Program Verification: An overview of the series

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The lecturers

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- Joel Ouaknine; University of Oxford

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- James Worrell; University of Oxford

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- Amy Felty; University of Ottawa

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- Joel Ouaknine; University of Oxford
- James Worrell; University of Oxford
- Amy Felty; University of Ottawa
- Stephen Brookes; Carnegie-Mellon University

The topics

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* Probabilistic systems (Worrell)

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* Theorem proving techniques (Felty)

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- Probabilistic systems (Worrell)
- Theorem proving techniques (Felty)
- Concurrent systems (Brookes)

Early history: formalisms

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- Scott, deBakker 1969: fixed-point induction
- Hoare 1969: axiomatic semantics
- Early 1970s: predicate transformers (Dijkstra)

The pace picks up

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* Abstract interpretation (1976) Cousot and Cousot
* Model checking (Clarke, Emerson, Sifakis)

Disasters and Opportunities

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- The attack on the Needham-Schroder protocol

Some areas of current research

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- Abstraction techniques

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- Probabilistic verification

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- Security

The Main Point

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- One should be able to extract the **relevant** aspects of the program through axioms that capture the semantics.

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- {P} S {Q}: If P holds before execution of S then Q will hold after S terminates, if S does indeed terminate.
- Compositionality: From {P} S {R} and {R} S' {Q} deduce {P} S;S' {Q}.

Dynamic Logic

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- Every program defines a modality.
- Challenging from the point of view of basic theory: the canonical model construction does not work.

Recursion

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- How to make sense of recursion compositionally?
- Fixed-point theory (Kleene).
- rec f. F[f] is the solution of f = F[f].
- Fixed-point induction for programs: Scott and deBakker, Park.

Denotational Semantics

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Continuous functions from D to itself have *least fixed points.*

The meaning of a recursively defined function from D to D is given by the least fixed point of a functional from $D \to D$ to $D \to D$.

Fixed-point Induction

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Let D be a dcpo. A subset $S \subseteq D$ is called *chain-closed* if for all chains

$$d_0 \leq d_1 \leq d_2 \leq \ldots$$

in D, we have

$$\forall n. d_n \in S \Rightarrow \bigvee_n d_n \in S.$$

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With this many properties of recursively defined functions can be proved.

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$$\begin{tikzcd}
D \arrow[r, "f"] \arrow[d, bend right, "\alpha_1"'] & E \arrow[d, bend left, "\alpha_2"] \\
A \arrow[r, "g"] \arrow[u, "\gamma_1"'] & B \arrow[u, "\gamma_2"]
\end{tikzcd}$$

Model Checking

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- Give the specification in a suitable (dynamic) logic.
- Show **automatically** that the system is a **model** of the specification.

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- Prove, using semi-automatic tools if possible, that Beh implies Spec.

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* Theorem proving is good when one doesn't have a complete picture of the model and one can capture some of their properties using axioms.
* Model checking allows a different formalism for describing the model and writing the specification. This allows one to use a rather restricted language for the specifications which has a better chance of being decidable.

Peace, flowers and love

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- Model checking can be a powerful tactic within a theorem proving environment.
- Abstraction is a vital tool in both cases.

Model Checking

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The system is a transition system

