Private Information via the Unruh Effect

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joint work with
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- If we are going to use quantum communication on a large scale, relativistic effects are essential.
- Relativistic effects in classical information theory had already been investigated as early as 1981.

Early Work

- Jarrett and Cover 1981: Relativistic classical information theory.
- Relativistic effects on transmission rates and energy requirements.
- Closely related to time dilation: special relativity.

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- Entanglement fidelity is preserved even though the finite dimensional Lorentz transformations are not unitary.
- Remarks on the effect of Unruh or Hawking radiation.

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We decided to investigate the information-theoretic properties of the Unruh effect.

Outline

- Review of quantum field theory: a biased view.
- QFT in curved spacetimes: the Unruh effect.
- Private capacity and quantum private capacity.
- Private information via the Unruh effect.

Geometrical Classical Mechanics

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$$\frac{\mathrm{d}A}{\mathrm{d}t} = \{A, H\}$$

Dynamics

Quantum Mechanics Recap

1. States are rays in a Hilbert space
2. Measurements are described by hermitian operators...
3. Evolution is given by a particular unitary operator $\exp(-iHt)$
4. The algebra of observables is non-commutative and is given by Dirac's rule

$$\{P, Q\} \longrightarrow [P, Q]$$

Wave Equations

What is the precise dynamical law?

Figure out H (and get Nobel prize) then time evolution is given by:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

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$$a|n\rangle = \sqrt{n}|n-1\rangle, \;\; a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$[a, a^\dagger] = 1, \;\; H = \hbar\omega(a^\dagger a + \frac{1}{2})$$

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A negative energy electron may be kicked upstairs and become an ordinary electron leaving a "hole". The hole will behave just like a positively charged electron: a positron.

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The mathematical complexity rises a whole level beyond that of ordinary quantum mechanics.

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$$\Omega(\phi_1,\phi_2)=\int_\Sigma(\phi_1\vec{\nabla}\phi_2-\phi_2\vec{\nabla}\phi_1)\cdot d\vec{\sigma}$$

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The vacuum is the state killed by all the a operators.

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One needs the canonical Fourier transform that one has in a flat spacetime.

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The "harmonic oscillators" give the creation and annihilation operators of QFT.

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- The notion of "particle" is not absolute.
- Particles may appear out of the vacuum: Leonard Parker, Stephen Hawking and Bill Unruh.
- Particles are a useful abstraction when talking about detectors coupled to quantum fields.

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We will use the classical data to guide the construction of the QFT.

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An abstract $*$-algebra can be represented as a concrete collection of operators on a Hilbert space: $*$-representation.

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Impose the Dirac condition:

$$[F[\phi], F[\psi]] := F[\phi]F[\psi] - F[\psi]F[\phi] = \Omega(\phi, \psi)$$

How should the the abstract $*$-algebra be realized as as operators on a Hilbert space?

We should have a Fock space built out of V, the classical solutions.

How can we make the real vector space V into a complex vector space?

Look for a *complex structure*: $J : V \to V$

$$J^2 = -I$$

But what *physical idea* will motivate the choice of J?

Polarizations and Complex Structures

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$$\overline{\phi^{(+)}} = \phi^{(-)}$$

Complex Structure $\equiv$ Polarization

$$J\phi = i\phi^{(+)} - i\phi^{(-)}$$

$$P^{(+)}\phi = -\frac{i}{2}[J\phi + i\phi]$$

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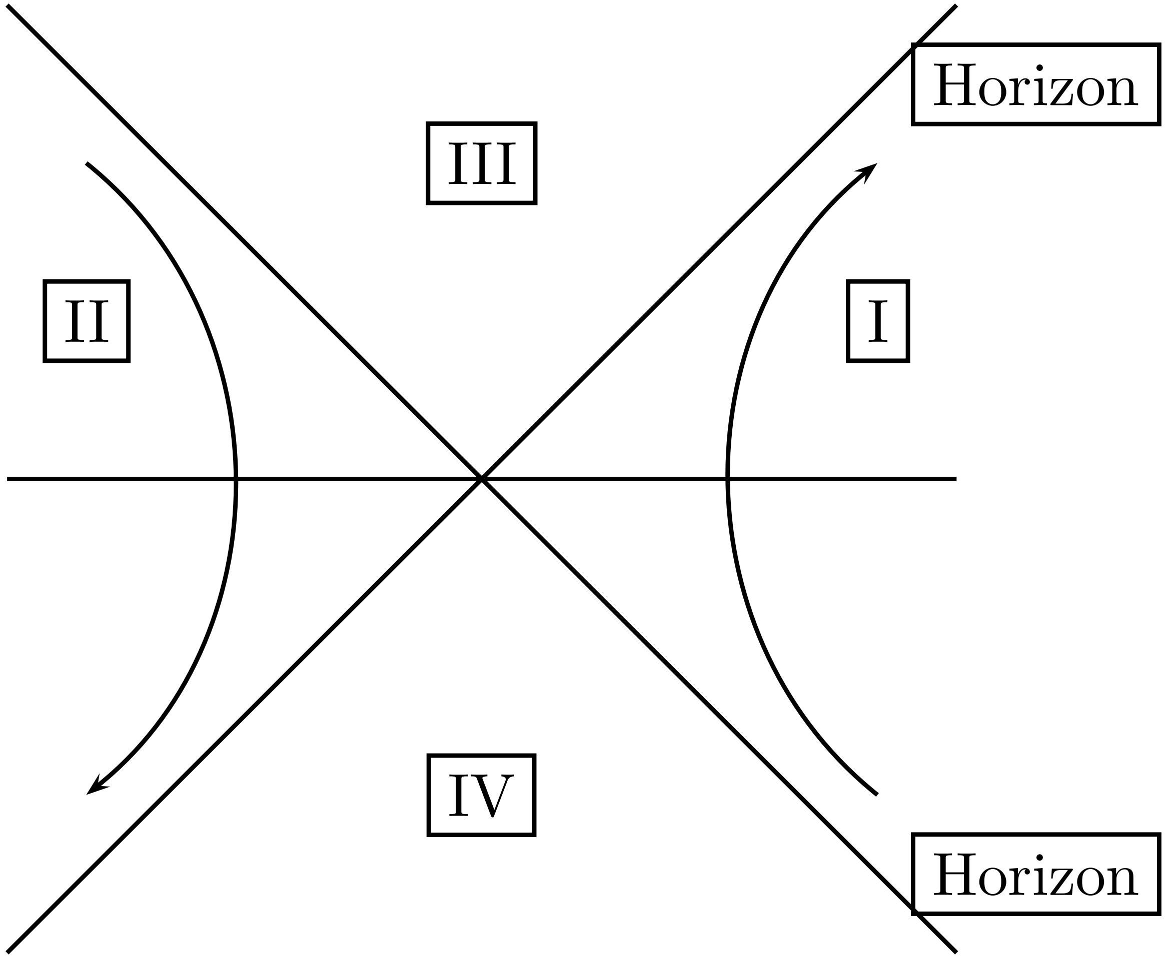
Choosing a decomposition into positive and negative frequencies is equivalent to choosing a complex structure.

In curved spacetime we have have no canonical choice of complex structure.

Hence no canonical choice of positive and negative frequencies.

Hence, one observer's vacuum may not be another observer's vacuum.

Thus there is a transformation from one observer's Fock space to another's.



Rindler spacetime.

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There will be modes corresponding to the inaccessible region,

so the accelerating observer's density matrix will involve a partial trace over the modes of the inaccessible region.

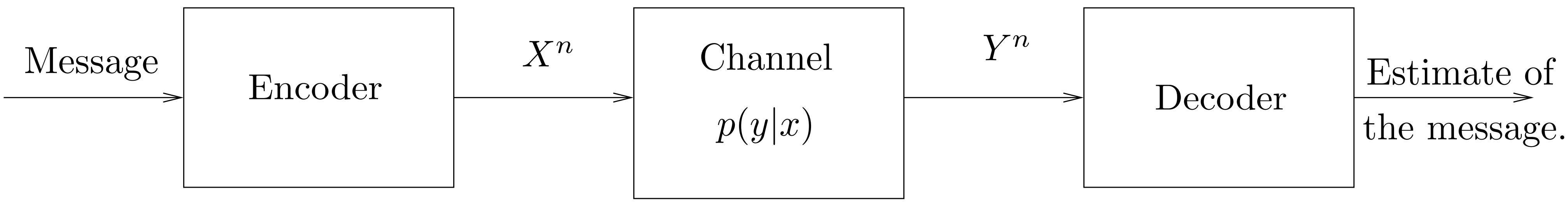
Unruh Effect

The inertial observer's vacuum will look like a bath of thermal radiation to the accelerating observer.

The notion of "particle" is not absoute:

it only refers to the effects of a detector interacting with a field.

