Population Dynamics

- Deductive modelling: based on physical laws
- Inductive modelling: based on observation + intuition
- Single species:
  Birth (in migration) Rate, Death (out migration) Rate

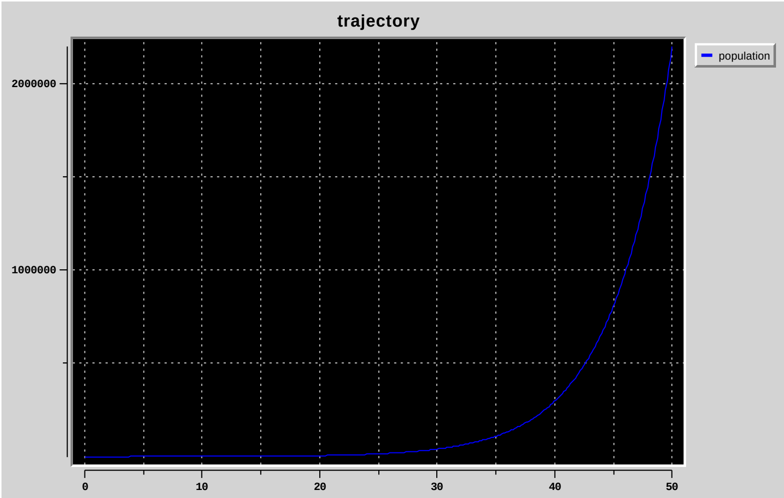
$$\frac{dP}{dt} = BR - DR$$

- Rates proportional to population

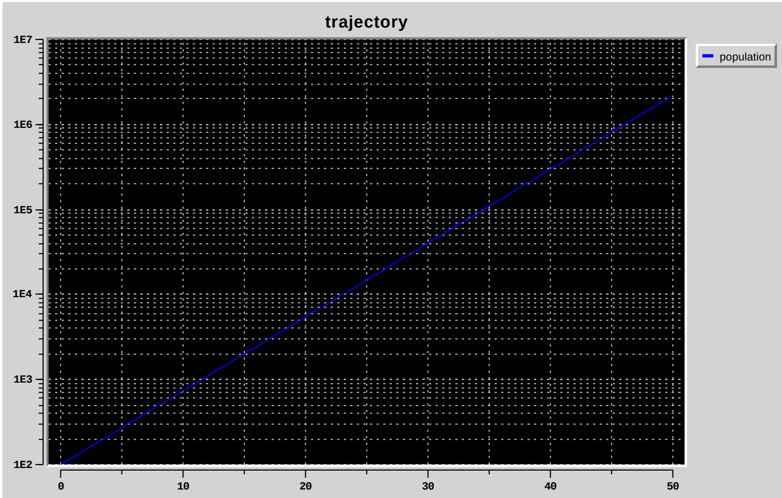
$$BR = k_{BR} \times P; DR = k_{DR} \times P$$

$$\frac{dP}{dt} = (k_{BR} - k_{DR})P$$

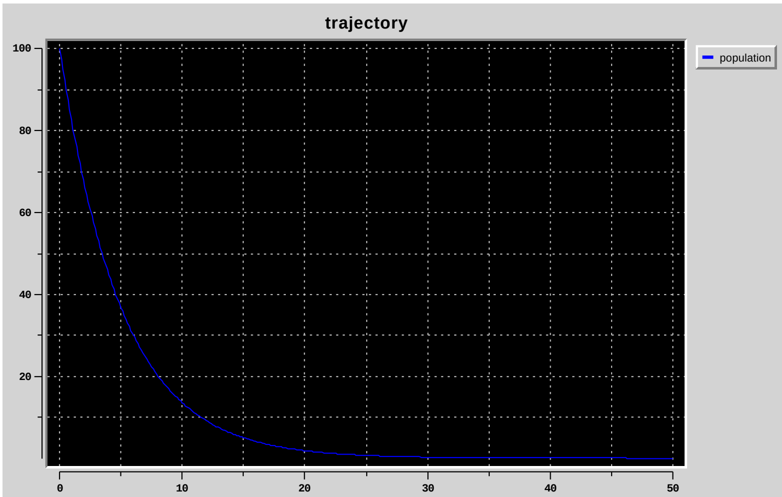
$k_{BR} = 1.4, k_{DR} = 1.2$: Exponential Growth



$k_{BR} = 1.4, k_{DR} = 1.2$: log(Exponential Growth)