S: States

P: Propositions

$\rightarrow \subset S \times S$: Transition relation

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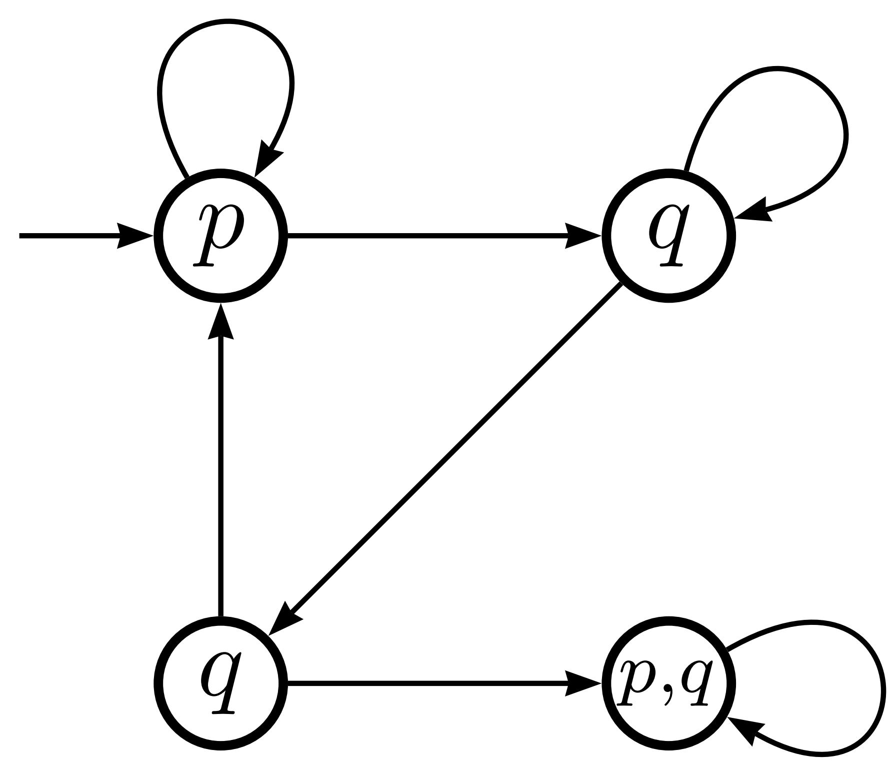

The Logic

$$\begin{array}{lll} \bigcirc & : & \text{Next} \\ \Diamond & : & \text{Eventually} \\ \Box & : & \text{Always} \\ \bigcup & : & \text{Until} \end{array}$$

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Usually temporal logic: Linear Temporal Logic (LTL) or Computation Tree Logic (CTL).

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State formulas:

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Path formulas:

$\psi ::== \bigcirc\phi||\Diamond\phi|\Box\phi|\phi_1 \bigcup \phi_2$

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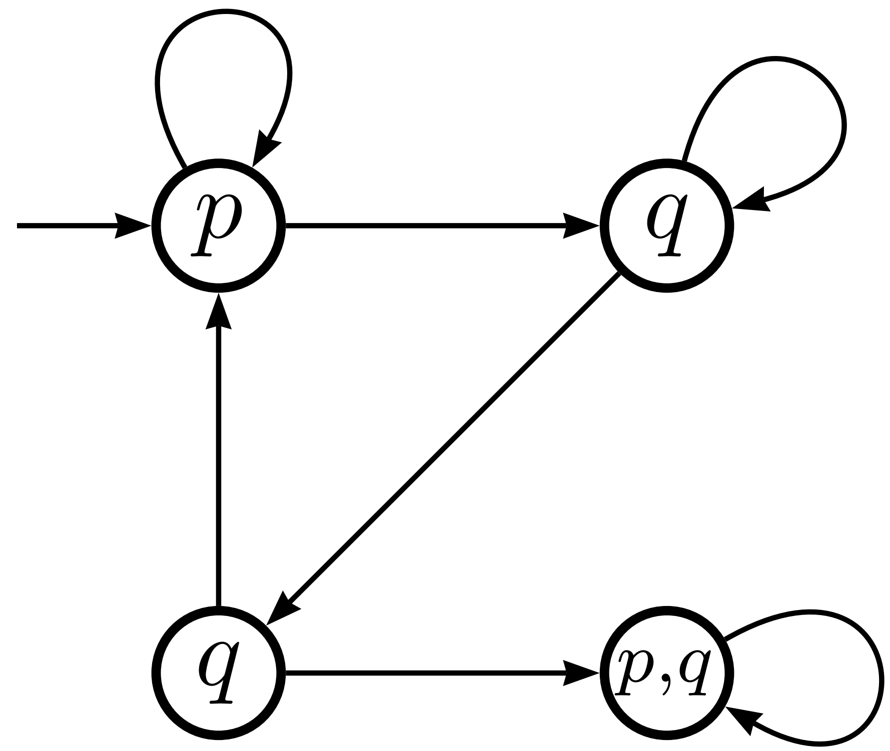
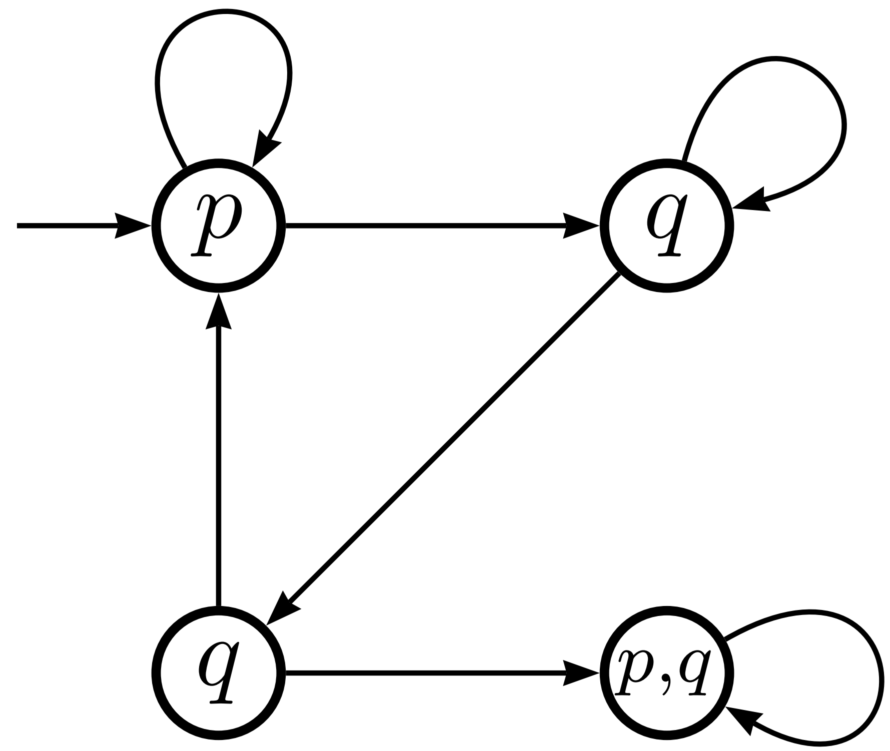