Channels



A typical channel.

How well can we estimate the intended message if the channel is noisy?

Channel Capacity

- The basic measure of information transmission.
- Shannon's coding theorem: All transmission rates below the capacity are achievable with asymptotically zero probability of error.

What is a quantum channel?

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where the A_i are linear maps
and $\sum_i A_i^\dagger A_i = I$

von Neumann Entropy

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Holevo bound: χ is an upper bound on accessible information in ρ.

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- Sending classical data: choose a basis to represent classical data and encode classical data in a quantum state. Bob has to extract the classical data from the quantum state.
- Sending quantum data: Alice wants to send the whole quantum state.

Quantum Channels 2

- New possibility: If Alice uses multiple copies of the channel she could entangle the quantum states across multiple uses of the channel.
- We do not know how to compute the capacity in this case!

Quantum Channels 3

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$C^{(1)}(\mathcal{E})$ the one-shot capacity

In this case we have the Holevo-Schumacher-Westmoreland theorem, which gives us a "formula" for the capacity.

The Holevo-Schumacher-Westmoreland Theorem

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$$C^{(1)}(\mathcal{E}) = \max_{(p_j,\rho_j)} [H(\mathcal{E}(\sum_j p_j \rho_j) - \sum_j p_j H(\mathcal{E}(\rho_j))]$$

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I will spare you hideous formulas in what follows!

Private Capacity

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What is the private capacity of a quantum channel for communicating quantum data? [Hayden et al. in progress]

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 & \cdot \arrow[r] & B \\
A \arrow[ur] \arrow[dr] & & \\
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\end{tikzcd}$$

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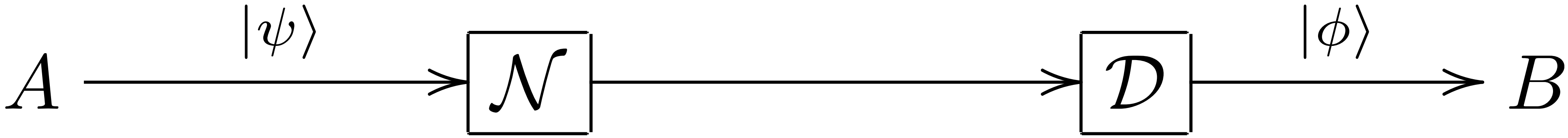
An (n, ϵ) private channel code
of rate R allows Alice to send one of 2^{nR}
messages,Bob can decode with error less than ϵ
and Eve cannot find out more than ϵ bits.

Private Quantum Communication

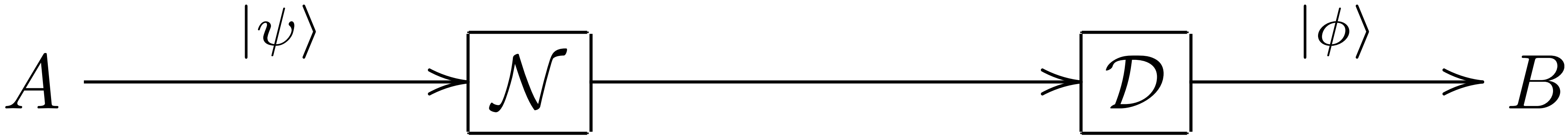
Private Quantum Communication



Noisy channel

Decoder

Private Quantum Communication



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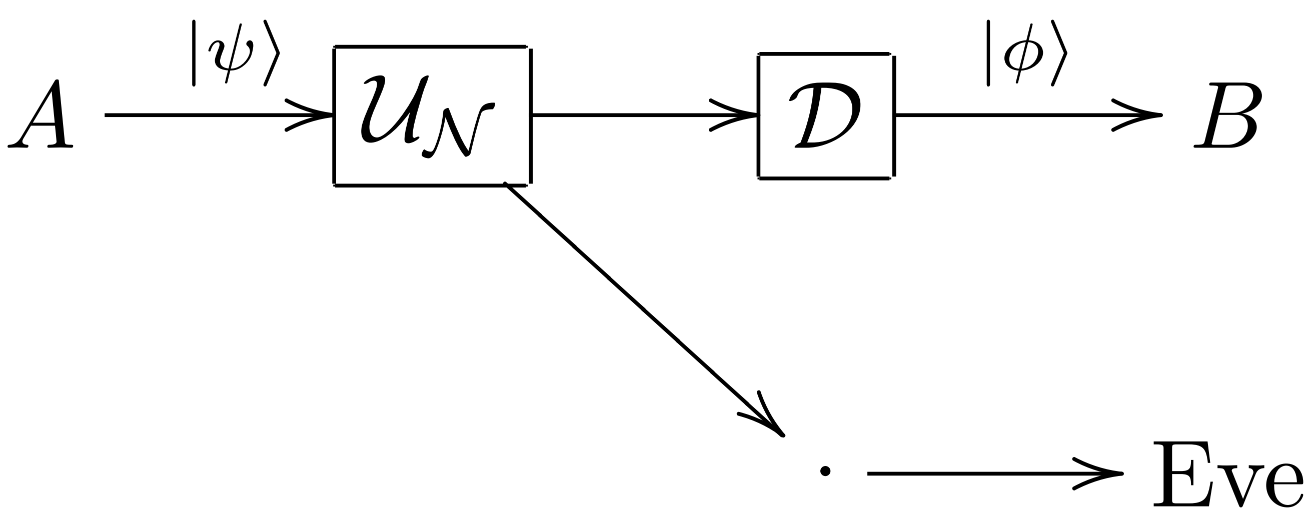
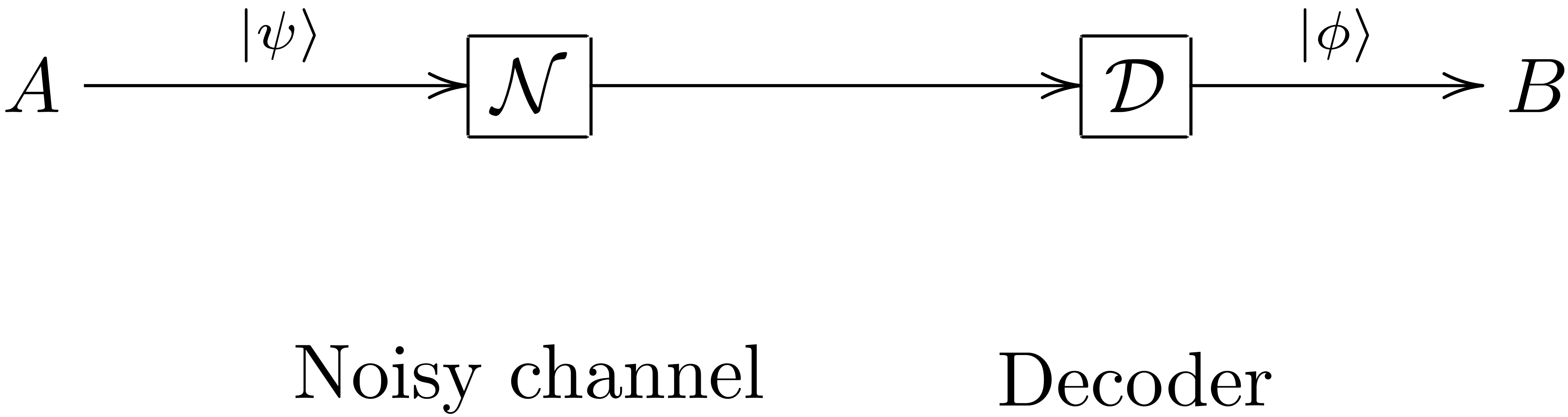
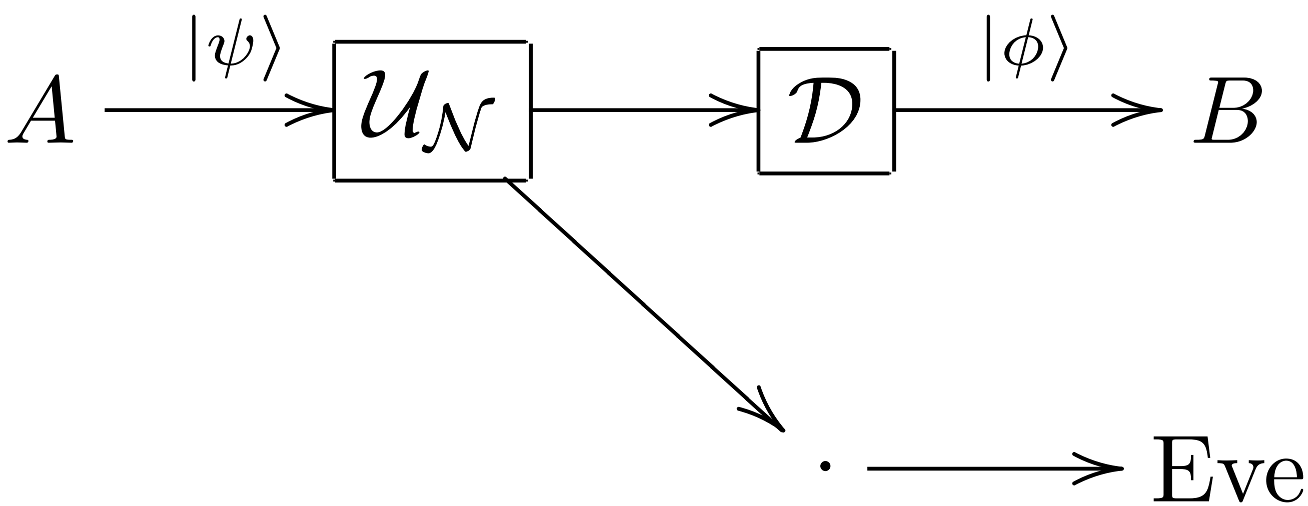


Private Quantum Communication





Eve cannot get a copy of ϕ: automatic privacy.

