BLACK HOLES AND INFORMATION

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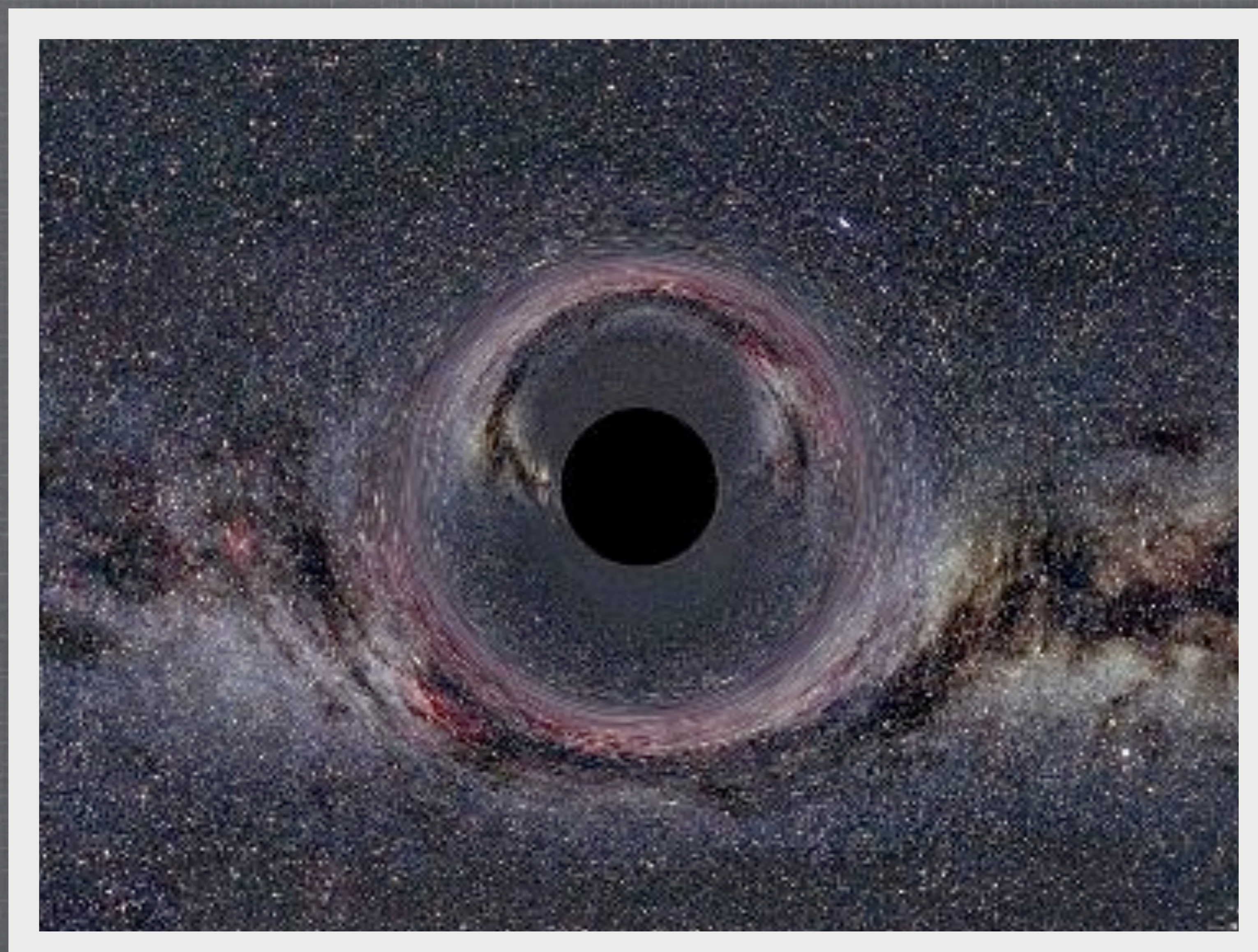


Early History of the Black Hole Concept

1783 John Michell of Cambridge University suggested the possibility.

1796 Laplace calculated the mass needed for the escape velocity to equal the velocity of light.

1916 Karl Schwarzchild discovered a static sperically symmetric solution to Einstein's equation. It is the modern description of a black hole.