p
q
q
p,q



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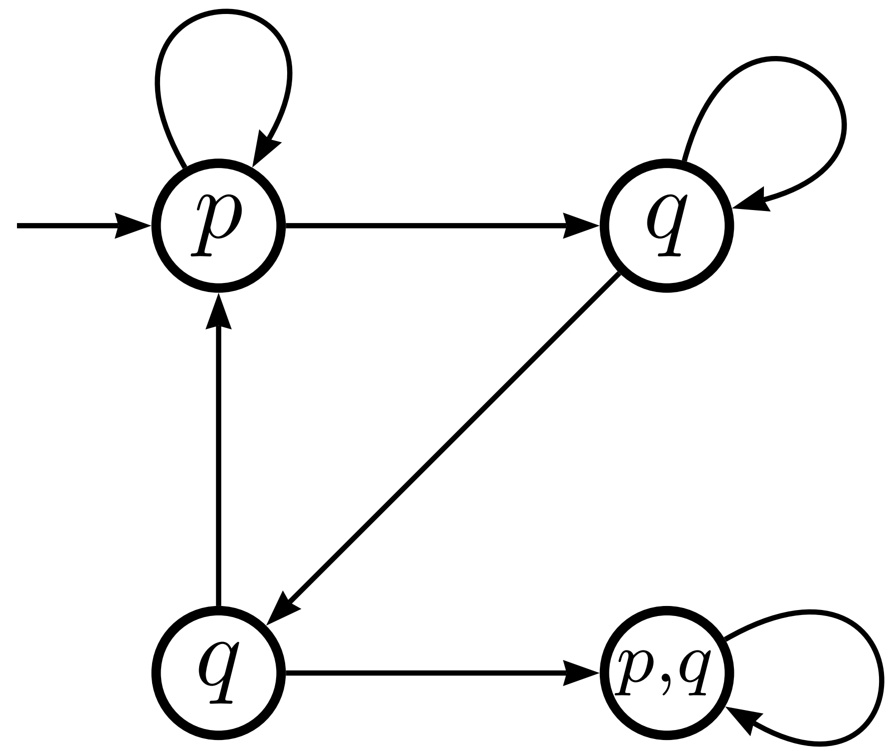

