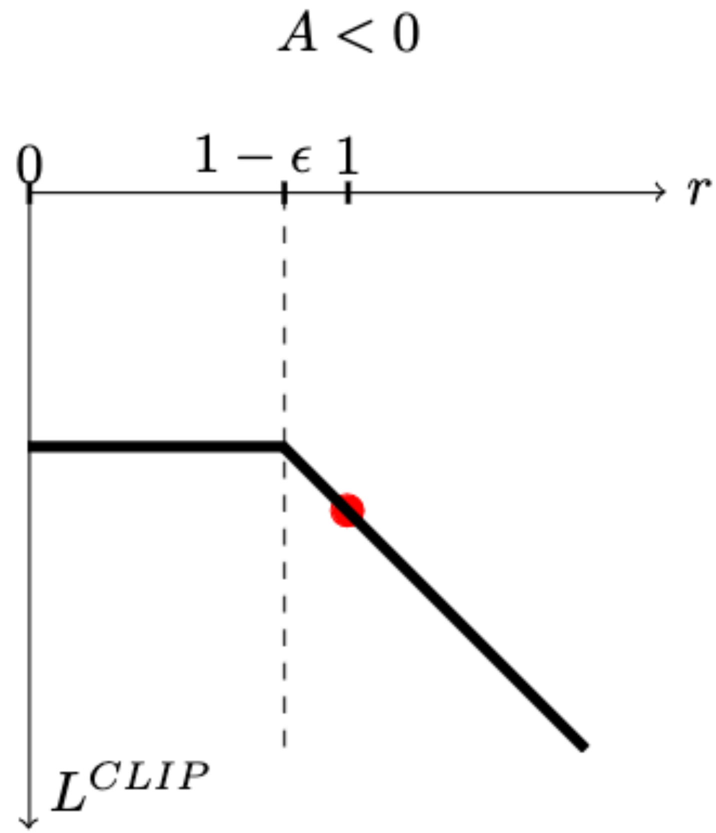

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Algorithm 1 PPO, Actor-Critic Style

```
for iteration=1, 2, ... do
    for actor=1, 2, ..., N do
        Run policy π_θold in environment for T timesteps
        Compute advantage estimates Â_1, ..., Â_T
    end for
    Optimize surrogate L wrt θ, with K epochs and minibatch size M ≤ NT
    θ_old ← θ
end for
```

PPO:

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\Big[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\Big]$$

algorithm	avg. normalized score
No clipping or penalty	-0.39
Clipping, $\epsilon = 0.1$	0.76
Clipping, $\epsilon = 0.2$	**0.82**
Clipping, $\epsilon = 0.3$	0.70
Adaptive KL $d_{\text{targ}} = 0.003$	0.68
Adaptive KL $d_{\text{targ}} = 0.01$	0.74
Adaptive KL $d_{\text{targ}} = 0.03$	0.71
Fixed KL, $\beta = 0.3$	0.62
Fixed KL, $\beta = 1.$	0.71
Fixed KL, $\beta = 3.$	0.72
Fixed KL, $\beta = 10.$	0.69

PPO:

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\Big[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\Big]$$

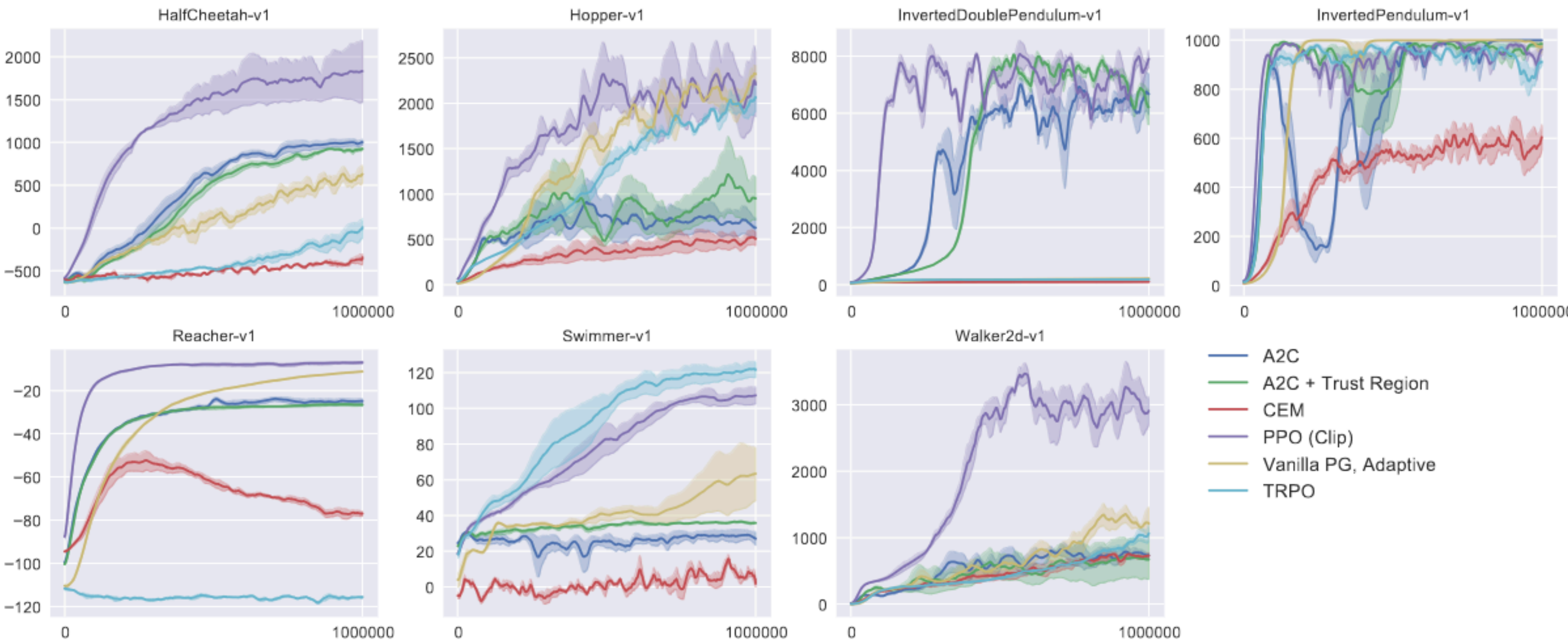

Figure 3: Comparison of several algorithms on several MuJoCo environments, training for one million timesteps.