1928 Subramanyam Chandrasekhar shows that stars above 1.4 times the mass of the Sun must collapse.

1933 Arthur Eddington drives Chandrasekhar out of England with a scathing attack on his work. Chandrasekhar moves to the University of Chicago for the rest of his career.

1963 Roy Kerr discovers the rotating black hole solution.

1969 Wheeler coins the term "black hole."

Causal Structure of Spacetime

"Quantum mechanics limits our ability to extract information and Relativity limits our ability to transmit it."

Keye Martin

At every point in spacetime a double cone of vectors is defined: the null cone or the "light" cone.

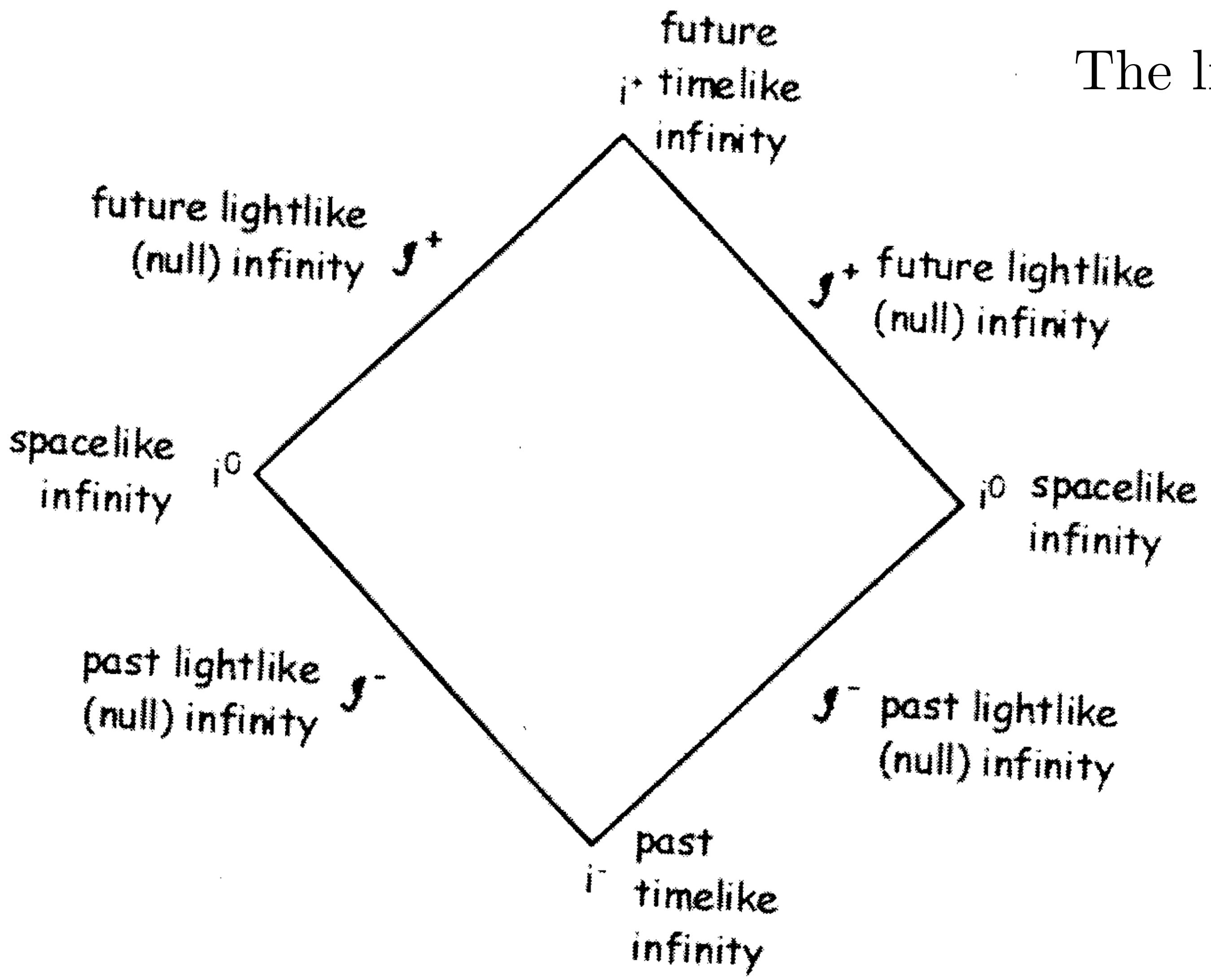
This is called the causal structure and determines limits on the propagation of physical effects or information.