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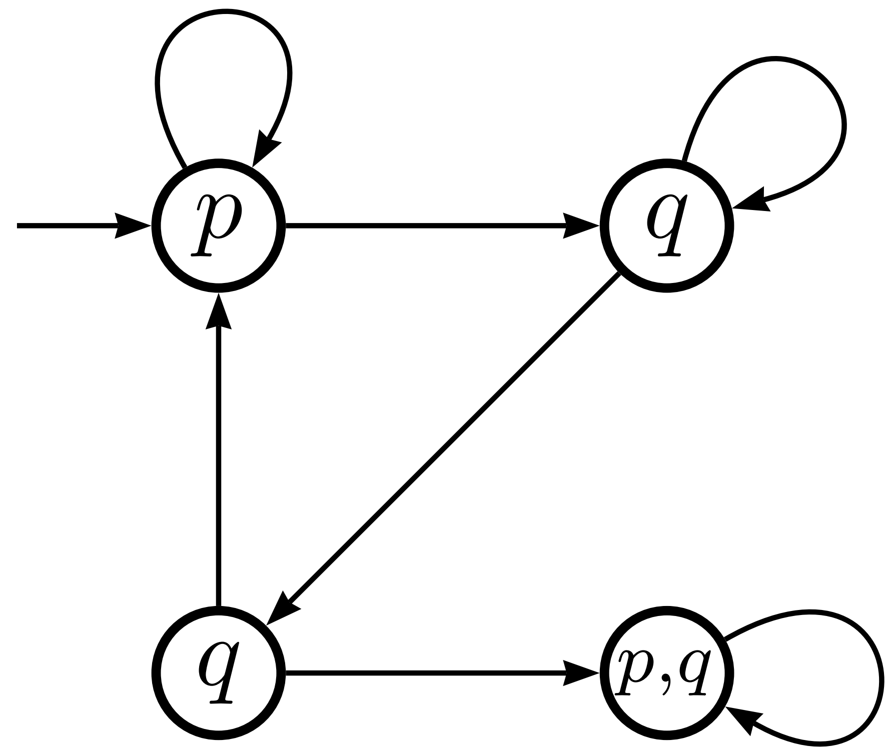

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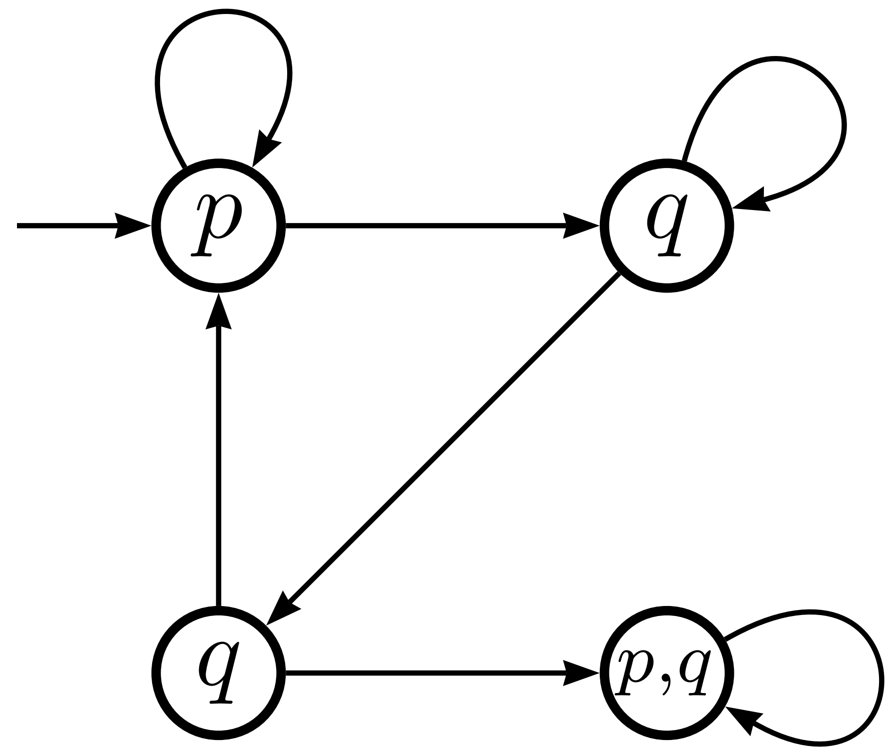

