Knowledge and Information in Probabilistic Systems

Prakash Panangaden

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- PODC: Algorithms, combinatorial arguments, expressiveness, complexity and impossibility results.
- BOTH care about interaction between agents.

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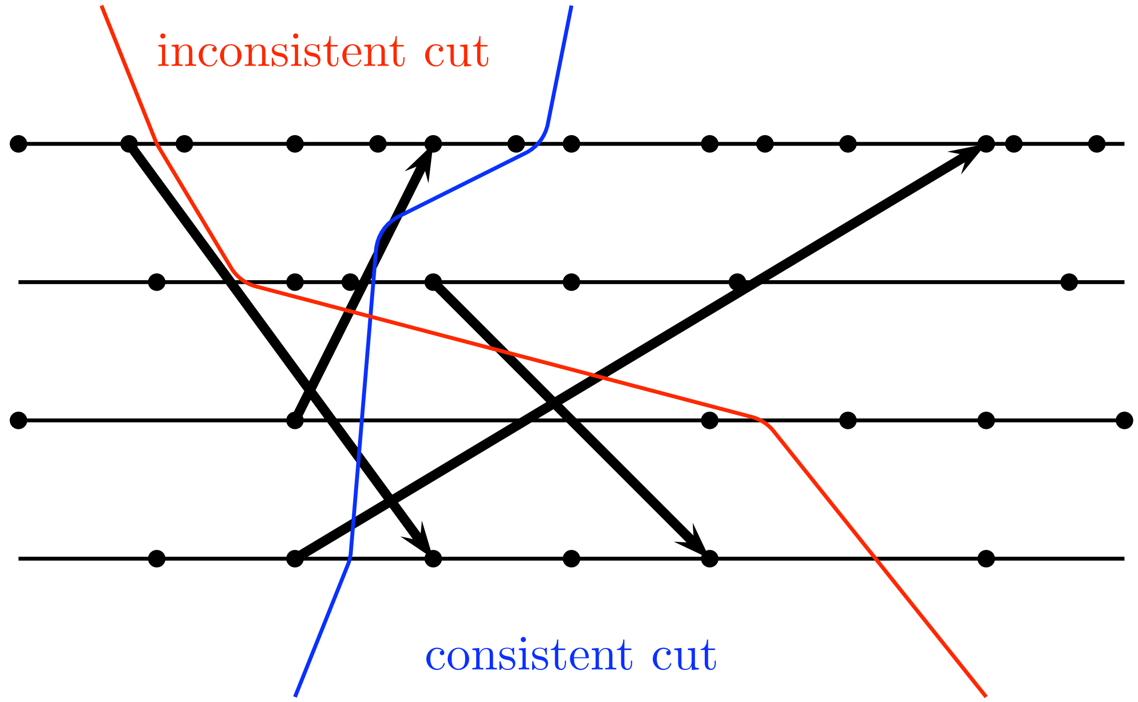
- If St is the set of states then the agent knows phi in state s if for all states t with s~t, phi is true in t.



A Lamport spacetime diagram

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$$(a, c) \models K_i\phi \text{ if}$$
$$\forall (a', c') \sim_i (a, c)\ (a', c') \models \phi$$

Axioms for Knowledge

1. All propositional tautologies
2. $(K_i\phi) \wedge (K_i(\phi \Rightarrow \psi)) \Rightarrow K_i\psi$
3. $K_i\phi \Rightarrow \phi$
4. $K_i\phi \Rightarrow K_iK_i\phi$
5. $\neg K_i\phi \Rightarrow K_i(\neg K_i\phi)$
6. *Modus Ponens*
7. From ϕ infer $K_i\phi$

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- The axioms given are for a static situation.
- Many combinations are possible: time, probability, dynamic update.

Co-algebras

- Intimately tied to transition systems and to modal logics.
- An algebra: op: A x A ---> A
- A co-algebra: co-op: A ---> A x A
- Split instead of combine.

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The usual notion of labelled transition system.

Bisimulation

An equivalence relation that is intimately related to transition systems.

$s \sim t$ means:

$\forall a \in \mathcal{L},\ s \xrightarrow{a} s'$ implies

$\exists t'$ such that $t \xrightarrow{a} t'$ with $s' \sim t'$

and vice versa.

May not be the most useful equivalence relation (far too fine) but it is mathematically natural and is intimately tied to the transition system.

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The modal formulas are the maximal set of first-order formulas invariant under bisimulation.

The trinity

- There is a close relation between
- transition systems and bisimulation
- modal logic and
- coalgebras.

Combining Modalities

- Combine coalgebras in a suitable way.
- Purely mathematical; one still needs a conceptual understanding.
- May have formidable technical difficulties: e.g. combining probability and nondeterminism.

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- How does it relate, if at all?

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- Security: information flow, anonymity.
- Suppose we model a protocol intended to maintain anonymity using a process algebra.
- Nondeterminism is resolved by a “scheduler”; in the end we quantify over all schedulers.
- But this includes schedulers that could leak information.