$k_{BR} = 1.2, k_{DR} = 1.4$: Exponential Decay



Logistic Model

- Are k_{BR} and k_{DR} really constant ?
- Energy consumption in a *closed* system $\rightarrow$ limits growth

$$E_{pc} = \frac{E_{tot}}{P}$$

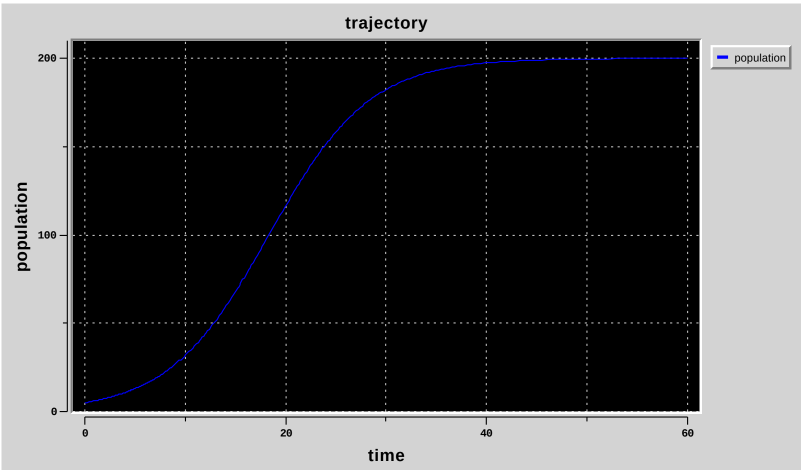
$P \uparrow \rightarrow E_{pc} \downarrow \rightarrow k_{BR} \downarrow$ and $k_{DR} \uparrow$ until equilibrium

- "crowding" effect:
  ecosystem can support maximum population P_{max}

$$\frac{dP}{dt} = k \times (1 - \frac{P}{P_{max}}) \times P$$

- crowding is a quadratic effect

$$k_{BR} = 1.2, k_{DR} = 1.4, crowding = 0.001$$



Disadvantages

- *NO* physical evidence for model structure !
- But, many phenomena can be well *fitted* by logistic model.
- P_{max} can only be *estimated* once steady-state has been reached. Not suitable for control, optimisation, ...
- Many-species system: P_{max}, steady-state ?

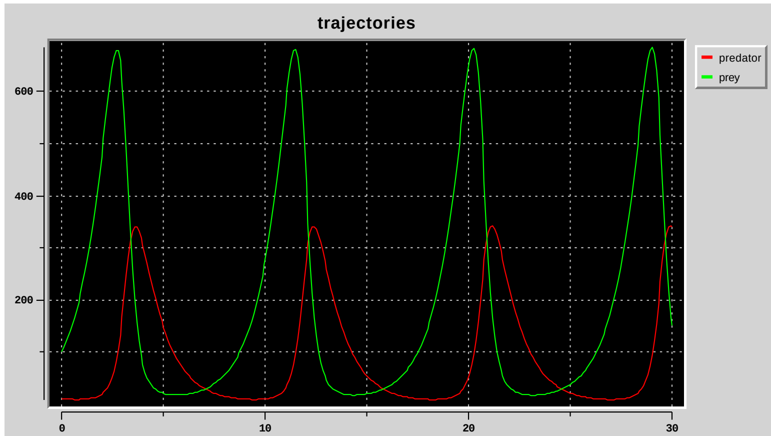
Multi-species: Predator-Prey

- Individual species behaviour + *interactions*
- Proportional to species, no interaction when one is extinct: *product* interaction $P_{pred} \times P_{prey}$

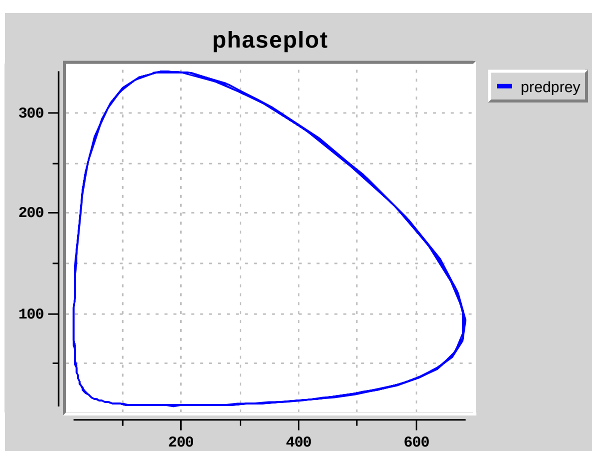
$$\begin{aligned} \frac{dP_{pred}}{dt} &= -a \times P_{pred} + k \times b \times P_{pred} \times P_{prey} \\ \frac{dP_{prey}}{dt} &= c \times P_{prey} - b \times P_{pred} \times P_{prey} \end{aligned}$$

