Midterm Exam, Computer Graphics (COMP 557)

Thursday October 15, 2009
1:05 - 2:25 (80 minutes)
Professor Paul Kry

- This exam has 3 parts and a total of 16 questions.
- There are a total of 33 marks.
- This is a closed book midterm.
- Basic calculators are permitted.
- Basic calculators are unnecessary.
- No other electronic devices are allowed.
- Use the provided exam booklet for your answers.
- Always indicate clearly the question you are answering.
- Show your work and write legibly.
- Write your name and student number on the front of the provided booklet.
- Please wait until instructed before turning the page.
Very Short Questions (11 marks)

Provide brief answers to the following questions. You should not spend more than a minute or two on any of these questions (i.e., if you find that you are taking longer, then skip it and come back to it).

1. Describe the main difference between affine transformations and linear transformations.

2. Let $P$ be an invertible homogeneous matrix representing a perspective projection that preserves the $z$ value of the $z = f_0$ near plane and $z = f_1$ far plane, i.e.,

$$
P = \begin{pmatrix}
f_0 & 0 & 0 & 0 \\
0 & f_0 & 0 & 0 \\
0 & 0 & f_0 + f_1 & -f_0f_1 \\
0 & 0 & 1 & 0
\end{pmatrix}
$$

Where do points behind the viewer go to after perspective projection?

3. Give a homogeneous representation of the point $p = (2, 5, 7)^T$. Give a homogeneous representation of the vector $v = (2, 5, 7)^T$. Give a non-homogeneous representation of the point expressed in homogeneous coordinates as $(2, 5, 7, 7)^T$.

4. Not all planar projections are perspective projections. Describe (or name) the two main types of a non perspective planar projections.

5. When can you convert a quadratic Bezier to a Cubic Hermite curve?

6. Given the bicubic Bezier patch $p(s, t) = \sum_{i=0}^{3} \sum_{j=0}^{3} B_{i,3}(s) B_{j,3}(t) g_{ij}$, $s \in [0, 1]$, $t \in [0, 1]$, with $g_{ij} \in \mathbb{R}^3$, write a simple expression for a vector that is parallel to the surface normal at $p(0, 0)$.

7. Describe the difference between $C^1$ and $G^1$ continuity.

8. Which parts of the teapot in Assignment 2 have a perfectly circular cross section? Explain your answer.

9. Describe one advantage and one disadvantage of Frenet Frames.

10. How many real values are necessary to describe the shape of a cubic rational Bezier curve in three dimensional space? Provide a short justification for your answer.

11. Given a polynomial $p(t) = \sum_{i=0}^{n} \alpha_i t^i$, for $t \in [0, 1]$ describe in words an easy method for computing bounds on the values obtained by this polynomial curve, i.e., $l$ and $h$ such that $p(t) \in [l, h]$ for all $t \in [0, 1]$. Explain why your method works.
Sketch and Describe Questions (5 marks)

12. **(4 marks total)** Consider the following picture, and let matrix $M$ specify how to change from object coordinates to world coordinates, and let

$$R_Z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The object coordinate frame and the happy face are drawn in world coordinates by first transforming by $M$.

(a) **(2 marks)** Draw a sketch and describe in words what happens if we replace $M$ with $R_Z(\theta)M$, for $\theta = \pi/2$.

(b) **(2 marks)** Draw a sketch and describe in words what happens if we replace $M$ with $MR_Z(\theta)$, for $\theta = \pi/2$.

13. **(1 mark)** Consider the following figure consisting of a piecewise linear curve used to define a smooth subdivision curve. Draw a diagram that clearly shows one level of subdivision of the curve using the subdivision mask $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ (i.e., the Lane-Riensfeld algorithm for producing quadratic B-spline curves). Assume that endpoints stay fixed.
Multi-Part Questions (17 marks)

14. (6 marks total) Let \( p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)^T \) be a point in non-homogeneous coordinates on a sphere with center at the origin and with radius equal to 1. Let \( n = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)^T \) be the normal vector in non-homogeneous coordinates of the sphere at point \( p \). Given transformations, \( T, S, R \),

\[
T = \begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad S = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad R = \begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

(a) (3 marks) What is the position of the point \( p \) on the sphere after the sphere is transformed by the product \( TSR \)? Show your work by applying each transformation separately rather than computing the product of the three matrices.

(b) (3 marks) What is the normal of the point \( p \) on the sphere after the sphere is transformed by the product \( TSR \)? Show your work by applying each transformation separately.

15. (3 marks total) Let \( p = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0) \) and \( q = (0, 1, 0, 0) \) be tuples representing two quaternions, which in turn represent rotations. Interpret the tuple \( (s, x, y, z) \) as \( s + xi + yj + zk \).

(a) (2 marks) What is the product \( pq \)?

(b) (1 mark) What is the axis of the rotation represented by the product \( pq \)?

16. (8 marks total) Let \( g_0 = (0, 0)^T, g_1 = (1, 1)^T, g_2 = (-1, 1)^T \) be the control points for a quadratic Bezier curve \( c(t) = \sum_{i=0}^{2} B_{i,2}(t)g_i \).

(a) (3 marks) Evaluate the Bernstein basis functions at \( t = 0.5 \), and compute the position of the point \( c(0.5) \).

(b) (3 marks) What is the derivative of the curve at \( t = 0.5 \)? Show your work.

(c) (2 marks) Make a sketch of the control polygon and apply De Casteljau’s algorithm to draw the position of \( c(0.5) \). Does your picture agree with the answer to the first two parts of this question?