A domain of spacetime intervals for General Relativity

Causal Structure, Topology and Domain Theory

Keye Martin[1] and Prakash Panangaden[2]

[1]Naval Research Laboratory

[2]School of Computer Science
McGill University

22 Aug 2006 / Dagstuhl Seminar on Computational Structures for Modelling Space, Time and Causality

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- Understand the role of order in analysing the causal structure of spacetime.
- Reconstruct spacetime topology from causal order: obvious links with domain theory.
- Not looking at the combinatorial aspects of order: continuous posets play a vital role; Scott, Lawson and interval topologies play a vital role.
- Everything is about classical spacetime: we see this as a step on Sorkin's programme to understand quantum gravity in terms of causets.

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Overview

- The causal structure of globally hyperbolic spacetimes defines a *bicontinuous* poset. The topology can be recovered from the order *and from the way-below relation* but with no appeal to smoothness. The order can be taken to be fundamental.
- The entire spacetime manifold can be reconstructed given a countable dense subset with the induced order: no metric information need be given.
- Globally hyperbolic spacetimes can be seen as the maximal elements of interval domains. There is an equivalence of categories between globally hyperbolic spacetimes and interval domains. The main theorem.

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Causality in Computer Science

- In distributed systems one loses synchronization and *absolute* global state just as in relativity. One works with causal structure.
- Causal precedence in distributed systems studied by Petri (65) and Lamport (77): clever algorithms, but the mathematics was elementary and combinatorial and did not reveal the connections with general relativity.
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The layers of spacetime structure

- Set of events: no structure
- Topology: 4 dimensional real manifold
- Differentiable structure: tangent spaces
- Causal structure: light cones (defines metric up to conformal transformations)
- Lorentzian metric: gives a length scale.

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Causal Structure of Spacetime I

- At every point a pair of “cones” is defined in the tangent space: future and past light cone. A vector on the cone is called **null** or **lightlike** and one inside the cone is called **timelike**.
- We assume that spacetime is *time-orientable*: there is a global notion of future and past.
- A *timelike* curve from x to y has a tangent vector that is everywhere timelike: we write $x \preceq y$. (We avoid $x \ll y$ for now.) A *causal* curve has a tangent that, at every point, is either timelike or null: we write $x \leq y$.
- A fundamental assumption is that $\leq$ is a partial order. Penrose and Kronheimer give axioms for $\leq$ and $\preceq$.

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Causal Structure of Spacetime II

- $I^+(x) := \{y \in M | x \preceq y\}$; similarly I^-
- $J^+(x) := \{y \in M | x \leq y\}$; similarly J^-.
- $I^\pm$ are always open sets in the manifold topology; $J^\pm$ are **not** always closed sets.
- Chronology: $x \preceq y \Rightarrow y \not\preceq x$.
- Causality: $x \leq y$ and $y \leq x$ implies $x = y$.

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Causality Conditions

- $$I^{\pm}(p) = I^{\pm}(q) \Rightarrow p = q.$$
- Strong causality at p: Every neighbourhood $\mathcal{O}$ of p contains a neighbourhood $\mathcal{U} \subset \mathcal{O}$ such that no causal curve can enter $\mathcal{U}$, leave it and then re-enter it.
- Stable causality: perturbations of the metric do not cause violations of causality.
- Causal simplicity: for all $x \in M$, $J^{\pm}(x)$ are closed.
- Global hyperbolicity: M is strongly causal and for each p, q in M, $[p, q] := J^{+}(p) \cap J^{-}(q)$ is compact.

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The Alexandrov Topology

Define

$$\langle x, y \rangle := I^+(x) \cap I^-(y).$$

The sets of the form $\langle x, y \rangle$ form a base for a topology on M called the Alexandrov topology.

Theorem (Penrose): TFAE:

(M, g) is strongly causal.

The Alexandrov topology agrees with the manifold topology.

The Alexandrov topology is Hausdorff.

The proof is geometric in nature.