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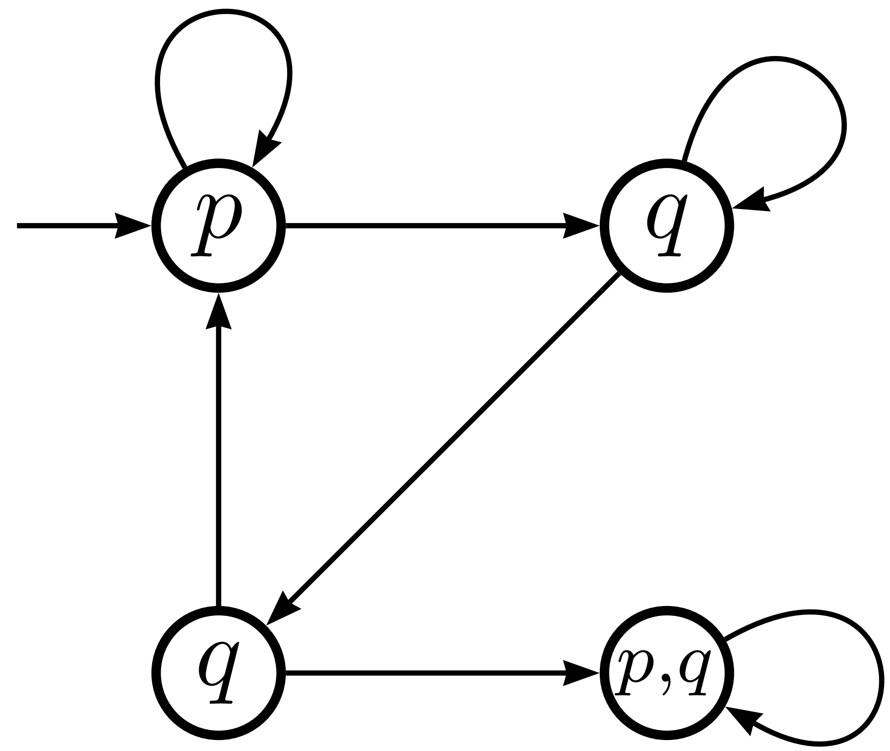

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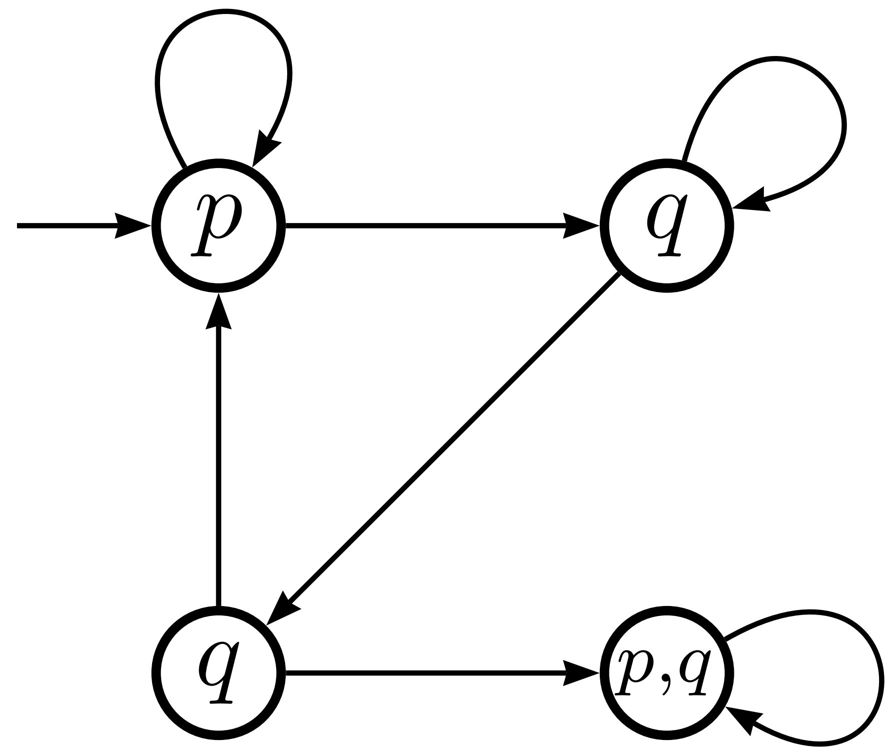