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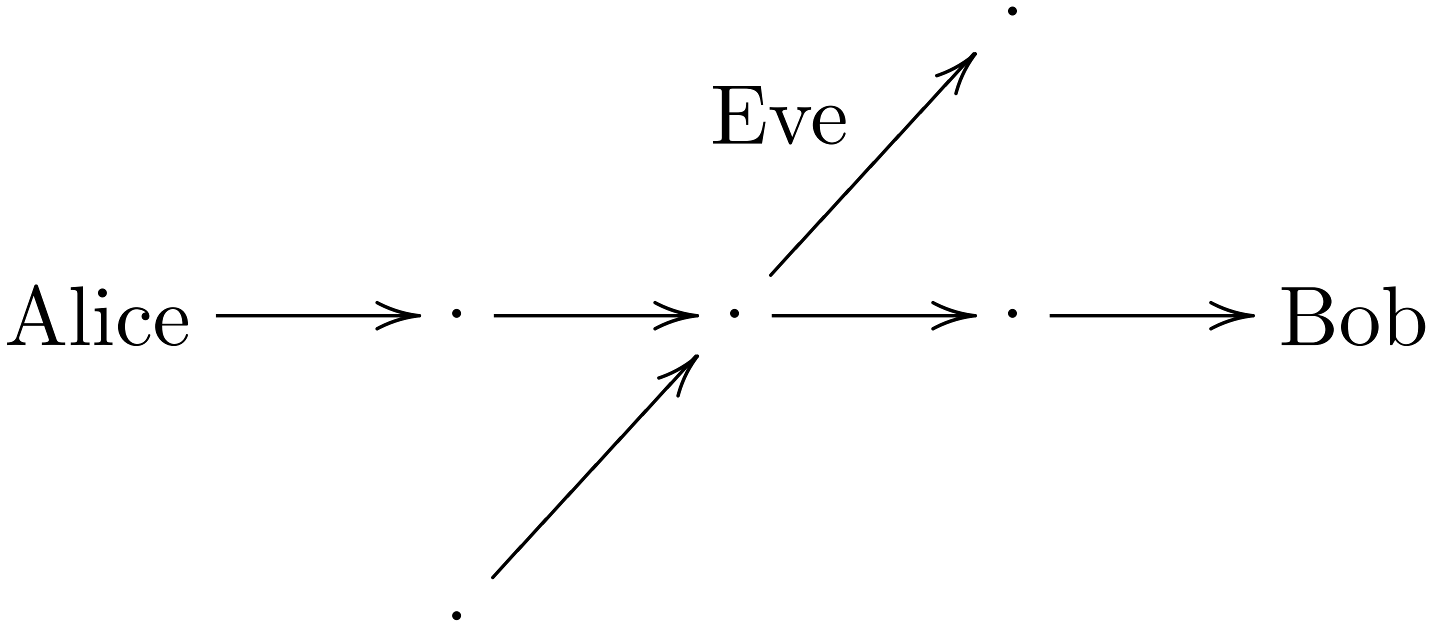
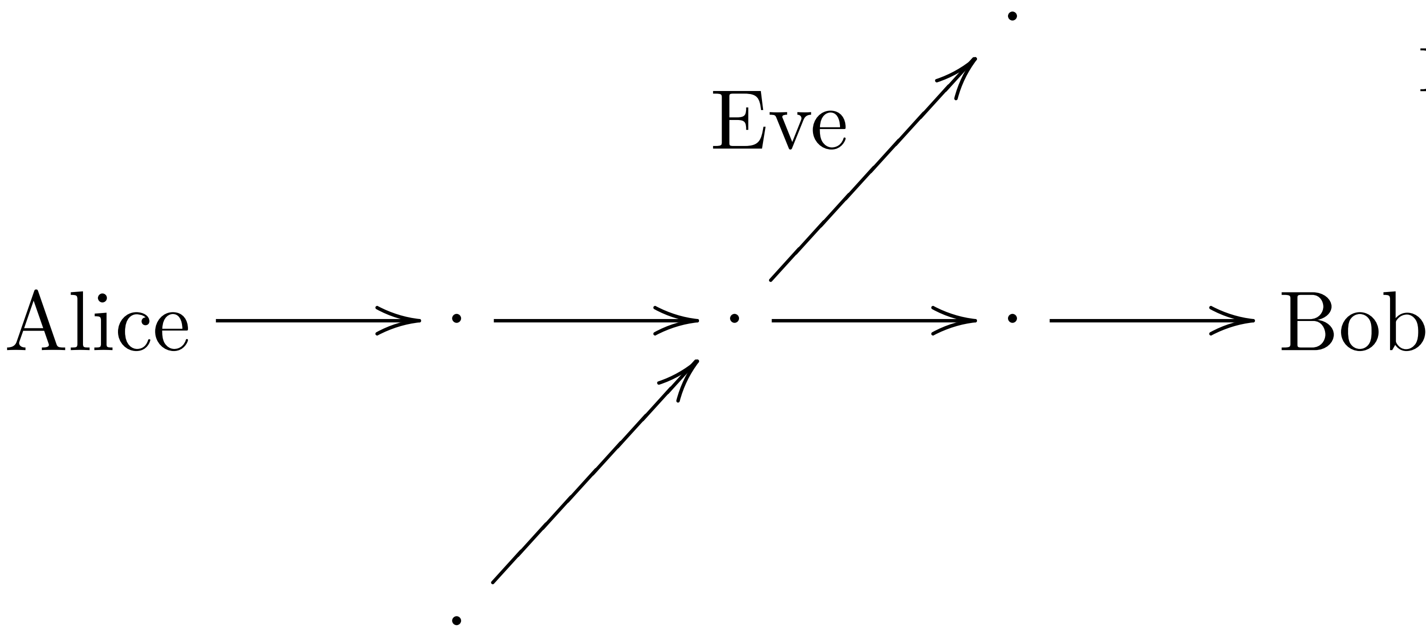
- What is the private capacity for Alice to Bob? Can we use the Unruh noise?


Eve
Alice
Bob

Eve intercepts.



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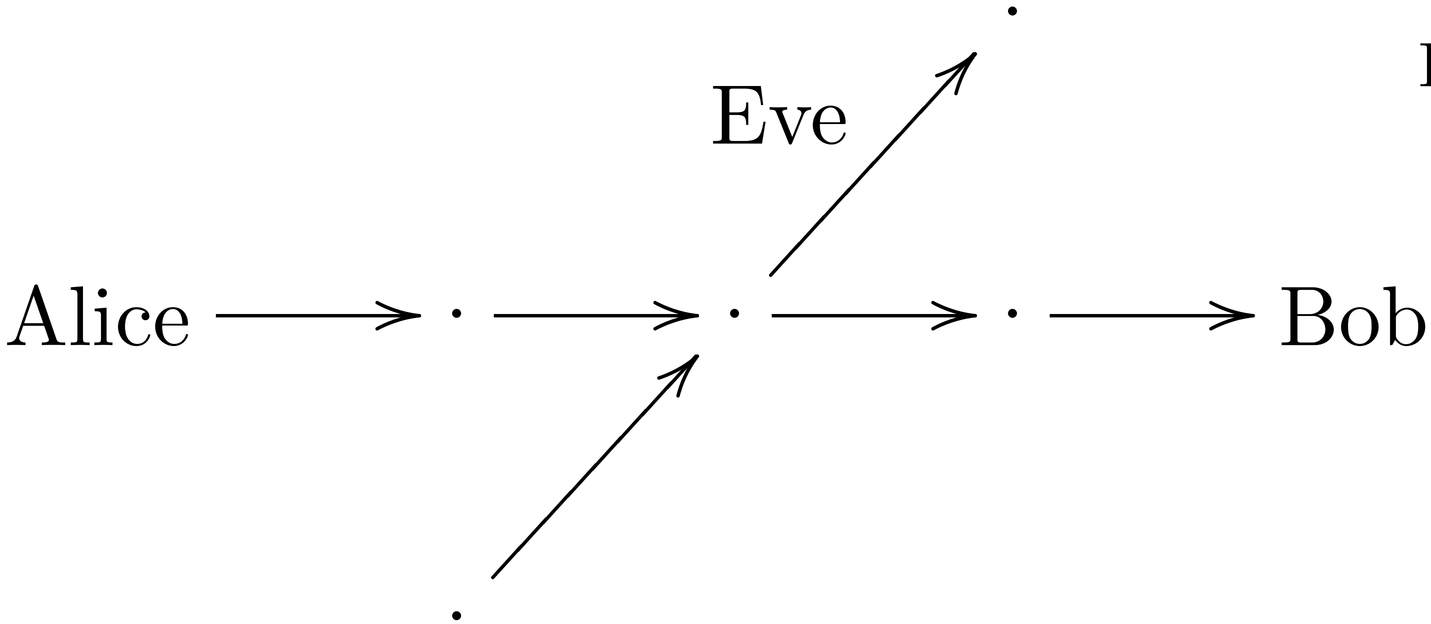