Keye Martin and I showed that this structure plays a fundamental mathematical role in Relativity. [CMP'06]

From the informatic point of view it is important to understand how distorted the causal structure can be in extreme situations.

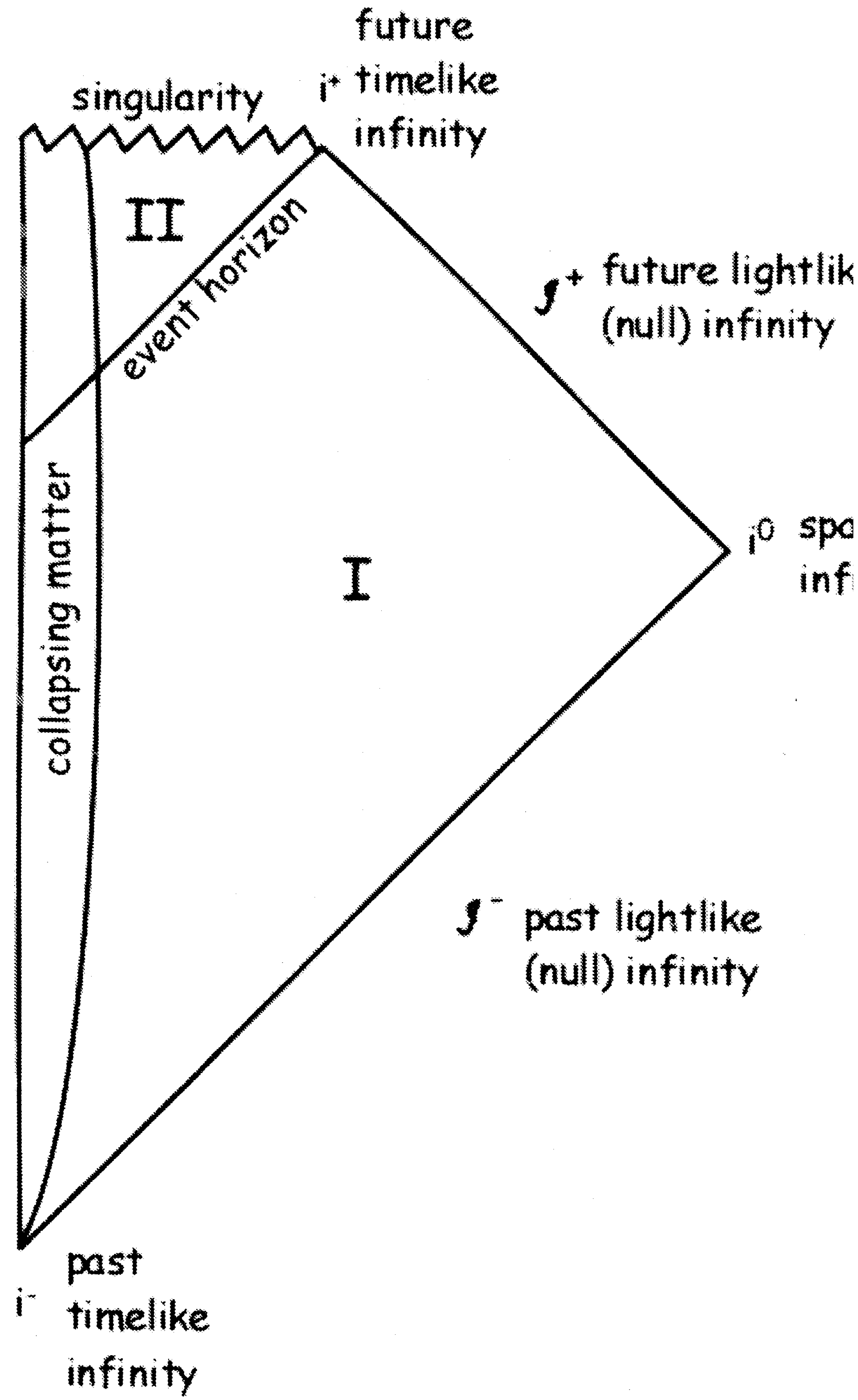
Penrose Diagrams

The light cones are at 45^0.

