Domain Theory and the Causal Structure of Spacetime

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Outline

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1. Introduction
2. Causal Structure
3. Domain Theory
4. Domains and Causal Structure
5. Interval Domains
6. Reconstructing spacetime

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Overview

- Causality is taken to be fundamental in physics and equally in computer science.
- In spacetime physics: Newton, Einstein, Penrose, Hawking, Sorkin.
- In computer science: Petri, Lamport, Pratt, Winskel.
- Causality forms a partial order: there are no causal cycles.
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Motivation

- We do not know how to reconcile relativity and quantum mechanics.
- The holy grail: a theory of quantum gravity.
- One approach [Sorkin] : discrete causal sets. Spacetime is a discrete poset. Causality is the fundamental structure.
- Our work is inspired by Sorkin but it is not the same framework.
- We are exploring the *classical* analogue of this question: in non-quantum relativity can the causal order be taken as fundamental?

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Where Does Computability Come In?

- Scott's vision: computability should be continuity in some topology.
- A finite piece of information about the output should only require a finite piece of information about the input.
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Summary of Results

- The causal order alone determines the topology of globally hyperbolic spacetimes. [CMP Nov'06]
- A (globally hyperbolic) spacetime can be given domain structure: approximate points. [CMP Nov'06]
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The first two items actually work for strongly causal spacetimes (Sumati Surya, in prep.).

The Newtonian View

- Spacetime is a 4-dimensional manifold $\mathcal{M}$.
- $\mathcal{M}$ has a canonical product structure: $\mathcal{M} = \mathbb{R} \times S$. $\mathbb{R}$ is "absolute" time and S is space.
- An event at $(t, \vec{\mathbf{x}})$ can influence any other event $(t', \vec{\mathbf{x}}')$ if $t < t'$.
- "Now" (a surface of simultaneity) is the boundary between the past and the future and defines "space."

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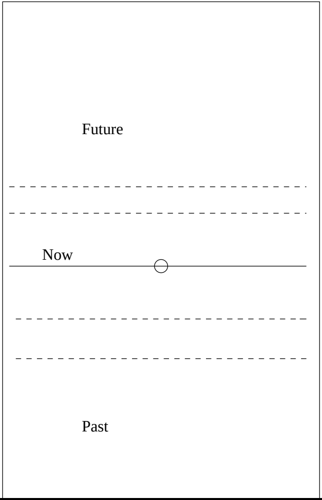
A Picture of Newtonian Spacetime



The layers of spacetime structure

- Set of events: no structure
- Topology: 4 dimensional real manifold, Hausdorff, paracompact,...
- Differentiable structure: tangent spaces
- Causal structure: light cones, defines metric up to conformal transformations. This is $\frac{9}{10}$ of the metric.
- Parallel transport: affine structure.
- Lorentzian metric: gives a length scale.

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The causal structure of spacetime

- At every point a pair of “cones” is defined in the tangent space: future and past light cone. A vector on the cone is called **null** or **lightlike** and one inside the cone is called **timelike**.
- We assume that spacetime is *time-orientable*: there is a global notion of future and past.
- A *timelike* curve from x to y has a tangent vector that is everywhere timelike: we write $x \prec y$. (We avoid $x \ll y$ for now.) A *causal* curve has a tangent that, at every point, is either timelike or null: we write $x \leq y$.

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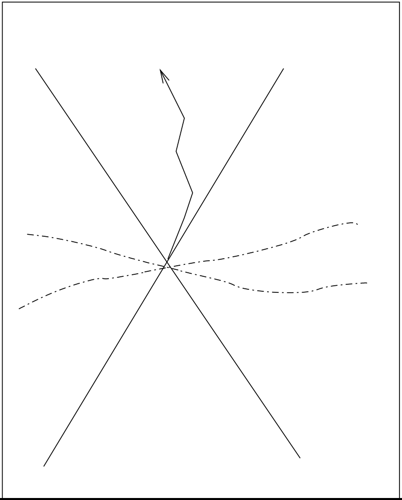
Relativistic Spacetime



Causal Structure of Spacetime II

- $I^+(x) := \{y \in M | x \prec y\}$; similarly I^-
- $J^+(x) := \{y \in M | x \leq y\}$; similarly J^-.
- $I^\pm$ are always open sets in the manifold topology; $J^\pm$ are **not** always closed sets.
- Chronology: $x \prec y \Rightarrow y \not\prec x$.
- Causality: $x \leq y$ and $y \leq x$ implies $x = y$.