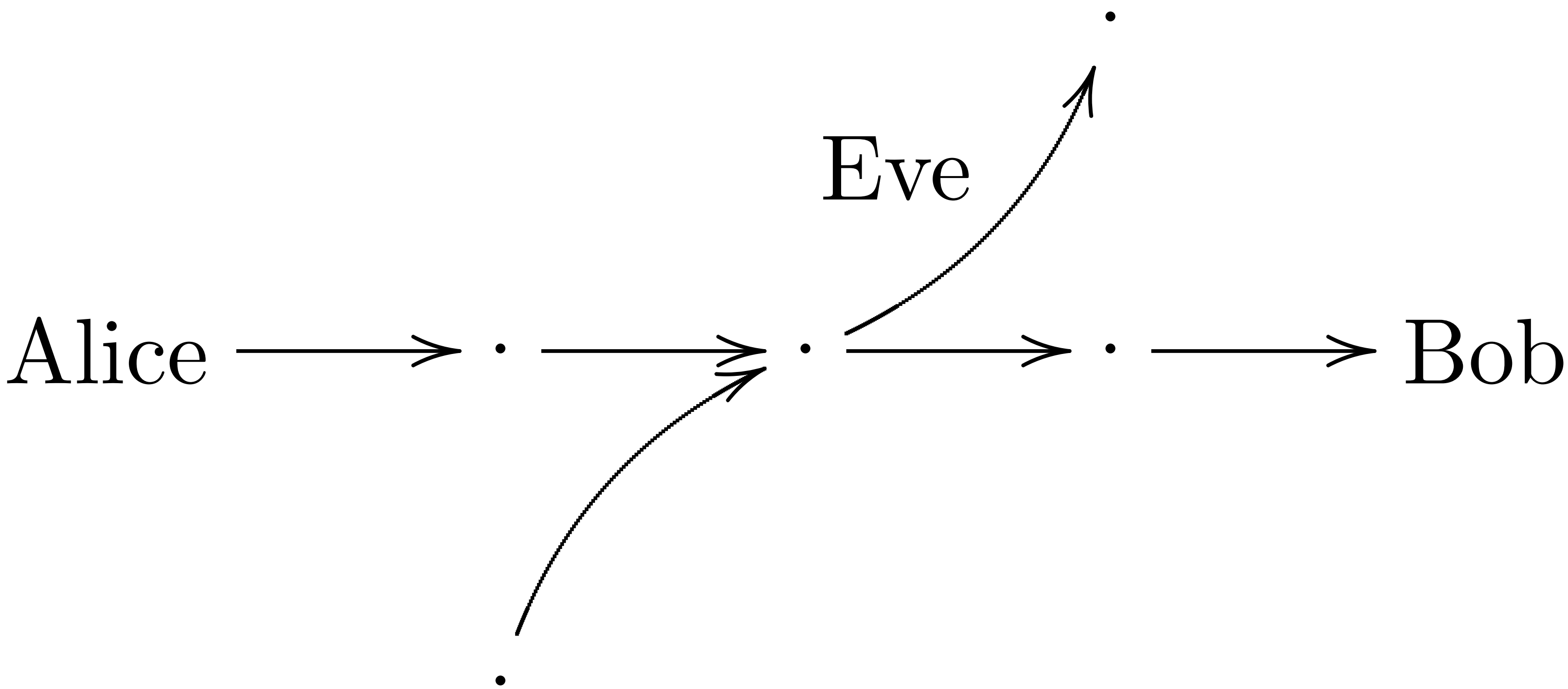

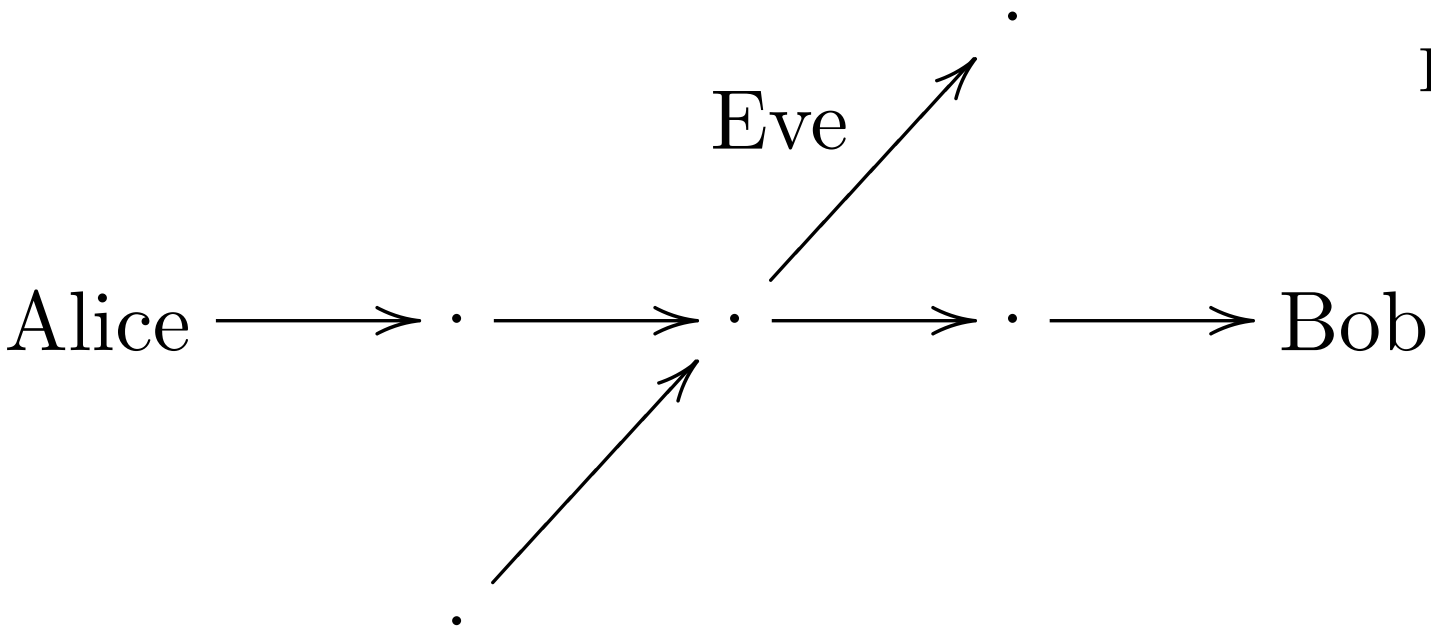

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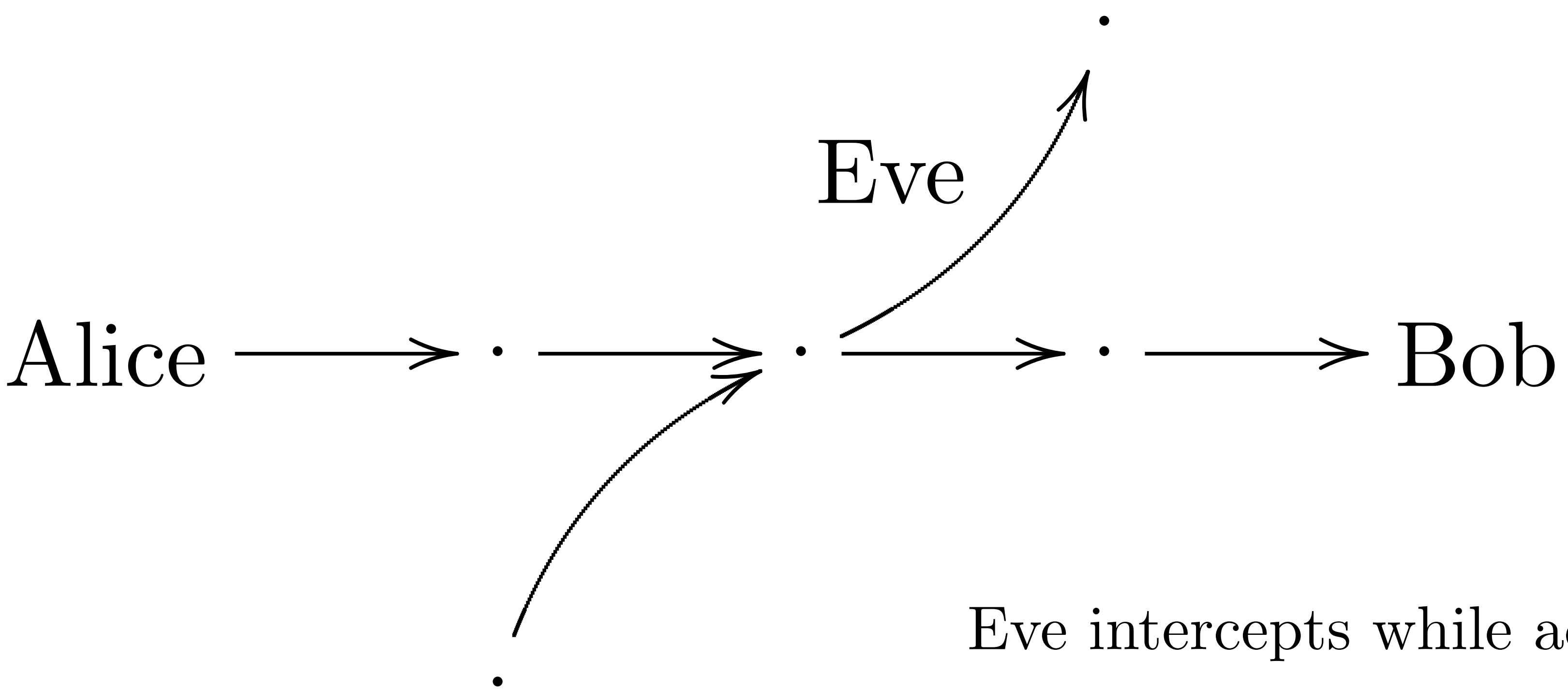

Eve intercepts while accelerating.

$$\text{Alice} \longrightarrow \mathcal{N} \longrightarrow \text{Bob}$$

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$$\text{Alice} \longrightarrow \mathcal{E} \longrightarrow \text{Eve}$$

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Eve is not a "part" of the environment $[Eve \not\subseteq Env]$

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Does the Unruh effect give a channel from Alice to Bob with nonzero quantum and classical private capacity?