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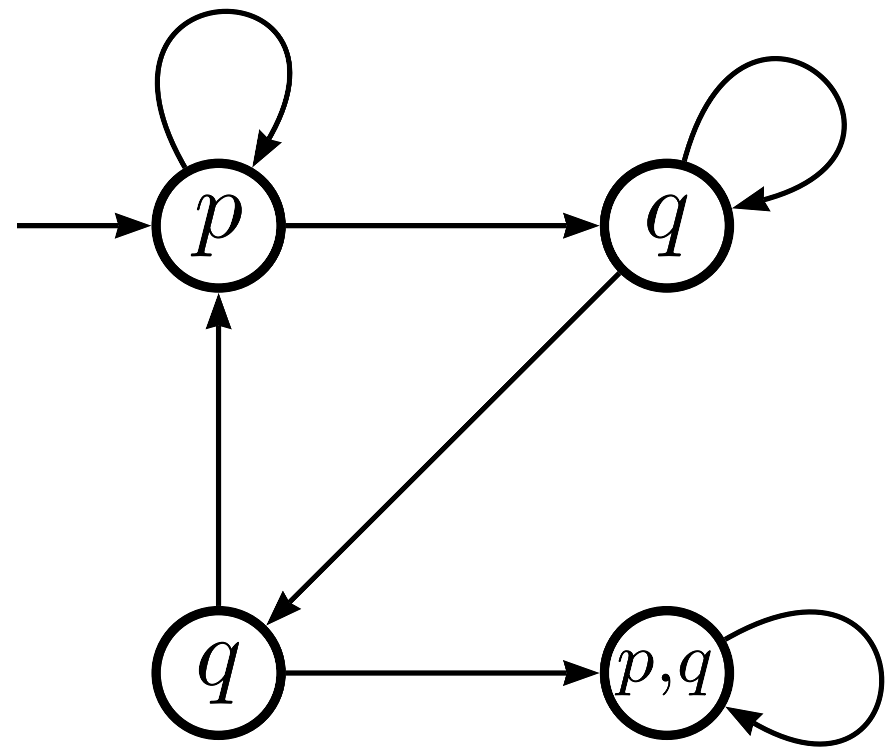

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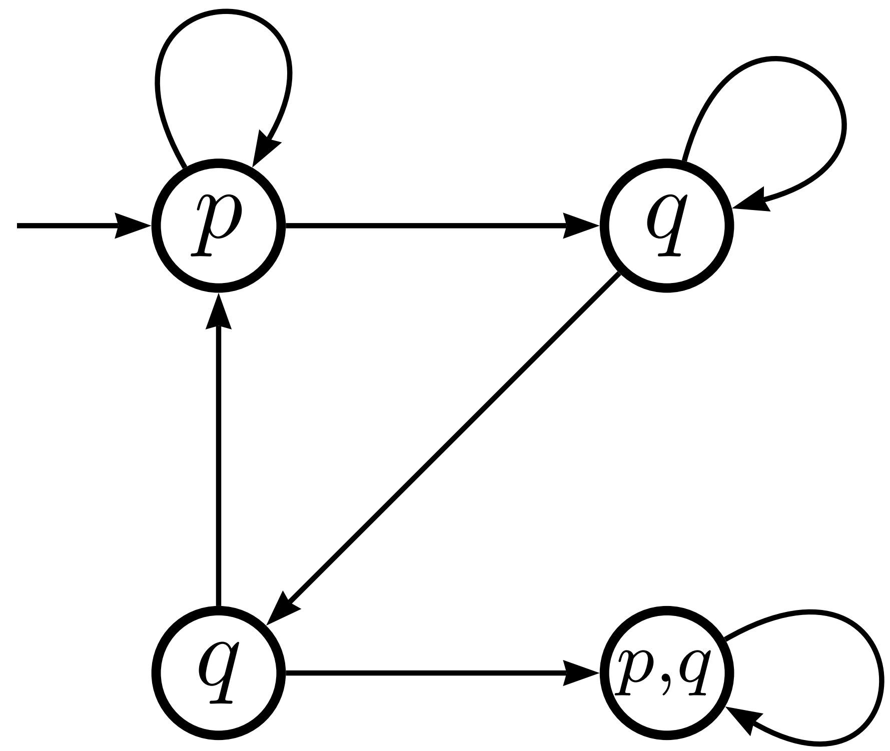