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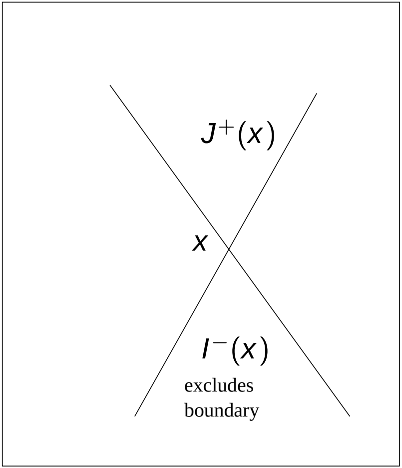
Fundamental Causality Assumption

$\leq$ is a partial order.

$J^+(x)$ and $I^-(x)$



Causality Conditions

- $$I^{\pm}(p) = I^{\pm}(q) \Rightarrow p = q.$$
- Strong causality at p: Every neighbourhood $\mathcal{O}$ of p contains a neighbourhood $\mathcal{U} \subset \mathcal{O}$ such that no causal curve can enter $\mathcal{U}$, leave it and then re-enter it.
- Stable causality: perturbations of the metric do not cause violations of causality.
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Global Hyperbolicity

- Spacetime has good initial data surfaces for global solutions to hyperbolic partial differential equations (wave equations). [Leray]
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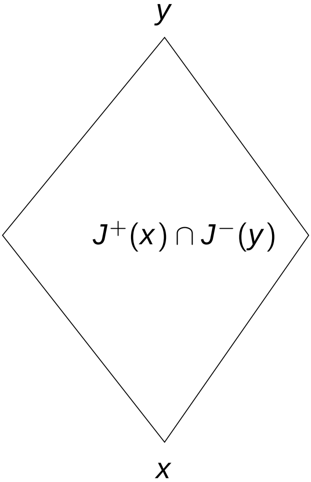
A Spacetime Interval



The Alexandrov Topology

Define

$$\langle x, y \rangle := I^+(x) \cap I^-(y).$$

The sets of the form $\langle x, y \rangle$ form a base for a topology on M called the Alexandrov topology.

Theorem (Penrose): TFAE:

1. (M, g) is strongly causal.
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Recursion as a Fixed Point

- Kleene had the idea of explaining recursion as a fixed point.
- Scott: how to obtain a model of the λ-calculus?

$$D \equiv [D \to D].$$

  No way!!
- Construct a model of the λ-calculus using posets and topologies on posets.
- Use the topology to cut down to the continuous maps.

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Domain theory

- Order as (qualitative) information content.
- data types are organized into so-called “domains”: directed-complete (directed sets have least upper bounds) posets
- For “directed set” think “chain.”
- computable functions are viewed as *continuous* with respect to a suitable topology: the Scott topology.
- ideal (infinite) elements are limits of their (finite) approximations.

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Examples of domains

- The integers with no relation between them and a special element $\perp$ below all the integers: a flat domain.
- Sequences of elements from $\{a, b\}$ ordered by prefix: the domain of streams.
- Compact non-empty intervals of real numbers ordered by *reverse* inclusion (with **R** thrown in).
- X a locally compact space with $K(X)$ the collection of compact subsets ordered by reverse inclusion.

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- One can view domain theory as a way of formalizing a theory of computability on topological spaces: Tucker and Stoltenberg-Hansen.
- Open sets are finitely checkable properties.
- More generally, one can seek a domain theoretic analogue of analysis seeking a computational handle on the subject: Edalat, Escardo, Alex Simpson, Keye Martin...

