The assignment is worth 4% of your final grade. Recall that the marking scheme has been changed to max(20% Assignments + 20% Midterm + 60% Final, 20% Assignments + 80% Final).

Answer two questions out of questions 1 to 5 (questions answered after the second one will not be marked). You may also answer the bonus question for extra marks but you cannot receive partial marks for the bonus question.

1. You are on a strange island where all inhabitants are either knights or knaves. Knights always tell the truth and knaves always lie. From what they say, write down propositions which must be true. Each variable should represent a statement of the form "Nameofperson is a knight". Then, determine what type each person is. If this is impossible to determine, give two possibilities for what they could be. If it is possible to determine their type, give a proof using rules of inference.

   For example, if you've determined that they are all knights, conclude $(p \wedge q) \wedge r$ from the propositions you have written. To shorten the proofs, for this question, you may use the following "special" inference rules.

Rule	Name
$\dfrac{\begin{array}{c} P \\ P \leftrightarrow Q \end{array}}{Q}$	special $\leftrightarrow \mathcal{E}$
$\dfrac{\begin{array}{c} Q \\ P \leftrightarrow Q \end{array}}{P}$	special $\leftrightarrow \mathcal{E}$
$\dfrac{\begin{array}{c} \neg P \\ P \leftrightarrow Q \end{array}}{\neg Q}$	special $\leftrightarrow \mathcal{E}$
$\dfrac{\begin{array}{c} \neg Q \\ P \leftrightarrow Q \end{array}}{\neg P}$	special $\leftrightarrow \mathcal{E}$

   (a) (3 points) Paul tells you that Ryan is a knight. Quincy then says "Paul is lying". Ryan says "Paul and Quincy are both knights".

   (b) (3 points) Pamela says something but you did not hear it. Quentin notices this and states that Pamela said she is a knave. Roland says "Quentin is lying".

   (c) (4 points) Pat says "It is not true that at least one of Quinn and Rita are knights". Quinn claims that Pat and Rita are the same type. Rita says "Quinn and I are both knaves."

2. For each of the following arguments, rewrite it using propositional logic. Determine if the argument is valid. If it is valid, give a proof using rules of inference. If it is invalid, give a set of values for the propositions you have defined which makes all premises true

but the conclusion false. Then, write down what these propositions with these values mean in English.

Make no assumption about the truth value of individual proposition variable (e.g., do not assume that $1 + 1 = 2$).

(a) (3 points)

If it is cloudy, then I have my umbrella whenever it is raining.

If a cloudy sky implies I have my umbrella then a cloudy sky also implies that it is raining.

(b) (3 points) Let n be a fixed integer.

If n is prime then $n + 2$ is prime.
$n + 2$ is prime if $n + 4$ is prime.

If $n + 4$ is prime then n is prime.

(c) (4 points)

I have an account.

Either I know my password whenever I can log in or I can log in whenever I know my password (or both).

3. For each of the following arguments, rewrite it using propositional logic. Determine if the argument is valid. If it is valid, give a proof using rules of inference. If it is invalid, give a set of values for the propositions you have defined which makes all premises true but the conclusion false. Then, write down what these propositions with these values mean in English.

Make no assumption about the truth value of individual proposition variable (e.g., do not assume that $1 + 1 = 2$).

(a) (3 points)

The gostak distims the doches.

If the gostak is in the delcot then the gostak distims the doches.

(b) (3 points)

If there is a polynomial time algorithm for 3-SAT then there is a polynomial time algorithm for 3-colouring.

Either there is no polynomial time algorithm for 3-SAT or there is a polynomial time algorithm for 3-colouring (or both).

(c) (4 points)

It is safe to silflay when there aren't hrair hombil outside.
If there are hrair hombil outside then the hrududu is embleer.
The hrududu is not embleer.

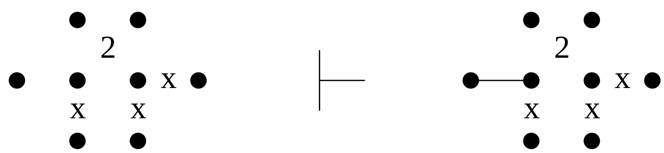
It is safe to silflay.

4. (a) (3 points) Draw the truth table for $\neg(p \wedge q)$ and $(\neg p \vee q)$.