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But "every second state satisfies q" cannot be expressed with these temporal formulas.

It can be expressed with *fixed-point* operators in the logic.

Semantics of the Logic

$s \models p$	iff $p \in L(s)$
$s \models \phi_1 \wedge \phi_2$	iff $s \models \phi_1$ and $s \models \phi_2$
$s \models \forall \psi$	iff $\forall$ paths $\pi = s s_1 s_2 \ldots$, $\pi \models \psi$
$s \models \exists \psi$	iff $\exists$ a path $\pi = s s_1 s_2 \ldots$, $\pi \models \psi$

A *path* is a sequence of states: $\pi = s_0 s_1 s_2 \ldots$

$\pi \models \bigcirc \phi$	iff $s_1 \models \phi$
$\pi \models \Diamond \phi$	iff $\exists j$ such that $s_j \models \phi$
$\pi \models \Box \phi$	iff $\forall j \;\; s_j \models \phi$
$\pi \models \phi_1 \bigcup \phi_2$	iff $\exists j$ such that $s_j \models \phi_2$ and $\forall i < j \; s_i \models \phi_1$

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Input: TS with states S, CTL state formula Φ
Output: $T(\subset S) = \{s | s \models \Phi\} = Sat(\Phi)$.

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$$\begin{aligned}
p: &\quad T = \{s | p \in L(s)\} \\
\phi_1 \wedge \phi_2: &\quad T = Sat(\phi_1) \bigcap Sat(\phi_2) \\
\neg\phi: &\quad T = S \setminus Sat(\phi) \\
\exists \bigcirc \phi: &\quad T = \{s | Post(s) \bigcap Sat(\phi) \neq \emptyset\} \\
\forall \bigcirc \phi: &\quad T = \{s | Post(s) \subseteq Sat(\phi)\}
\end{aligned}$$

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Note that $\phi = \phi_2 \vee \exists \bigcirc \phi$; a fixed-point formula!

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Iterative algorithm to compute this (least) fixed point:

$$
\begin{array}{l}
T := Sat(\phi_2) \\
\textbf{for all} \\
\quad s \in Sat(\phi_1) \setminus T \\
\qquad \textbf{do} \\
\qquad\quad \textbf{if } Post(s) \bigcap T \neq \emptyset \\
\qquad\quad \textbf{then } T := T \bigcup \{s\}.
\end{array}
$$

Similarly,

$$\exists\Box\phi = \phi \wedge \exists \bigcirc \exists\Box\phi,$$

so we have a *greatest* fixed point.

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An iterative algorithm for computing the greatest fixed point.

$T := Sat(\phi)$
repeat
  choose $s \in T$;
    if $Post(s) \bigcap T = \emptyset$
    then $T := T \setminus \{s\}$
until
  $\forall s \in T, Post(s) \bigcap T \neq \emptyset.$

For a transition system with n states and t transitions and a CTL formula ϕ of size k, the model-checking problem can be solved in time

$$O((n + t).k).$$

Further directions

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- Model checking with fairness assumptions

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* Finding counterexamples and witnesses

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- Symbolic model checking: dealing with large systems by working with sets of states symbolically

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- Finding counterexamples and witnesses
- Symbolic model checking: dealing with large systems by working with sets of states symbolically
- Using BDDs to represent sets and set operations efficiently

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- mu-calculus, allows general fixed-point operators.

LTL model checking

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- So what? Still very useful!
- Handles fairness nicely.
- CTL* not significantly harder.

Extensions

* Timed automata
* Probabilistic transition systems

THE END