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Computational Intuition: Streams

- A box with input streaming in and output streaming out: may proceed forever.
- The more we see the better we know the "complete" output.
- Anything that is output cannot be retracted.
- Computable: to see a finite "portion" of the output only a finite amount of input can be required.
- Typical non-computable function: output "yes" if there are infinitely many zeros in the input.

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The Way-below relation

- In addition to $\leq$ there is an additional, (often) irreflexive, transitive relation written $\ll$: $x \ll y$ means that x has a "finite" piece of information about y or x is a "finite approximation" to y. If $x \ll x$ we say that x is *finite*.
- The relation $x \ll y$ - pronounced x is "way below" y - is directly defined from $\leq$.
- Official definition of $x \ll y$: If $X \subset D$ is directed and $y \leq (\sqcup X)$ then there exists $u \in X$ such that $x \leq u$. If a limit gets past y then some finite stage of the limiting process already got past x.

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- In addition to $\leq$ there is an additional, (often) irreflexive, transitive relation written $\ll$: $x \ll y$ means that x has a "finite" piece of information about y or x is a "finite approximation" to y. If $x \ll x$ we say that x is *finite*.
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The role of way below in spacetime structure

- **Theorem:** Let (M, g) be a spacetime with Lorentzian signature. Define $x \ll y$ as the way-below relation of the causal order. If (M, g) is globally hyperbolic then $x \ll y$ iff $y \in I^+(x)$.
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We can stop being coy about notational clashes: henceforth $\ll$ is way-below *and* the timelike order.

Bicontinuity and Global Hyperbolicity

- The definition of continuous domain - or poset - is biased towards approximation from below. If we symmetrize the definitions we get bicontinuity (details in the CMP paper).
- Theorem: If (M, g) is globally hyperbolic then $(M, \leq)$ is a bicontinuous poset.
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We *define* a globally hyperbolic poset $(X, \leq)$ to be

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An Important Example of a Domain: $\mathbf{I}\mathbb{R}$

- The collection of compact intervals of the real line

$$\mathbf{I}\mathbb{R} = \{[a,b] : a,b \in \mathbb{R} \;\&\; a \leq b\}$$

ordered under reverse inclusion

$$[a,b] \sqsubseteq [c,d] \Leftrightarrow [c,d] \subseteq [a,b]$$

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$$\max(\mathbf{I}X) \simeq X$$

where the set of maximal elements has the relative Scott topology from $\mathbf{I}X$.

Spacetime from a discrete ordered set

If we have a countable dense subset $\mathcal{C}$ of $\mathcal{M}$, a globally hyperbolic spacetime, then we can view the induced causal order on C as defining a discrete poset. An ideal completion construction in domain theory, applied to a poset constructed from $\mathcal{C}$ yields a domain $\mathbf{I}C$ with

$$\max(\mathbf{I}C) \simeq \mathcal{M}$$

where the set of maximal elements have the Scott topology. Thus from a countable subset of the manifold we can reconstruct the manifold as a topological space.

Globally Hyperbolic Posets and Interval Domains

- One can define a category of globally hyperbolic posets
- and an abstract notion of “interval domain”: these can also be organized into a category.
- These two categories are equivalent.
- Thus globally hyperbolic spacetimes *are* domains - not just posets - but
- not with the causal order but, rather, with the order coming from the notion of intervals; i.e. from notions of approximation.

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Spacetime as a domain

- The domain consists of intervals $[x, y] = J^+(x) \cap J^-(y)$.
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Other layers of structure

- We would like to put differential structure on the domain and
- metric structure as well.
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- Keye Martin defined a concept called a "measurement." This is designed to capture quantitative notions on domains.
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Keye's measurements

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- The fact that globally hyperbolic posets are interval domains gives a sensible way of thinking of "approximations" to spacetime points in terms of intervals.
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- How do we understand differential structure in this framework? This is vital for understanding how to describe dynamics of systems living on spacetime.

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Thanks! and welcome back to Earth.