Alice

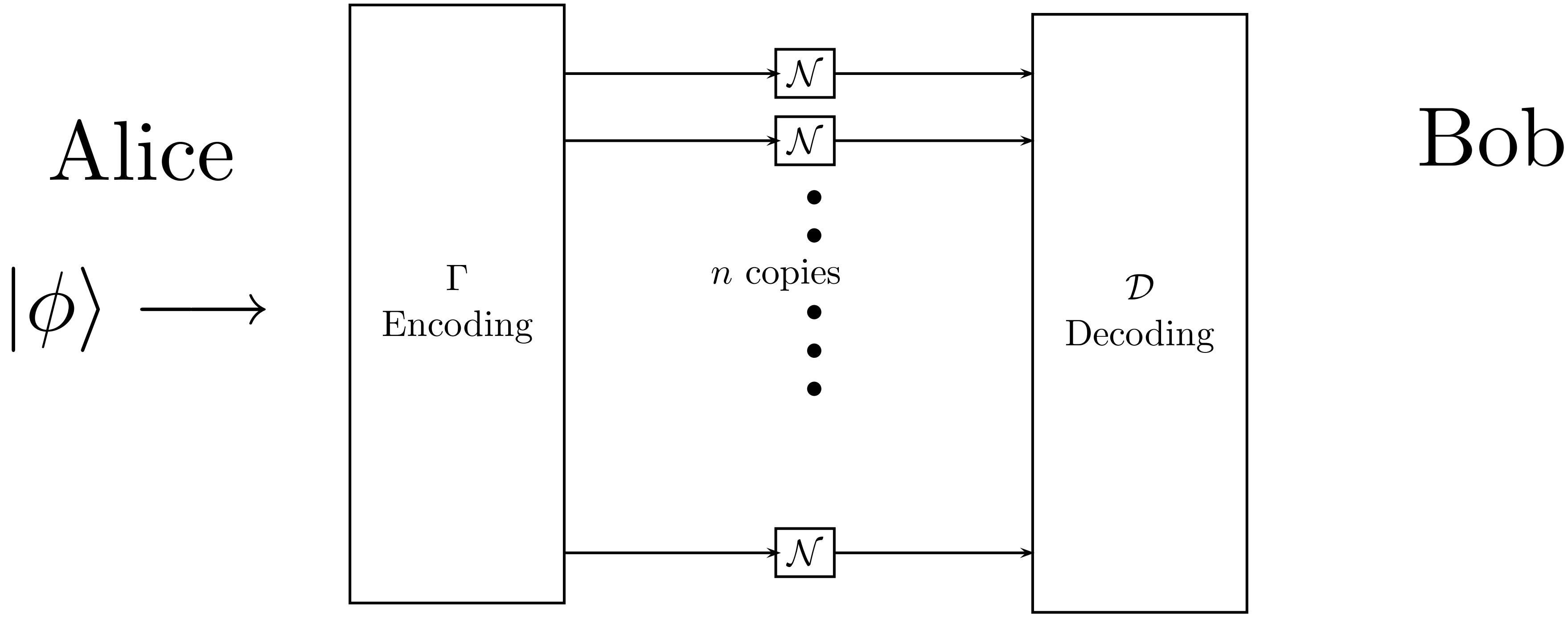
Alice

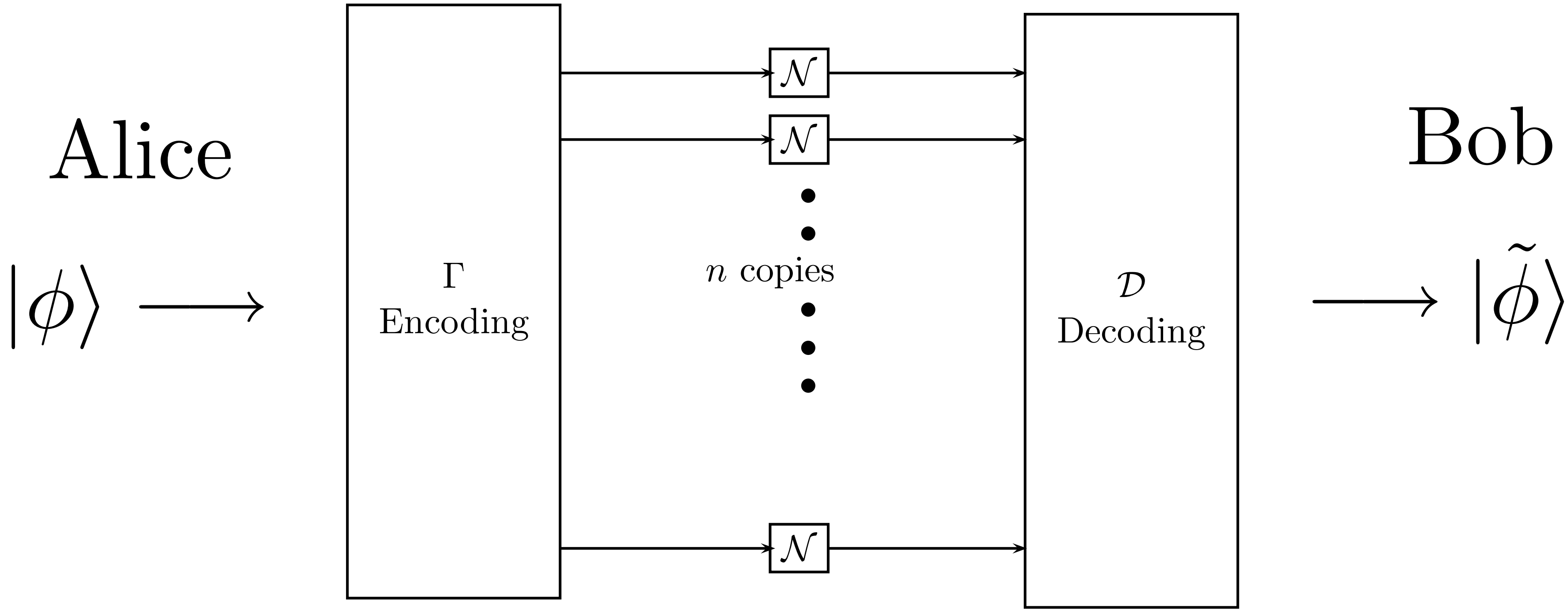
Bob

Alice

Bob

$|\phi\rangle \longrightarrow$


Alice
Bob
$|\phi\rangle \longrightarrow$
Γ
Encoding
$\mathcal{N}$
$\mathcal{N}$
n copies
$\mathcal{N}$
$\mathcal{D}$
Decoding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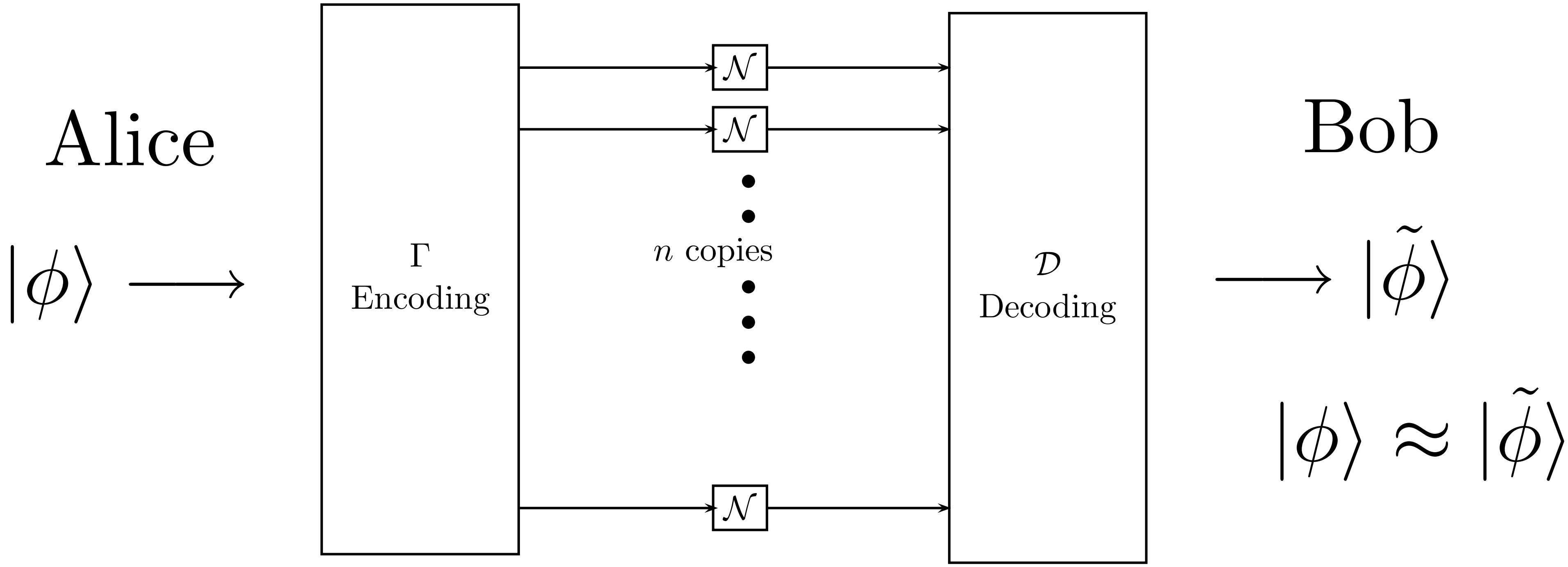
Bob