- Excess death rate $a > 0$, excess birth rate $c > 0$, grazing factor $b > 0$, efficiency factor $0 < k \leq 1$
- *Lotka-Volterra* equations (1956): periodic steady-state

Predator Prey (population)



Predator Prey (phase)



Competition and Cooperation

- Several species competing for the *same* food source

$$\begin{aligned}\frac{dP_1}{dt} &= a \times P_1 - b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= c \times P_2 - d \times P_1 \times P_2\end{aligned}$$

- Cooperation of different species (symbiosis)

$$\begin{aligned}\frac{dP_1}{dt} &= -a \times P_1 + b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= -c \times P_2 + d \times P_1 \times P_2\end{aligned}$$

Grouping and *n*-species Interaction

- Grouping (opposite of crowding)

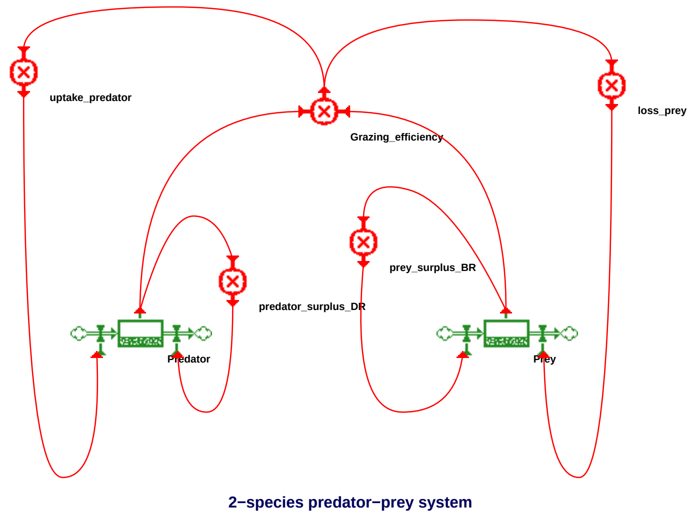
$$\frac{dP}{dt} = -a \times P + b \times P^2$$

- *n*-species interaction

$$\frac{dP_i}{dt} = (a_i + \sum_{j=1}^{n} b_{ij} \times P_j) \times P_i, \forall i \in \{1, \ldots, n\}$$

- Only *binary* interactions,
  no $P_1 \times P_2 \times P_3$ interactions

Forrester System Dynamics



Experiments

- Simulation
- Parameter Fit
- Optimisation
- Shooting

Experiment Scripts (Resources)

```
# Simulation: Integrator.

exp_simul_control_send simul_integ_set_start_time 0;
exp_simul_control_send simul_integ_set_stop_time 31.5;
exp_simul_control_send simul_integ_set_method 9;
exp_simul_control_send simul_integ_set_accuracy 1e-06;
exp_simul_control_send simul_integ_set_max_no_steps 0;
exp_simul_control_send simul_integ_set_initial_step_size 0.1;
exp_simul_control_send simul_integ_set_min_step_size 0.001;
exp_simul_control_send simul_integ_set_max_step_size 1;
```


Chaining

```
exp_read \
 $env(WESTPP_DATA_PATH)/examples/nonPhysical/Circle/Circle.RK4ASC.exp.tcl;
exp_start;
exp_wait;

exp_read \
 $env(WESTPP_DATA_PATH)/examples/systemDynamics/PredPrey/PredPrey.exp.tcl;
exp_start;
exp_wait;
```

Iteration

```
exp_read \
 $env(WESTPP_DATA_PATH)/examples/nonPhysical/Circle/Circle.exp.tcl;
# if no plots are wanted
#exp_simul_control_send simul_output_plot_set_flag 0;
# if plots are wanted (just for demos)
exp_simul_control_send simul_output_plot_set_flag 1;
set values {0.1 0.8 1.2 2.0 5.0};
for {set i 0} {$i < 5} {incr i} \
{
  set value [lindex $values $i];
  exp_message \
    1 \
    ".Circle.x initial value = $value" \
    "";
  exp_simul_control_send simul \
    .Circle.x set_initial_value $value;
  exp_simul_control_send simul_output_file_set_name \
    $env(WESTPP_DATA_PATH)/examples/nonPhysical/Circle/Circle.simul.out.$i.txt;
  exp_start;
  exp_wait;
}
```