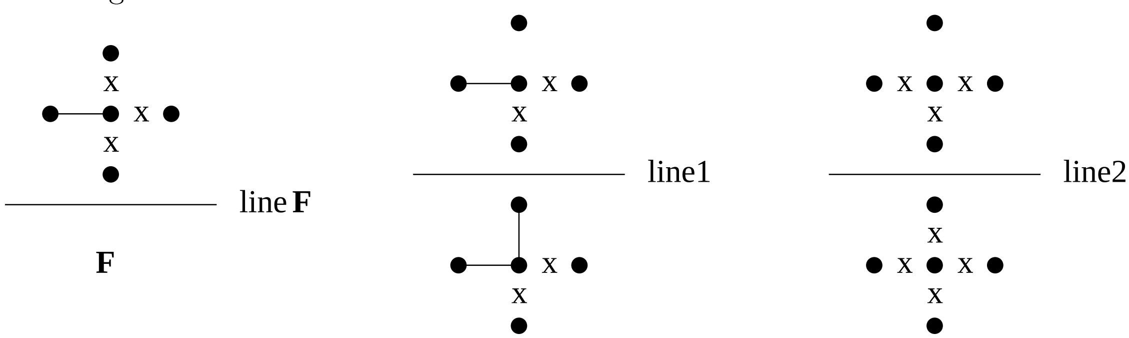
   (b) (3 points) Prove $\neg p \vee \neg q \vdash \neg(p \wedge q)$ using inference rules.

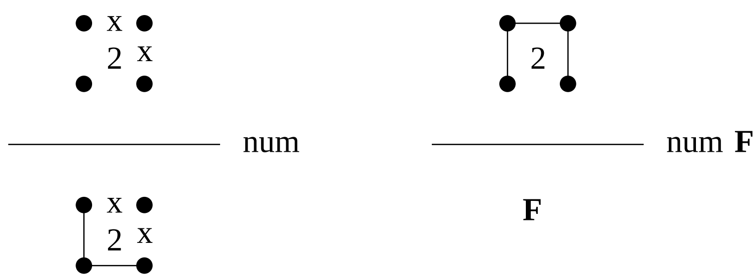
   (c) (4 points) Show that the rule of inference $\neg\mathcal{I}$ (negation introduction) is redundant. That is, prove $p \rightarrow \mathbf{F} \vdash \neg p$ using only the other rules of inference.

5. (a) (5 points) Prove the following slitherlink theorem. That is prove that if the configuration to the left of $\vdash$ appears **in the middle of some board** then we can deduce the extra line on the left. For example, you may not make assumptions about where the boundary of the board are.



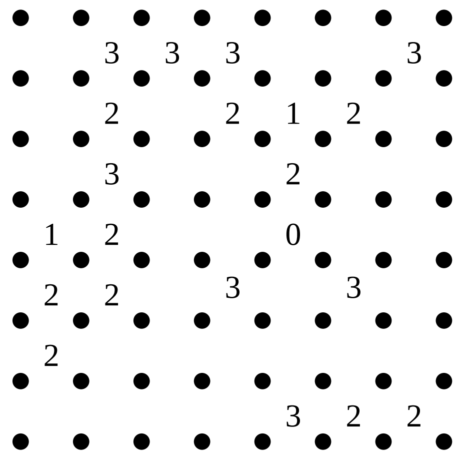
You may only use rules of inference of propositional logic given in class and the following rules.





Only one "representative" per rule is drawn and here is what they are meant to represent (if the diagrams are already clear, there's no need to read this). The "line**F**" rule states that if there is exactly 1 or 3 lines around a point then we can infer a contradiction. The "line2" rule states that if there are 3 non-lines (X's) around a point then there is a non-line in the fourth direction around that point. The "line2" rule states that if there is one line and two non-lines around a point then there is another line around that point. The "num" rule states that if there is already enough non-lines or around a number then we can complete the rest with lines (and vice versa). "num**F**" states that if there are too many lines or non-lines then we can infer a contradiction.

(b) (5 points) Solve the following Slitherlink puzzle.



6. (Bonus) (5 points) Prove that the rules of inference we have seen in class are sound. That is, prove that if $P_1, \ldots, P_k \vdash Q$ then $P_1, \ldots, P_k \models Q$ where $P_1, \ldots, P_k, Q$ are propositional formulas.