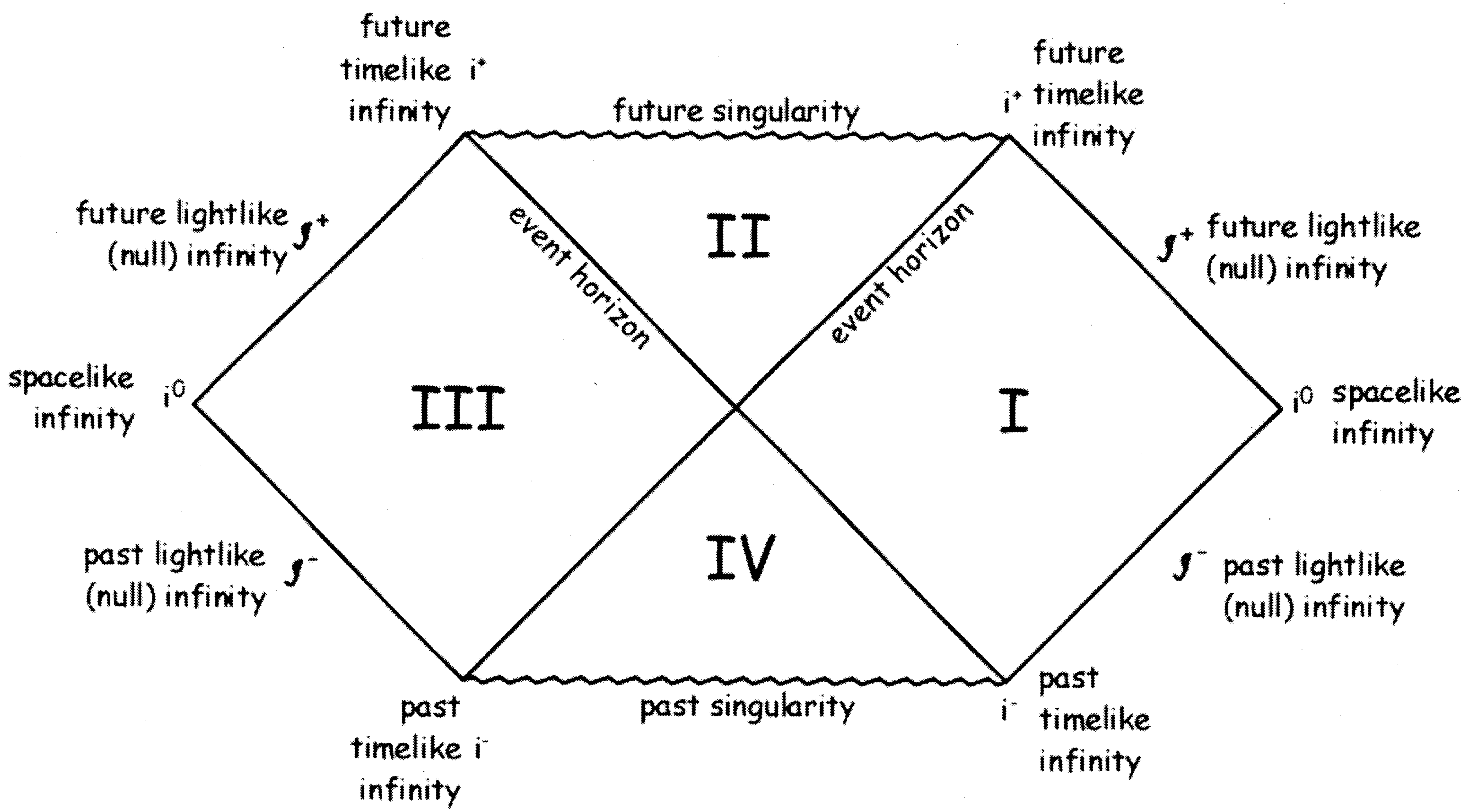


A Penrose diagram of flat spacetime. It is a compactification (not one point!) showing future and past timelike infinity $i^{\pm}$, spacelike infinity i^0 and future and past null infinity.

Penrose Diagram of a Collpasing Star



Penrose Diagram of an Eternal Black Hole



The Kerr Solution

$$c^2 d\tau^2 = \left(1 - \frac{r_s r}{\rho^2}\right) c^2 dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 -$$

$$\left(r^2 + \alpha^2 + \frac{r_s r \alpha^2}{\rho^2} \sin^2\theta\right) \sin^2\theta \ d\phi^2 + \frac{2 r_s r \alpha \sin^2\theta}{\rho^2} c\, dt\, d\phi$$