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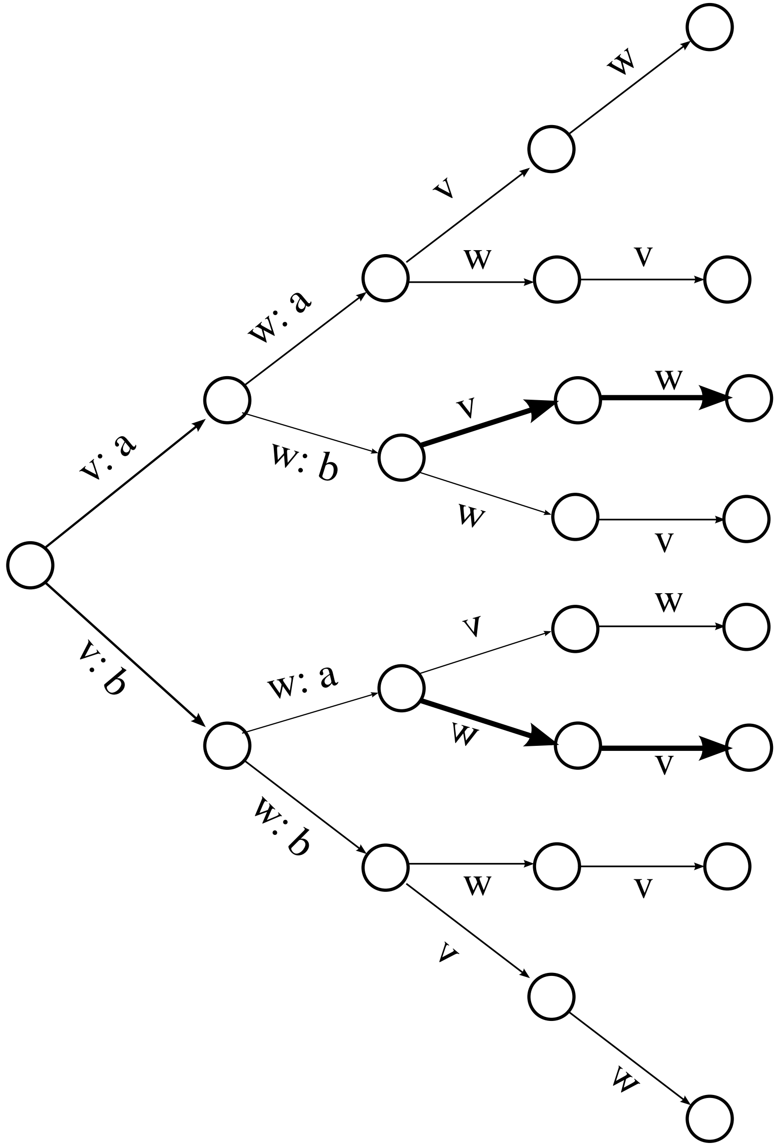
- It must **not** reveal who voted for whom; unless the vote is unanimous.
- A scheduler can leak the votes!



A scheduler that leaks voting preferences.

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Games and Knowledge

- Games are an ideal setting to explore epistemic concepts.
- Economists have been particularly active in developing these ideas.

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- Games in logic: model theory, EF, Lorenzen,...

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- The concurrent process is the "board" and the moves end up choosing the action.
- We control what the schedulers "know" by putting restrictions on the allowed strategies.

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- Easy to impose epistemic restrictions on strategies.

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- Games are already used in many ways in concurrency, semantics, logic and economics.
- But we still do not have a systematic way of describing and reasoning about interacting agents algebraically.

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- Use this to reason about information flow.

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- There is a beautiful relation between probability, bisimulation and logic.
- We shall review this quickly.

Fully Probabilistic Transition Systems

- A transition system with probabilities and actions (labels) associated with the transitions.

- $$(S, \mathcal{L}, \forall a \in \mathcal{L}\ T_a : S \times S \to [0,1])$$

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Extended to systems with *arbitrary measurable* state spaces in LICS'97 by Blute, Desharnais, Edalat and P. We called them *Labelled Markov Processes*, very similar to *Markov Decision Processes* without the rewards.

Larsen-Skou Bisimulation

- Let $\mathcal{S} = (S, \mathcal{L}, T_a)$ be a probabilistic transition system.
- An equivalence relation R on S is a **bisimulation** if whenever sRs', with $s, s' \in S$, we have that for all $a \in \mathcal{L}$ and every R-equivalence class, A, $T_a(s, A) = T_a(s', A)$.
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Some painful technical problems had to be overcome to extend this to LMPs.

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Notice that there is no negation not even any negative propositions!
Simpler than the non-probabilistic case.

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Probability theory itself is a kind of logic!

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Probability theory as a kind of "logic."

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- What happens when we combine probability and knowledge or belief?

