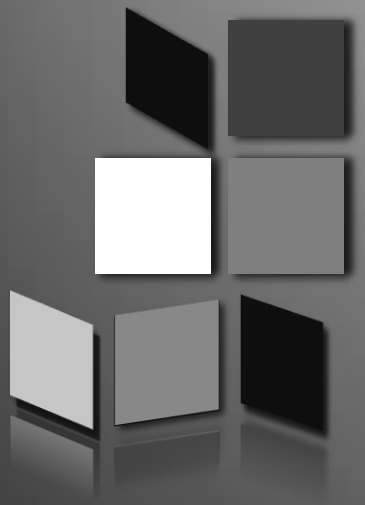

COMP/MATH 553 Algorithmic Game Theory
Lecture 3: Myerson's Lemma

Sep 10, 2014

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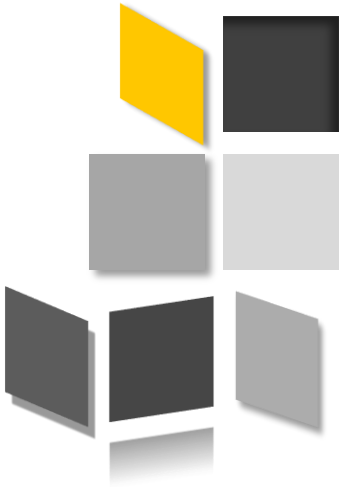
An overview of today's class

- *Case Study: Sponsored Search Auction*
- *Myerson's Lemma*
- *Back to Sponsored Search Auction*



Case Study:

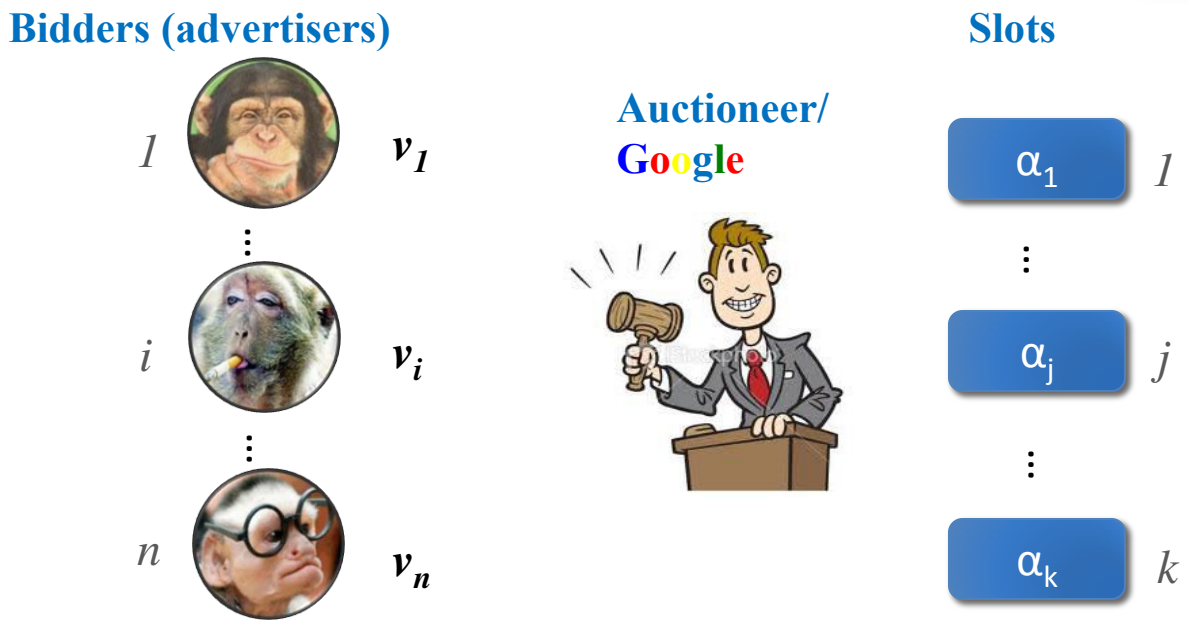
Sponsored Search Auction

Sponsored Search Auction

bing Ads

Google AdWords

Sponsored Search **Auctions: Set-up**



- ***k*** slots for sale.
- Slot ***j*** has click-through-rate (CTR) α_j.
- Bidder ***i***'s value for slot ***j*** is $\alpha_j v_i$.

Sponsored Search Auction: Goal

(1)DSIC. That is, truthful bidding should be a dominant strategy, and never leads to negative utility.

(2) Social welfare maximization. That is, the assignment of bidders to slots should maximize $\Sigma v_i x_i$,

where x_i now denotes the CTR of the slot to which i is assigned (or 0 if i is not assigned to a slot). Each slot can only be assigned to one bidder, and each bidder gets only one slot.