$$\alpha = \frac{J}{Mc}$$

$$r_s = \frac{2GM}{c^2}$$

$$\rho^2 = r^2 + \alpha^2 \cos^2\theta$$

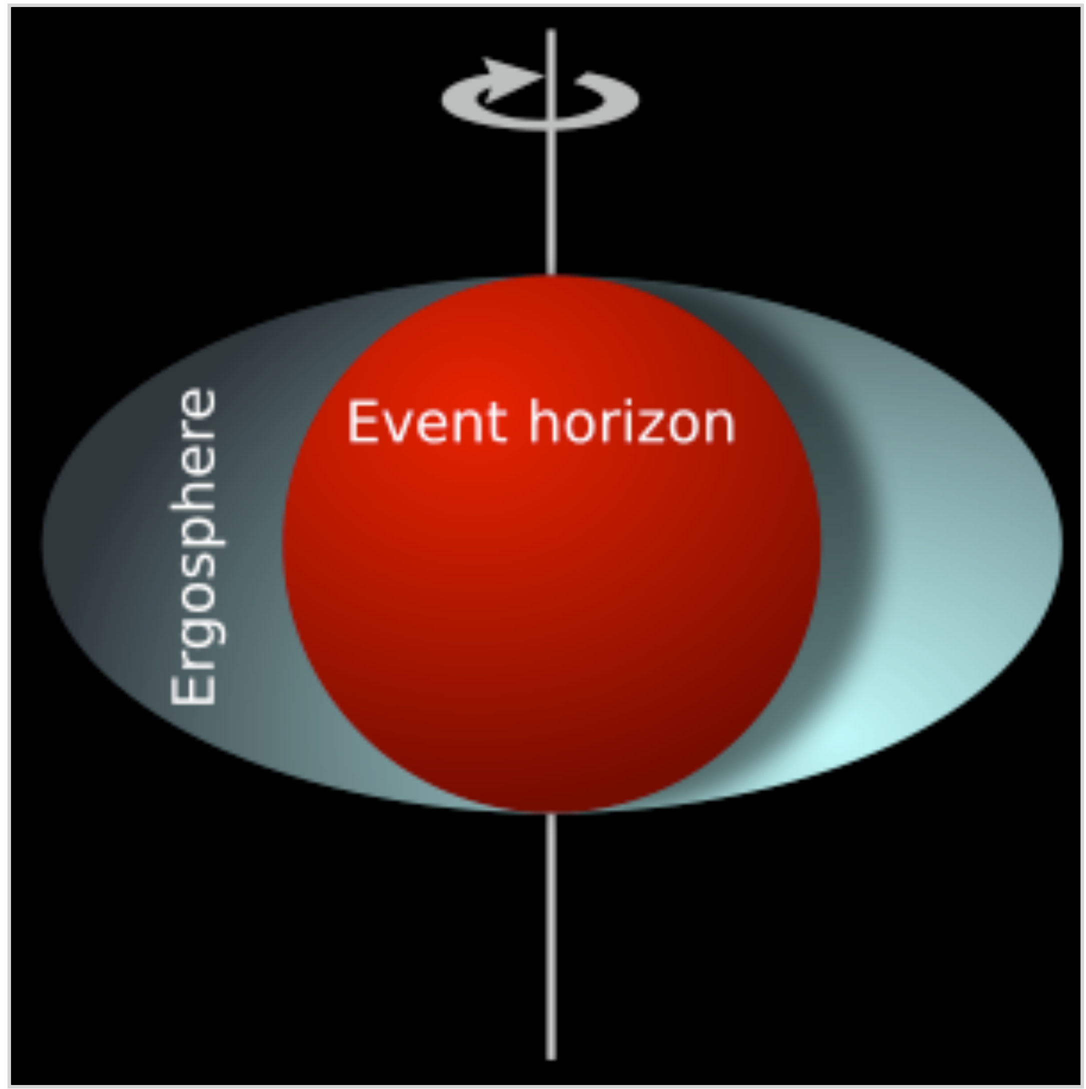
$$\Delta = r^2 - r_s r + \alpha^2$$

M is the mass and J is the angular momentum.

Of course, I do not expect you to read this!

The key point: there are only 2 parameters.

A Better Picture



There are two horizons.

The Penrose Process

Nothing can escape from a black hole.

Yet one can extract energy from a rotating black hole.

Send in a rocket ship into the ergosphere – but outside the event horizon – and fling some junk into the black hole in a direction counter to the rotation.

This will give your spaceship a kick and it will emerge with greater energy than when it entered.

The mass of the black hole increases but the angular momentum decreases and the total energy of the black hole decreases.

Of course, it is unlikely that any realistic astrophysical process can do this.

Some Uniqueness Theorems

Birkhoff: Any spherically symmetric to Einstein's equations must be static.

Israel: Any static solution to the vacuum Einstein equations must be spherically symmetric, hence Schwarzchild.

Carter-Robinson: The unique stationary axisymmetric vacuum solution to Einstein's equations is the Kerr metric.

Can be extended to charged rotating black holes.

Black holes are described by just two (three if you count charge) parameters. "Black holes have no hair!"

They are perfect astrophysical objects. Of course, they can be perturbed and slightly distorted, but basically they are very simple objects.

What happened to the information about the star that collapsed to form the black hole?