The Way-below relation

- In domain theory, in addition to $\leq$ there is an additional, (often) irreflexive, transitive relation written $\ll$: $x \ll y$ means that x has a “finite” piece of information about y or x is a “finite approximation” to y. If $x \ll x$ we say that x is *finite*.
- The relation $x \ll y$ - pronounced x is “way below” y - is directly defined from $\leq$.
- Official definition of $x \ll y$: If $X \subset D$ is directed and $y \leq (\sqcup X)$ then there exists $u \in X$ such that $x \leq u$. If a limit gets past y then, at some finite stage of the limiting process it already got past x.

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Continuous Domains and Topology

- A continuous domain D has a basis of elements $B \subset D$ such that for every x in D the set $x\downarrow := \{u \in B | u \ll x\}$ is directed and $\sqcup(x\downarrow) = x$.
- The Scott topology: the open sets of D are upwards closed and if $\mathcal{O}$ is open, then if $X \subset D$, directed and $\sqcup X \in \mathcal{O}$ it must be the case that some $x \in X$ is in $\mathcal{O}$.
- The Lawson topology: basis of the form
$$\mathcal{O} \setminus [\cup_i (x_i \uparrow)]$$
where $\mathcal{O}$ is Scott open. This topology is metrizable if the domain is ω-continuous.
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The role of way below in spacetime structure

- **Theorem:** Let (M, g) be a spacetime with Lorentzian signature. Define $x \ll y$ as the way-below relation of the causal order. If (M, g) is globally hyperbolic then $x \ll y$ iff $y \in I^+(x)$.
- One can recover I from J without knowing what smooth or timelike means.
- Intuition: any way of approaching y must involve getting into the timelike future of x.
- We can stop being coy about notational clashes: henceforth $\ll$ is way-below *and* the timelike order.

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Bicontinuity and Global Hyperbolicity

- The definition of continuous domain - or poset - is biased towards approximation from below. If we symmetrize the definitions we get bicontinuity (details in the paper).
- Theorem: If (M, g) is globally hyperbolic then $(M, \leq)$ is a bicontinuous poset. In this case the interval topology is the manifold topology.
- We feel that bicontinuity is a significant causality condition in its own right; perhaps it sits between globally hyperbolic and causally simple.
- Topological property of causally simple spacetimes: If (M, g) is causally simple then the Lawson topology is contained in the interval topology.

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An “abstract” version of globally hyperbolic

We *define* a globally hyperbolic poset $(X, \leq)$ to be

1. bicontinuous and,
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Properties of globally hyperbolic posets

- A globally hyperbolic poset is locally compact and Hausdorff.
- The Lawson topology is contained in the interval topology.
- Its partial order $\leq$ is a closed subset of X^2.
- Each directed set with an upper bound has a supremum.
- Each filtered set with a lower bound has an infimum.

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Second countability

- Globally hyperbolic posets share a remarkable property with metric spaces, that separability (countable dense subset) and second countability (countable base of opens) are equivalent.
- Let $(X, \leq)$ be a bicontinuous poset. If $C \subseteq X$ is a countable dense subset in the interval topology, then:
  (i) The collection
  $$\{(a_i, b_i) : a_i, b_i \in C, a_i \ll b_i\}$$
  is a countable basis for the interval topology.
  (ii) For all $x \in X$, $\twoheaddownarrow x \cap C$ contains a directed set with supremum x, and $\twoheaduparrow x \cap C$ contains a filtered set with infimum x.

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An Important Example of a Domain: $\mathbf{I}\mathbb{R}$

- The collection of compact intervals of the real line

$$\mathbf{I}\mathbb{R} = \{[a,b] : a,b \in \mathbb{R} \;\&\; a \leq b\}$$

ordered under reverse inclusion

$$[a,b] \sqsubseteq [c,d] \Leftrightarrow [c,d] \subseteq [a,b]$$

is an ω-continuous dcpo.

- For directed $S \subseteq \mathbf{I}\mathbb{R}$, $\bigsqcup S = \bigcap S$,
- $I \ll J \Leftrightarrow J \subseteq \mathrm{int}(I)$, and
- $\{[p,q] : p,q \in \mathbb{Q} \;\&\; p \leq q\}$ is a countable basis for $\mathbf{I}\mathbb{R}$.
- The domain $\mathbf{I}\mathbb{R}$ is called the interval domain.
- We also have $\max(\mathbf{I}\mathbb{R}) \simeq \mathbb{R}$ in the Scott topology.