(3) Polynomial running time. Remember zillions of these auctions need to be run every day!

Sponsored Search Auction

- Two things to consider: who wins what and how much to charge?
  - Make the "correct" choice for only the first one is not enough, e.g. single item auction.

- Tackle this one step at a time:

1) Assume that bidders bid truthfully. Then, how should we assign bidders to slots so that property (2) and (3) hold?

2) How do we set prices so that truthful is a dominant strategy?

Sponsored Search Auction

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Greedy Alg. ✔

1) How do we set prices so that being truthful is a dominant strategy?

Can we run k Vickrey auctions?

Sponsored Search Auction

- NO! It's not truthful!
- Example: 3 bidders 2 slots.

  $v_1=7$, $v_2=6$ and $v_3=1$; $\alpha_1=1$ and $\alpha_2=0.4$.

- Instead of being truthful, it's better for bidder 1 to bid 5 and win the second slot.

Sponsored Search Auction

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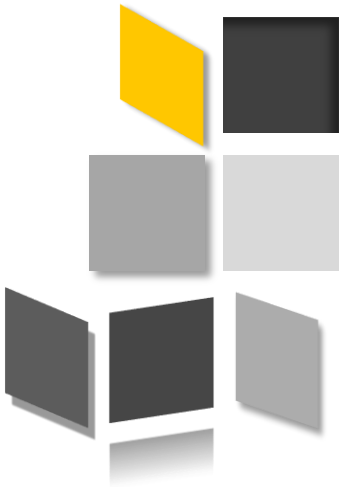
Myerson's Lemma!



Myerson’s Lemma

Single-dimensional Environment

- Definition:
  - n bidders
  - Each bidder ***i*** has a private valuation v_i, its value "per unit of stuff" that she gets.
  - A feasible set X. Each element of X is an n-dimensional vector $(x_1, x_2, \ldots, x_n)$, where x_i denotes the "amount of stuff" given to bidder i.

Single-dimensional Environment

- Examples:
  - Single-item auction: X is the set of 0-1 vectors that have at most one 1
  - k units of the same items for sale: each bidder wants only one item. X is the 0-1 vectors satisfying $\Sigma_i x_i \leq k$.
  - Sponsored search auction: X is the set of n-dimensional vectors corresponding to assignments of bidders to slots. If bidder i is assigned to slot j, then the component x_i equals the CTR α_j of its slot.

Seal-Bid Auction in Single-dimensional Settings

- Make Two choices:
  - Allocation rule
  - Payment rule

Sealed-Bid Auctions:

1. Collect bids $b=(b_1, ..., b_n)$
2. **[allocation rule]** Choose a feasible allocation $x(b)$ in X as ***a function of the bids***
3. **[payment rule]** Choose payments $p(b)$ as ***a function of the bids***.

Two important definitions

Definition 1: (Implementable Allocation Rule) An **allocation rule *x*** for a single-dimensional environment is **implementable** if there is a **payment rule *p*** such the sealed-bid auction ***(x, p)*** is **DSIC**.

- Example: The allocation rule that gives the item to the highest bidder is implementable
- Is the Greedy allocation rule implementable for Sponsored Search Auctions?
- How about giving the item to the second highest bidder? Lowest bidder?

Two important definitions

Definition 2: (Monotone Allocation Rule) An **allocation rule** $\boldsymbol{x}$ for a single-dimensional environment is **monotone** if for every bidder $\boldsymbol{i}$ and bids $\boldsymbol{b_{-i}}$ by the other bidders, the allocation $\boldsymbol{x_i(z,b_{-i})}$ to $\boldsymbol{i}$ is ***nondecreasing in its bid z.***

- Example: The allocation rule that gives the item to the highest bidder is monotone.
- The Greedy allocation rule for Sponsored Search Auctions is monotone.
- Giving the item to the second highest bidder or the lowest bidder is not monotone.

Myerson's Lemma

[Myerson '81  **]** Fix a single-dimensional environment.

(a) An allocation rule x is implementable if and only if it is monotone.

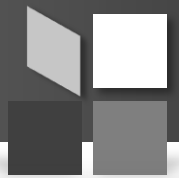
(b) If x is monotone, then there is a unique payment rule such that the sealed-bid mechanism (x, p) is DSIC [assuming the normalization that $b_i = 0$ implies $p_i(b) = 0$].

(c) The payment rule in (b) is given by an explicit formula.

Myerson's Lemma

Corollary: The greedy allocation rule for sponsored search is Implementable. Thus, there is a truthful auction that maximizes social welfare in sponsored search.

Myerson's Lemma



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Proof: See the Board.