Recap of Thermodynamics

Macroscopic systems can be described by a few bulk parameters: Pressure, Temperature, Entropy (S), Free energy (H), ...

Zeroth law: a system in thermal equilibrium has a uniform temperature.

First law: The energy is conserved: $\delta E = T\delta S +$ "terms representing work done."

Second law: S always increases, or at least never decreases.

These laws have a statistical interpretation.

Macrostate: a crude description of the state of a system in terms of a few macroscopic parameters.

Microsate: a precise characterization of all details of the state.

All microstates are equally likely.

A macrostate that describes many microstates is more likely to occur than one corresponding to relatively few microstates.

The entropy of a macrostate measures how many microstates there are associated with it.

The Laws of Black Hole Mechanics

The energy of a Kerr black hole can be changed by dropping something in it and can even by decreased by a trick called the Penrose process.

This can affect the size, area and other geometrical quantities.

Bardeen, Carter and Hawking showed that:

The surface gravity κ is constant on the horizon.

$$\delta m = \tfrac{\kappa}{8\pi}\delta A_h + \omega \delta J$$

The area A_h cannot decrease.

Black Hole Thermodynamics

There is a striking *formal* analogy between the laws of black hole mechanics and the laws of ordinary thermodynamics.

The first law is like the statement that a system in thermal equilibrium has a constant temperature. So by this analogy we should think of κ as a "temperature."