$\longrightarrow |\tilde{\phi}\rangle$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$|\phi\rangle \approx |\tilde{\phi}\rangle$

Alice

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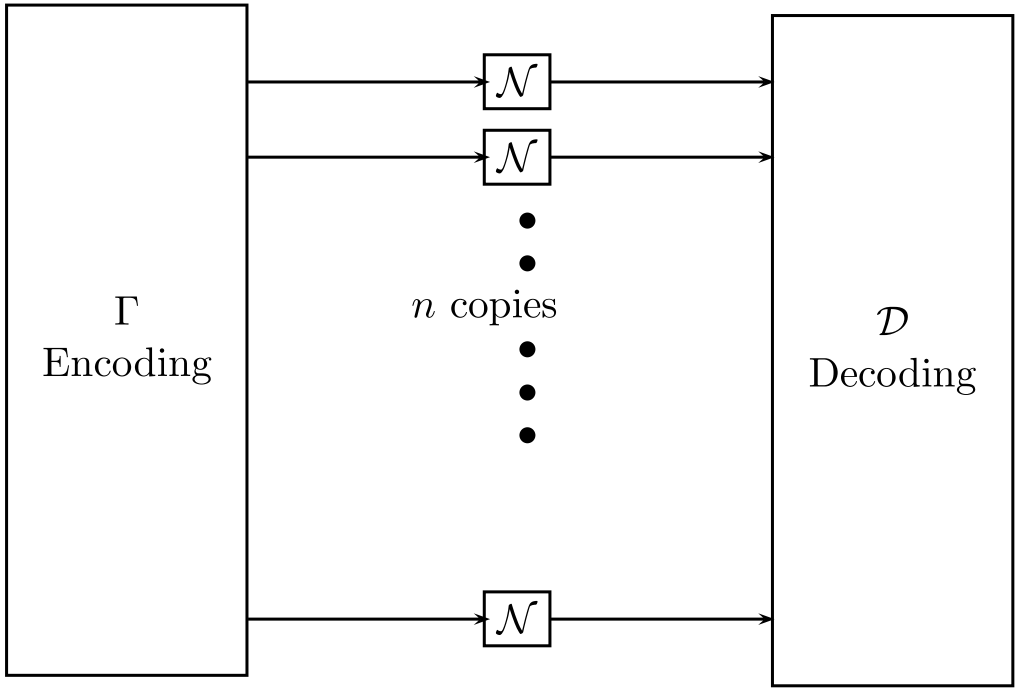

Bob

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Alice

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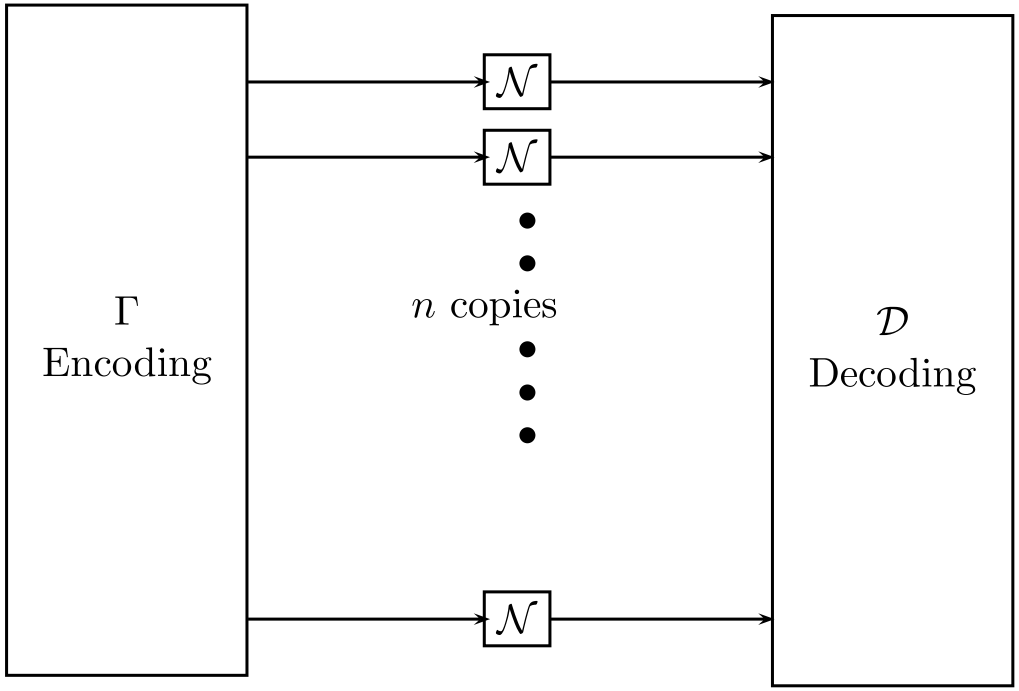