Probabilistic Epistemic Logic

- One can combine modalities for probability and knowledge.
- Interpret them using epistemic probability frames.
- Can be generalized to non measure-theoretic formalisms for modelling uncertainty.
- See “Reasoning About Uncertainty” by J. Halpern

Information vs Knowledge

- Information theory measures information in bits. No attempt to say which bits are important.
- Information theory gives "inference rules" for reasoning about how information is updated.
- Is information theory a kind of logic?

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- $H(0, 0, \ldots, 1, 0, \ldots, 0) = 0$
- $H(\frac{1}{n}, \ldots, \frac{1}{n}) = \log_2 n.$
- Clearly continuous.

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Entropy is the unique continuous function that is:

- maximized by the uniform distribution
- minimized by the point distribution
- additive when you combine systems
- and ....

What does it tell us?

If you have a distribution $p(s)$ on a set S,
you can define a code such that it takes $H(p)$
bits on the average to encode the members of the set.

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It would require $H(p) + KL(p \mapsto q)$ bits.

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$KL(p \mapsto q) = 0.2075$ and $KL(q \mapsto p) = 0.1887$.

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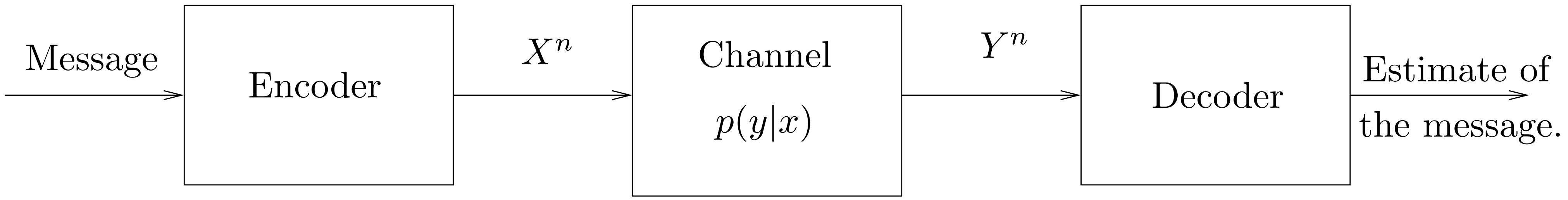
- Clearly an epistemic concept.

Channels



A typical channel.

How well can we estimate the intended message if the channel is noisy?

Channel Capacity

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$$C = \max_{p(x)} I(X;Y).$$

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- Need probabilistic notions of anonymity: Halpern and O'Neill; Bhargava and Palamidessi, Chatzikokolakis and Palamidessi

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- Perfect anonymity corresponds to zero capacity.

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- What if there are two agents with slightly different partitions of the possible worlds?
- Epistemic logic may tell us that nothing is common knowledge!
- Intuitively, there should be high probability of one agent guessing what the other knows.

- Key point noted by Krasucki et al.: How much information is acquired when probabilities change as a result of new data?

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- They define **information gain** and develop its properties and its relations with mutual information and common knowledge.

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- They define **information gain** and develop its properties and its relations with mutual information and common knowledge.
- Information gain turns out to be exactly the same as relative entropy!

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- One needs metrics and metric reasoning principles: Jou and Smolka, DGJP, van Breugel and Worrell, ...
- What are appropriate "approximate" concepts of knowledge? Surely something information theoretic.
- But, beware of applying information theory naively: there are many counter-examples.

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- This will lead to a mixture of probability and nondeterminism.
- Additivity is lost: one has capacities instead. What information-theoretic concepts apply?
- Relate the dynamics with an appropriate quantitative logic: perhaps a suitable multi-agent generalization of information theory.

Information Theory as a Logic

- Why does it matter?
- It would give us compositional ways of reasoning about information flow.
- Perhaps a duality theory lurking underneath the surface.

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- Entirely new phenomena are possible: entanglement and teleportation, knowing "less than nothing!" [Andreas Winter]
- Can it do anything for standard distributed protocols?

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- With a GHZ state one can do distributed consensus.
- Both these protocols are trivial: can we use these resources to do more clever things?

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- At the end of the protocol Alice will not have the quantum state anymore.

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- Another approach [d'Hondt, P]: there is no such thing as quantum knowledge!
- Measurements create classical knowledge.

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- Reason compositionally about quantum protocols: process algebras, equivalences, resource inequalities [Devetak et al.]
- Develop interesting protocols for distributed computing tasks.

Conclusions

- Concurrency theory should incorporate the idea of games between agents and investigate richer modes of interaction than currently available: some key ideas due to Abramsky.
- Epistemic ideas should come to the fore.
- Information theory should be developed as a kind of quantitative epistemic logic.

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Wild ideas

- What is information theory in a "relativistic" setting?
- Mutual information is very dependent on absolute time, what does it mean when we do not have absolute global states?
- Believe it or not "relativistic quantum information theory" is alive and well!!

Thanks for listening!

Thanks to Samson Abramsky, Vincent Danos, Josee Desharnais, Vineet Gupta, Radha Jagadeesan, Sophia Knight, Dexter Kozen, Keye Martin, Catuscia Palamidessi, Caitlin Phillips, Doina Precup and many others.