The second really looks like the statement that "area is entropy." It reinforces the view that κ is a temperature because $\kappa\delta A$ looks formally like $T\delta S$ in the first law.

This co-incidence prompted Jacob Bekenstein (1973) to suggest that this analogy should be taken seriously.

Of course it was pointed out that if black holes should have a temperature they ought to radiate like a black body at that temperature would.

But in 1974 Hawking discovered that, according to Quantum Field Theory, they do indeed radiate at just the temperature suggested by Bekenstein.

Recap of Quantum Field Theory

Particles can be created and destroyed.

The space of states has the structure of a Fock Space
$\mathcal{F}(\mathcal{H}) = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H} \otimes_S \mathcal{H}) \oplus (\mathcal{H} \otimes_S \mathcal{H} \otimes_S \mathcal{H}) \ldots$

However, the notion of "particle" is not absolute.

In curved spacetimes particles may appear out of the vacuum: L. Parker 1966.

Particles are a useful abstraction when talking about detectors coupled to fields.

The Vacuum in Quantum Field Theory

The zero-particle state is filled with activity!

Casimir effect: Two *neutral* conducting plates are placed close together. The vacuum modes between the plates are modified and this produces a measurable force between the plates.

If a particle detector is accelerated in a pure vacuum it will interact with the vacuum fluctuations and detect "thermal" radiation at a temperature that depends on the acceleration.

This is the Unruh effect (1976) and came hard on the heels of the Hawking effect.

Hawking Radiation

This is an inherently quantum effect which depends on the presence of a horizon.

The vacuum state at past null infinite is not the same as the vacuum state at future null infinity. A past vacuum evolves to a future thermal state.

The initial calculation was done by Hawking and rederived by Wald, Parker, Hartle and Hawking and others.

The bottom line: the black hole radiates like a black body at exactly the same temperature as suggested by Bekenstein.

Black hole behaves like a black body even with respect to stimulated emission. [P. and Wald 1977]

Eventually the black hole will evaporate. Now what happened to the information inside it?

Summary

Information falls into a black hole and is not accessible from the outside.

A black hole is only described by 2 (3) parameters (M, J, Q)

The information content must be inside the black hole.

Quantum mechanically, a black hole will evaporate producing thermal radiation.

Is the information in the radiation?

Has it disappeared from our universe?

Can information be destroyed?

"... a fundamental limitation to our ability to predict the future, a limitation that is analogous but additional to the limitation imposed by the normal quantum-mechanical uncertainty principle."

S. W. Hawking, Phys. Rev. D14, 2460 (1976).

Pure and Mixed States

A density matrix ρ is a positive Hermitian operator with $Tr(\rho) = 1$.

A pure state has $Tr(\rho^2) = 1$ and $-Tr(\rho \ln \rho) = 0$.

A mixed state has $Tr(\rho^2) < 1$ and $-Tr(\rho \ln \rho) > 0$.

A pure state is of the form $|\psi\rangle\langle\psi|$ for some ψ in the Hilbert space.

A mixed state is of the form $\sum_n p_n |\psi_n\rangle\langle\psi_n|$ for some ψ_ns in the Hilbert space and some real p_ns.

A pure state and only a pure state allows one to predict all the values of a *complete* set of observables with certainty.

For any pure state there are observables that give definite results. (They may be difficult to measure in a real lab.)

For a mixed state there is no observable that can be predicted with certainty. In this sense a mixed state has less than complete information.

Hawking's 1976 Proposal

Usually one has: $\rho^{init} \mapsto \rho^{fin} = S\rho^{init}$

S is called the "scattering" matrix or just S-matrix. It is unitary.

Hawking proposed a "superscattering" matrix: $\$$

$\$$ maps pure states to mixed states and is *not* unitary.

One can have examples where $\$$ is not unitary but it can nevertheless be inverted.

However, partial information is degraded and most people refer to the existence of a $\$$ as a loss of information.

Culture Wars

Relativists tend to believe that unitarity can be abandoned but the impenetrability of the event horizon is sacred.

Quantum field theorists tend to believe that unitarity is sacred and that the location of the event horizon cannot be pinned down anyway.