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- The collection of compact intervals of the real line

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Generalizing $\mathbf{I}\mathbb{R}$

- The closed segments of a globally hyperbolic poset X

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Spacetime from a discrete ordered set

- If we have a countable dense subset $\mathcal{C}$ of $\mathcal{M}$, a globally hyperbolic spacetime, then we can view the induced causal order on C as defining a discrete poset. An ideal completion construction in domain theory, applied to a poset constructed from $\mathcal{C}$ yields a domain $\mathbf{I}C$ with

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Compactness of the space of causal curves

- A fundamental result in relativity is that the space of causal curves between points is compact on a globally hyperbolic spacetime. We use domains as an aid in proving this fact for any globally hyperbolic poset. This is the analogue of a theorem of Sorkin and Woolgar: they proved it for K-causal spacetimes; Keye did it for globally hyperbolic posets; the paper is now published in *Classical and Quantum Gravity*.
- The Vietoris topology on causal curves arises as the natural counterpart to the manifold topology on events, so we can understand that its use by Sorkin and Woolgar is very natural.
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Globally Hyperbolic Posets and Interval Domains

- One can define categories of globally hyperbolic posets and an abstract notion of “interval domain”: these can also be organized into a category.
- These two categories are equivalent.
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Interval Posets

- An *interval* poset D has two functions left : $D \rightarrow \max(D)$ and right : $D \rightarrow \max(D)$ such that

$$(\forall x \in D)\, x = \text{left}(x) \sqcap \text{right}(x).$$

- The union of two intervals with a common endpoint is another interval and
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- $(D, \mathrm{left}, \mathrm{right})$ with D a continuous dcpo
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Globally Hyperbolic Posets are an Example

- For a globally hyperbolic $(X, \leq)$, we define $\mathrm{left} : \mathbf{I}X \to \mathbf{I}X :: [a, b] \mapsto [a]$ and $\mathrm{right} : \mathbf{I}X \to \mathbf{I}X :: [a, b] \mapsto [b]$.
- Lemma: If $(X, \leq)$ is a globally hyperbolic poset, then $(\mathbf{I}X, \mathrm{left}, \mathrm{right})$ is an interval domain.
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The category of Interval Domains

The category **IN** of interval domains and commutative maps is given by

- **objects** Interval domains $(D, \text{left}, \text{right})$.
- **arrows** Scott continuous $f : D \to E$ that commute with left and right, i.e., such that both

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From **GlobHyP** to **IN**

The correspondence $\mathbf{I} : \mathbf{GlobHyP} \to \mathbf{IN}$ given by

$$(X, \leq) \mapsto (\mathbf{I}X, \mathrm{left}, \mathrm{right})$$

$$(f : X \to Y) \mapsto (\bar{f} : \mathbf{I}X \to \mathbf{I}Y)$$

is a functor between categories.

Summary

- We can recover the topology from the order.
- We can reconstruct the spacetime from a countable dense subset.
- We can characterise causal simplicity order theoretically.
- We can prove the Sorkin-woolgar theorem on compactness of the space of causal curves.
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- The fact that globally hyperbolic posets are interval domains gives a sensible way of thinking of “approximations” to spacetime points in terms of intervals.
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Related Work

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Causal Simplicity

For a spacetime (M, g), the following are equivalent:
(1) (M, g) is causally simple, every increasing sequence with a sup is convergent, every decreasing sequence with an inf is convergent.
(2) M is bicontinuous.
We need a spacetime that is not globally hyperbolic or causally simple but satisfies (1). Ideas anyone?

Conjecture

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Usually E is $[0, \infty)^{rev}$ and the number is the “degree of uncertainity” of the element.

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- There is a notion of “informatic” derivative which could be used to set up discrete differential geometry on domains and ultimately to consider “fields” living on domains.
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Can anyone get me to Stop?

Stop babbling and do some of these things already!