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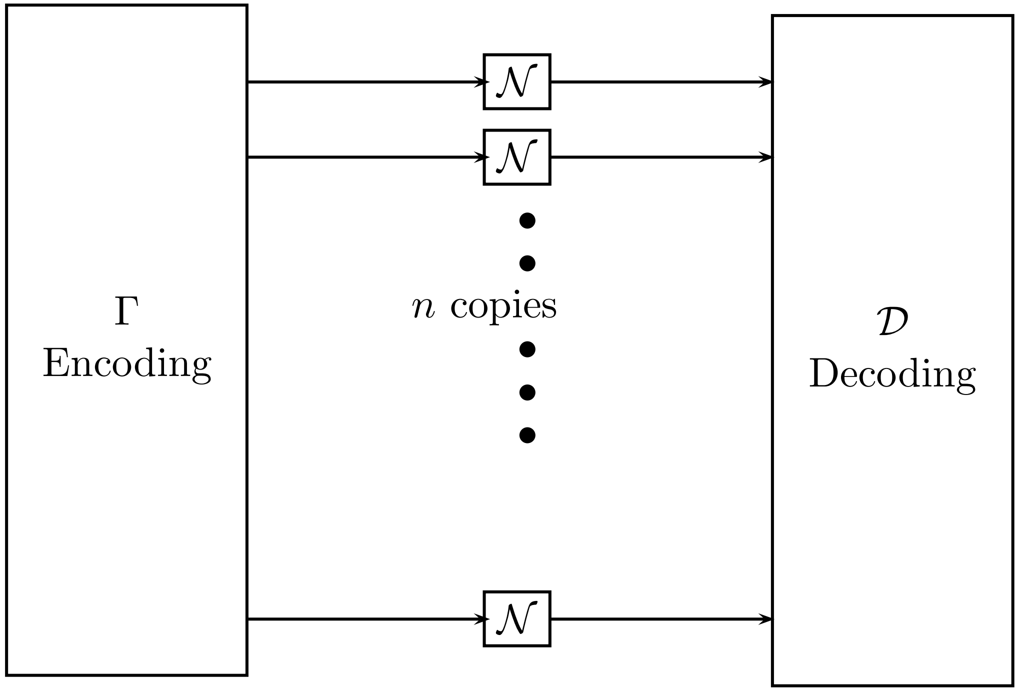

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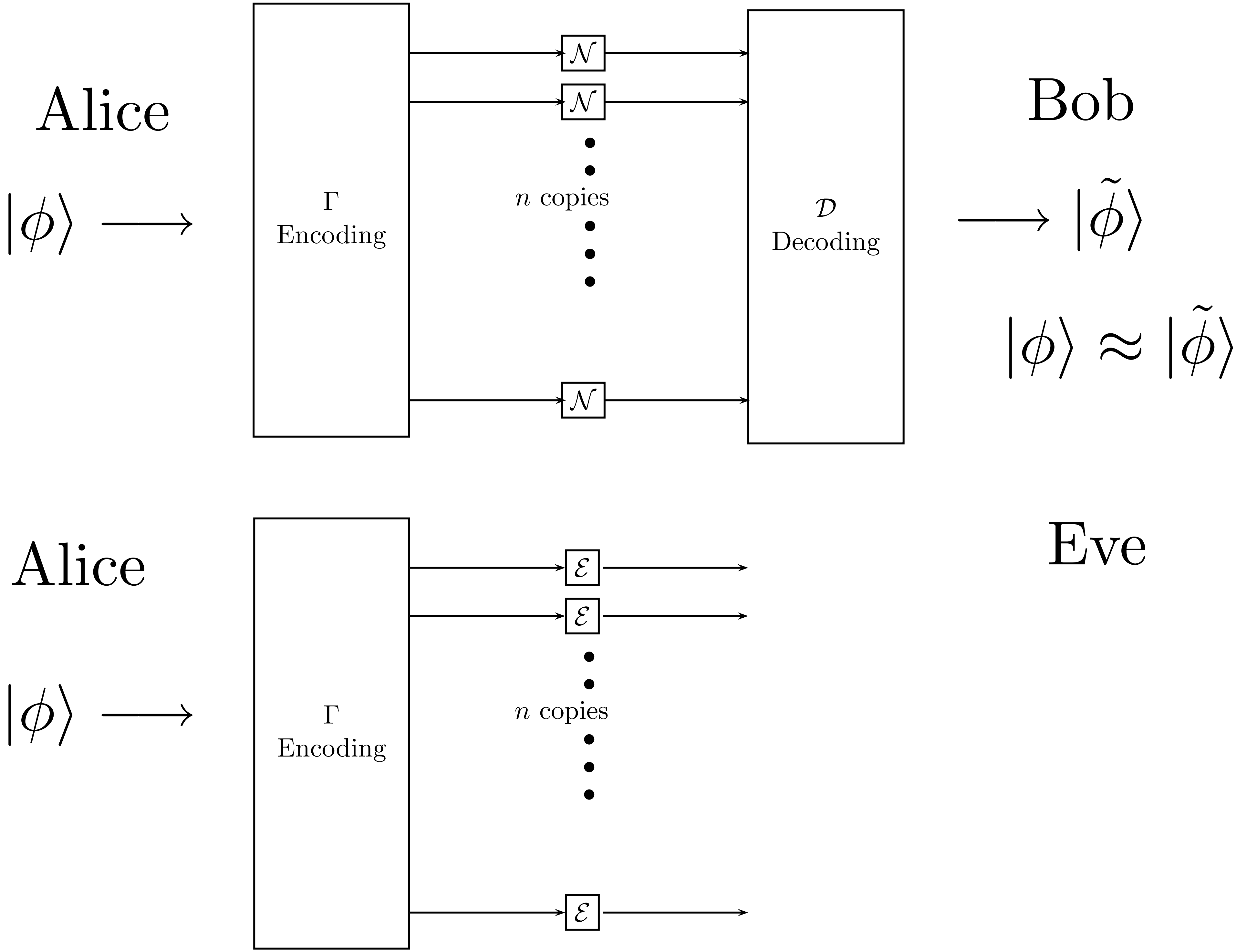
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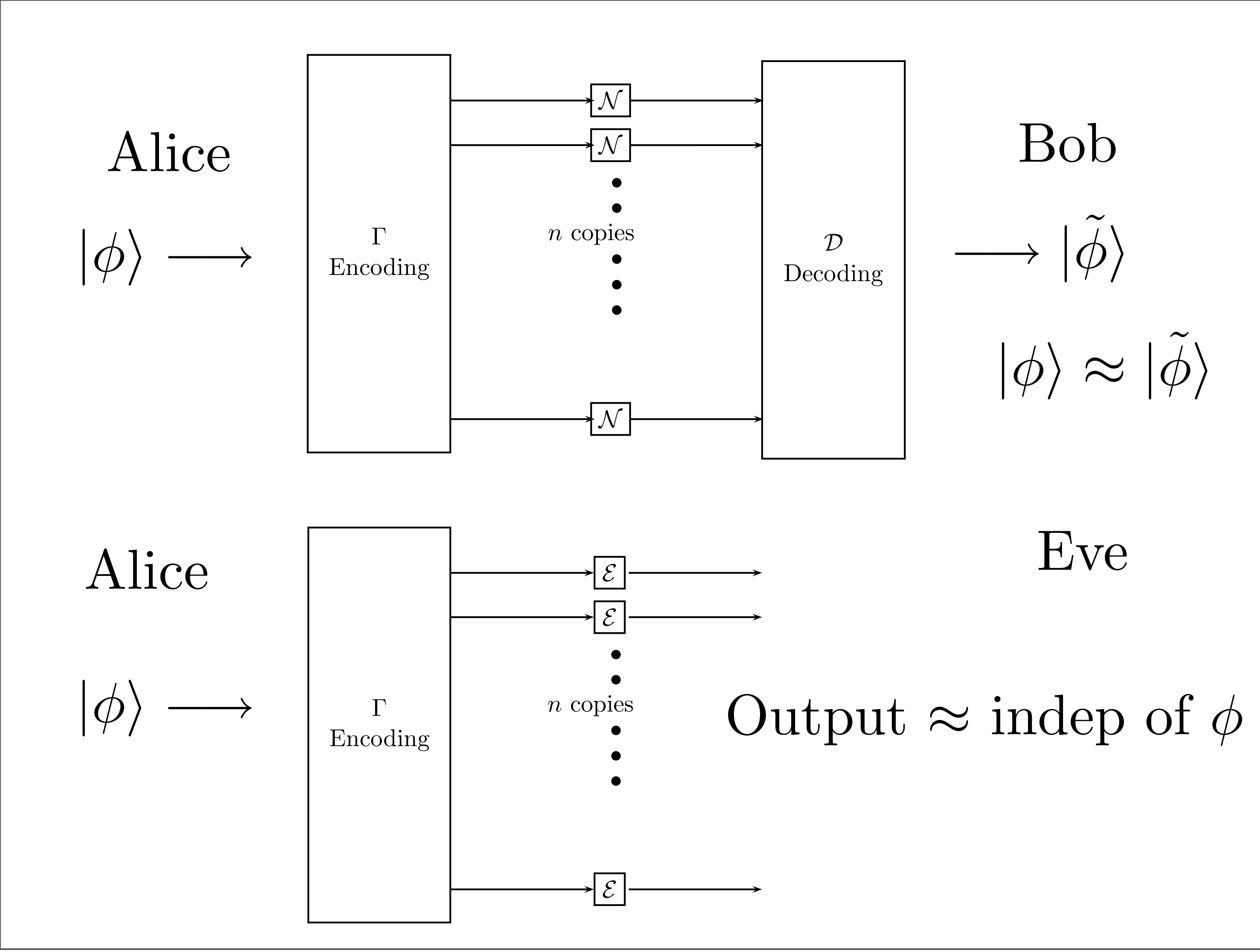

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Eve
Output $\approx$ indep of ϕ

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1. Easier to compute
2. Essentially using the law of large numbers to get better behaviour

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where $\mathcal{Q}$ in an ensemble of pure states on n copies of the channel.