"Although Hawking's conclusion is undoubtedly wrong, it played a central role in replacing the old ideas of locality with a new paradigm."

Leonard Susskind 2005

Objections to Hawking's Scenario

There is a fundamental symmetry in nature: CPT invariance.

If in any physical process we flip all the charges (C), film it in a mirror (P) and run the film backwards (T), we should get a physically realizable process.

In fact this is a theorem! It does not depend on any detailed theory of interactions.

The CPT transformation of $ should send mixed states to pure states.

Hawking's calculation is based on a semiclassical approximation. This should break down near the end stages of the evaporation process.

There were many arguments back and forth about CPT and "weak" variations of this concept.

Some Possibilities

Evolution by a unitary S-matrix. All the information comes back out in our universe.

Evolution by \$, which is not CPT invariant but the resulting mixed state is predictable.

Nonlinear evolution (i.e. we pass the buck!).

Some fragment remains with all the information trapped in it.

Information escapes to another universe.

Some fundamental new nonlocality ideas come into play.

Summary of Possibilities

Information Loss

Information trapped in a remnant.

Information emerges and evolution is unitary.

Sorkin's Viewpoint

Black hole entropy is finite because of fundamental discreteness of spacetime.

This entropy is "objective" because the horizon provides a preferred notion of coarse graining.

The second law if obeyed because the effective dynamics outside the horizon is not unitary.

Non-unitarity should be welcomed!

Reference: Ten Theses on Black Hole Entropy. hep-th/050437

Hayden and Preskill

Assume internal dynamics is unitary and rapidly mixing

Internal dynamics is an instantaneous random unitary.

Assume also that outside observers have access to all the emitted Hawking radiation!

Assume that at the halfway point, internal state is maximally entangled with the previously emitted Hawking radiation

Then new bits dropped into the black hole will emerge almost immediately.

Black holes are information mirrors!

In fact black holes are quantum cloners! Alice falls in with her quantum state but Bob gets a copy of it in the Hawking radiation.

Adami and Steeg 2006: Black Holes are optimal quantum cloners!

Black hole complementarity principle: Alice and Bob can never verify that cloning has happened.

Hayden and Preskill calculations suggest that this is OK but only just!

A Fundamental New Principle?

New principles guide the development of new theories: Galilean invariance, Lorentz invariance, The Principle of Equivalence.

A new principle that might guide the development of quantum gravity: The Holographic principle.

The principle states that the description of a 3D space is determined by its 2D boundary: a hologram!

Why does anyone believe such a thing?

Because of entropy bounds results.

Entropy Bounds

Bekenstein "showed" that in a weakly gravitating spherically symmetric spacetime the entropy of a region of space is limited by the area of the smallest sphere enclosing it.

For spherical black holes the entropy bound is saturated.

There is much controversy about whether this bound is necessary or sufficient for the generalized second law of thermodynamics.

Naively one may try to generalize the Bekenstein bound to a spacelike entropy bound, but this has many counter-examples.

Covariant entropy bound: Busso [1999] proposed a bound based on light sheets emanating from an area. This survives all the counter-examples to the spacelike entropy bound.

The entropy bound limits the number of degrees of freedom of *any* physical system in a fixed region.

It says that this is *much smaller* than predicted by *local* quantum field theory.

The entropy bound is a relation between the number of states of (quantum) matter, the (causal) structure of spacetime and information.

Any future theory that incorporates this bound must combine all these concepts.

Conclusions

Fundamental connections between information, quantum mechanics and space-time geometry.

Black holes figured prominently in the story but the implications of holography and entropy bounds are much deeper and far reaching.

Possible ideas that may be relevant: domains and measurements, event structures, categorical quantum mechanics (and qft), algebraic information theory,

It is something for us (yes us! not just them) to think about.

References

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Papers by Hawking and by Page.

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Papers by Don Page, Preskill, Hayden and Preskill,

Black Holes, Information and the String Theory Revolution: Susskind and Lindesay.

Reviews of Modern Physics article by Raphael Busso: The Holographic Principle.