1. What is a depth buffer, and how it is used in hidden surface removal?

2. What is Phong shading used for?

3. Describe how the accumulation buffer can be used to produce the following:
   
   (a) soft shadows
   (b) depth of field
   (c) motion blur

4. Describe at least two situations where aliasing can arise when rendering a scene.

5. What is mipmapping?

6. Can Gouraud shading produce a highlight in the interior of a face?

7. Can Phong shading produce a highlight in the interior of a face?

8. Can the standard OpenGL pipeline easily handle light scattering from object to object?

9. The rendering equation can be written as

   \[ L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega} f_r(x, \omega_o, \omega_i) L_i(x, \omega_i)(-\omega_i \cdot n) \ d\omega_i. \]

   What are the assumptions of this model, and what further approximations are made for the Radiosity rendering method?

10. What property of the eye allows us to represent a color with 3 values?

11. Give the advantages and disadvantages of implicit and parametric curves.

12. What are the Cohen-Sutherland outcodes for clipping a line in \((x, y)\) space? Sketch an example of a line segment that can be neither trivially accepted, nor rejected. What are the outcodes of its endpoints prior to clipping? (You do not need to clip the line.)

13. Consider the ellipsoid \((x - 3)^2 + y^2 + 4z^2 = 20.\)

   (a) Write the ellipsoid in the form \(x^T Q x = 0\) where \(x^T = (x, y, z, 1).\)

   (b) What is the equation of this ellipsoid after it has undergone a projective transformation to the view volume between planes \(z = -1\) and \(z = -8.\)

   Hint: It is sufficient for you to write your answer as a product of matrices and vectors. One of these matrices should be of the form:

   \[
   M = \begin{pmatrix}
   f_0 & 0 & 0 & 0 \\
   0 & f_0 & 0 & 0 \\
   0 & 0 & f_0 + f_1 & -f_0 f_1 \\
   0 & 0 & 1 & 0
   \end{pmatrix}.
   \]
14. Show that if \( Q \) is symmetric (\( Q = Q^t \)), the matrix \( (M^t)^{-1}QM^{-1} \) is also symmetric (hence, with respect to the previous question, the transformed points \( Mx \) also lie on a quadric surface).

15. One can define an ellipsoid surface in \( \mathbb{R}^3 \) by scaling, rotating, and translating a unit sphere. Define an ellipsoid whose:
   - center is \((8, 1, 3)\) in world coordinates,
   - \(x, y, z\) axes are of length 1, 1, 4, respectively.
   - \(z\) axis is in direction \((1, 1, 0)\) in world coordinates,

For your answer, it is sufficient that you specify suitable matrices \( S, R, T \), and then use them to define a quadric matrix \( Q \).

16. Write pseudocode that perform one level of subdivision using the subdivision mask \((\frac{1}{4}, \frac{1}{2}, \frac{1}{4})\) (i.e., the Lane-Riensfeld algorithm for producing quadratic B-spline curves). Assume the additional rule for the boundary that leaves the endpoints fixed.

17. Write pseudocode that performs a vertex split on a half edge data structure. Your method should take as parameters two half edges, \( e_1 \) and \( e_2 \) (that both have as head the same vertex \( v \)), and two vertex positions \( v_1 \) and \( v_2 \). The new edge between \( v_1 \) and \( v_2 \) has two incident faces that should be inserted between \( e_1 \) and its twin, and \( e_2 \) and its twin. Be sure to also correctly change the head vertex of all the half edges that were originally arriving at \( v \). Draw a diagram to show the local structure of the mesh before and after to convince yourself that your pseudocode is correct.

How many half edges do you need to create? Write comments in your pseudo code to describe exactly what is happening.

18. Suppose three faces of a cube meet at a vertex \( v_0 \) at the origin with normals pointing in each of the positive axis directions.

(a) For each plane, write a simple expression of the form \( q^TK_rq \) for the squared distance of a point \( q \) to the plane.
(b) Write the quadric error function for the vertex where these three planes meet. That is, give $Q_{v_0}$. What can you say about the shape of this quadric?

(c) Suppose that 3 other faces of this cube meet at the vertex $v_1$, which is at the point $(-1, 0, 0, 1)^T$ in homogeneous coordinates. The faces have normals in positive y, positive z, and negative x directions. Write the quadric error function for this vertex. That is, give $Q_{v_1}$. What can you say about the shape of this quadric?

(d) What can you say about the shape of the quadric defined by sum $Q_{v_0} + Q_{v_1}$.

19. How can you decide if two triangles intersect? Write pseudocode to outline your intersection test.

20. Given a half edge data structure and a half edge pointing at a given vertex, as shown in the grid below, write a sequence of pointer dereferences that will bring you to any half edge with the destination vertex as head. Use n for next half edge, and t for the twin half edge, so if the destination was at the head of the next half edge’s twin, then your answer would be $he = current.n.t$;

21. What does the Warnock algorithm do to solve the problem of Painter’s algorithm.

22. What property of the eye allows us to represent a color with 3 values?

23. About ten percent of human males are missing one of the three types of visual cones needed for seeing in color. Such people are said to be color blind.

Consider the following statement. “$I_1(\lambda)$ and $I_2(\lambda)$ be any two color spectra. If a color blind person who is missing the L cones can distinguish these two spectra, then a normal color vision person can distinguish these spectra.”

Is the statement true? If so, then justify it using a mathematical argument. If not, then explain why.

24. Why are the opposite ends of the spectrum in the CIE horse shoe diagram connected by a straight line?

25. (a) Suppose that a shiny ground plane $y = 0$ is illuminated by sunlight. Let the sun be in direction $(4, 3, 0)$ and let the camera be at position $(x, y, z) = (2, 3, 1)$. Determine the position on the ground plane at which the peak of the highlight occurs.
(b) Briefly describe how you would answer the same question if the shiny plane were of the more general form

\[ Ax + By + Cz = D. \]

** Hint: for a mirror, \( l + r = 2(n \cdot l)n. \)

26. Consider a rectangle in the image plane. The intensities \( I(x,y) \) at the four corner vertices are:

\[ I(20,20) = 50, \quad I(60,20) = 80, \quad I(20,40) = 100 \quad I(60,40) = 110. \]

Use bilinear interpolation to calculate the intensity \( I(30,30) \).

27. Calculate the result of the following compositing operations on \( rgb\alpha \) values:

(a)

\[ \left( \frac{4}{5}, \frac{2}{5}, \frac{4}{5}, 1 \right) \text{ over } \left( \frac{3}{5}, 0, \frac{1}{5}, 1 \right) \]

(b)

\[ \left( \frac{4}{5}, \frac{2}{5}, \frac{4}{5}, \frac{4}{5} \right) \text{ over } \left( \frac{3}{5}, 0, \frac{1}{5}, \frac{4}{5} \right) \]

28. How are *shadow buffers* used in point source lighting? In particular, compare the case of a point source in the scene versus a directional source (sunlight). Why is it difficult to accurately compute cast shadows using a shadow buffer? Sketch an example of a case where shadow buffers perform poorly.