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Uses $\mathcal{N}_2$ and compares the output with the maximally *mixed* state.

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Achievable rates: *Given any pure state* $|\varphi\rangle^{AA'}$, *the rate* $\min(I(A\rangle B)_\rho, H(A|E)_\sigma)$ *is achievable, where* $\rho = (\mathrm{id} \otimes \mathcal{N}_1)(\varphi)$, $\sigma = (\mathrm{id} \otimes \mathcal{N}_2)(\varphi)$, $I(A\rangle B)_\rho = H(B)_\rho - H(AB)_\rho$ *is the coherent information and* $H(A|E)_\sigma = H(AE)_\sigma - H(E)_\sigma$ *the conditional entropy.*

A rate Q is *achievable* if for all δ, ϵ and sufficiently large n there exists an $(n, \lfloor nQ \rfloor, \delta, \epsilon)$ code.

The private quantum capacity is the sup over all achievable rates.

Achievable rates: *Given any pure state* $|\varphi\rangle^{AA'}$, *the rate* $\min(I(A\rangle B)_\rho, H(A|E)_\sigma)$ *is achievable, where* $\rho = (\mathrm{id} \otimes \mathcal{N}_1)(\varphi)$, $\sigma = (\mathrm{id} \otimes \mathcal{N}_2)(\varphi)$, $I(A\rangle B)_\rho = H(B)_\rho - H(AB)_\rho$ *is the coherent information and* $H(A|E)_\sigma = H(AE)_\sigma - H(E)_\sigma$ *the conditional entropy.*

So calculating the private capacity involves computing conditional entropies and then minimzing.

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The only hope: deal with it block by block.

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The blocks are irreducible representations of $SU(2)$!!

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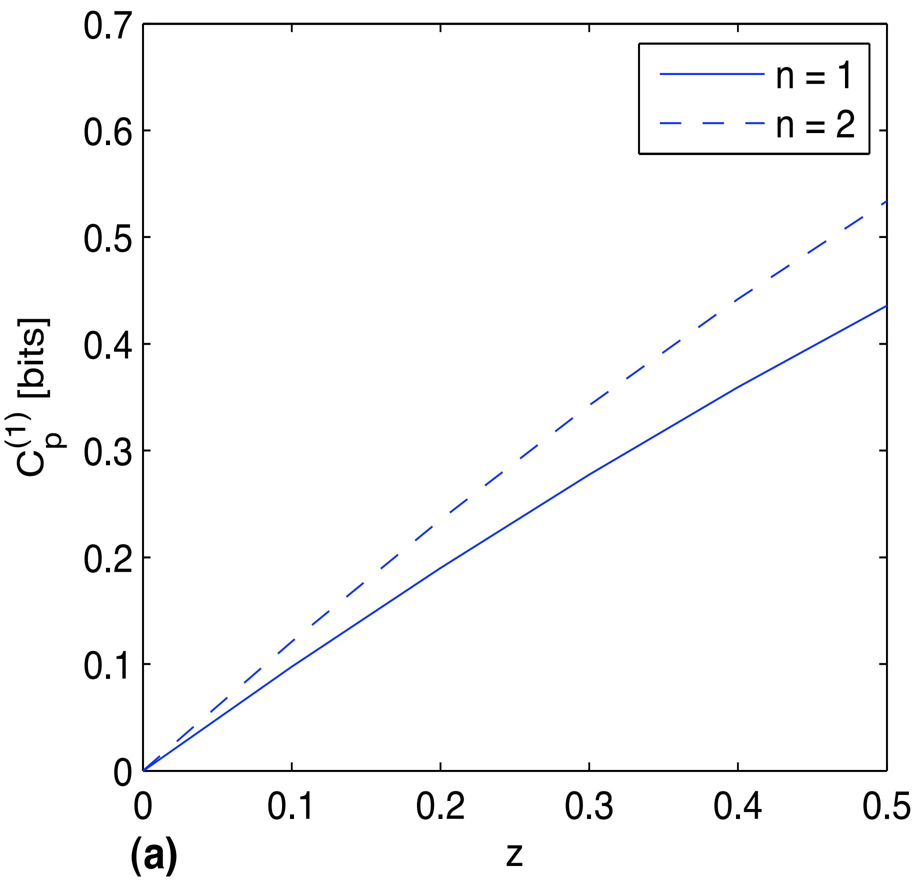
Now we can calculate the entropies and the private capacities.

with a bit of work

The classical private capacitiy is not zero and depends on the acceleration.



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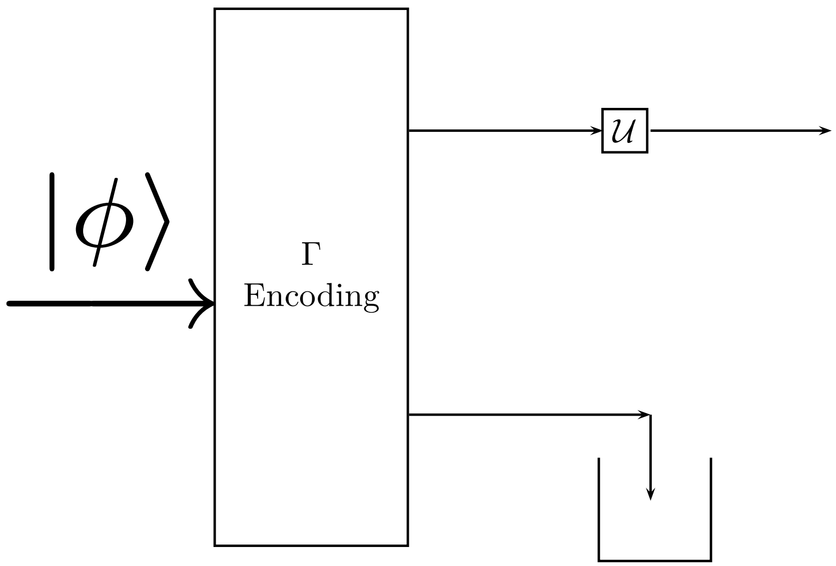

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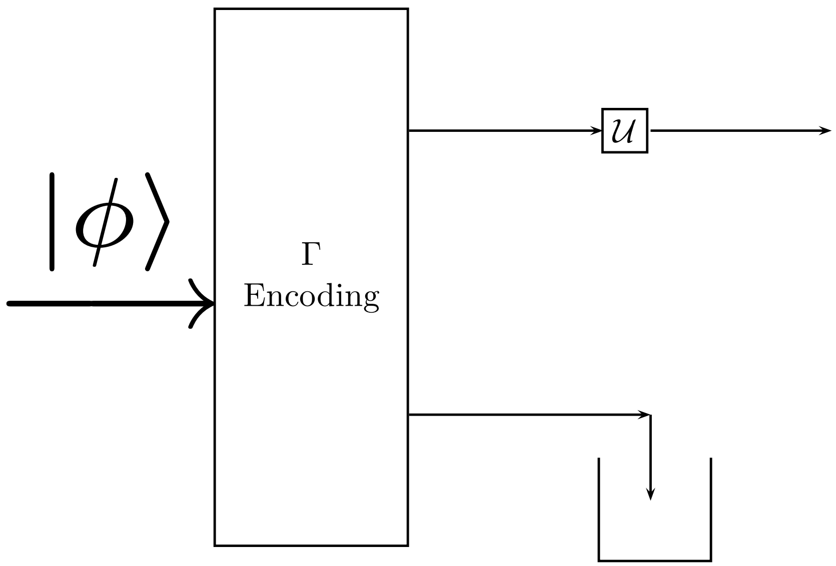

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Next stop: Hawking radiation from black holes.