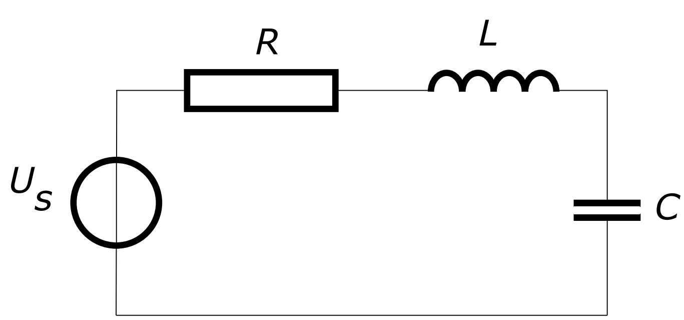
Bond Graphs (Paynter)

- *domain-independent*, *graphical* description
  multi-domain: electric, mechanical (translational, rotational), hydraulic, thermodynamic
- *dynamic* behaviour of *physical* systems
- basis of *power* bond: energy and *energy exchange*, *analogy*
- *non-causal* $\rightarrow$ *re-use* of Bond Graph models
- *hierarchy*, shows physical structure
- *causality* assignment $\rightarrow$ equations
- systematic *procedure*: physical system $\rightarrow$ equations

Electrical Example: RLC circuit



Equations

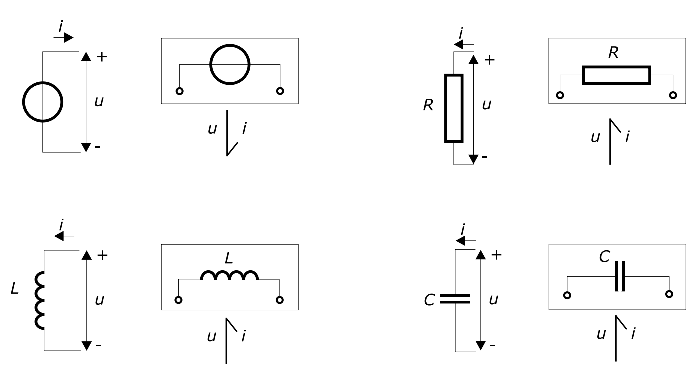
$$P = ui$$

$$u_R = iR$$

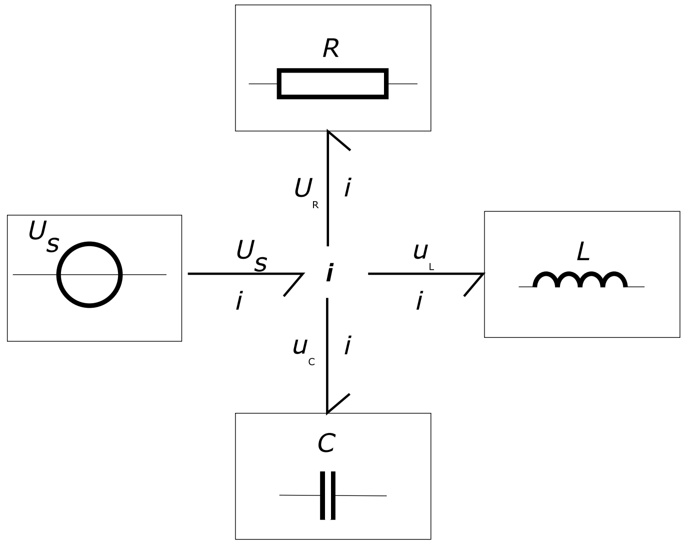
$$u_C = \frac{1}{C}\int i dt$$

$$u_L = L\frac{di}{dt} \; or \; i_L = \frac{1}{L}\int u_L dt$$

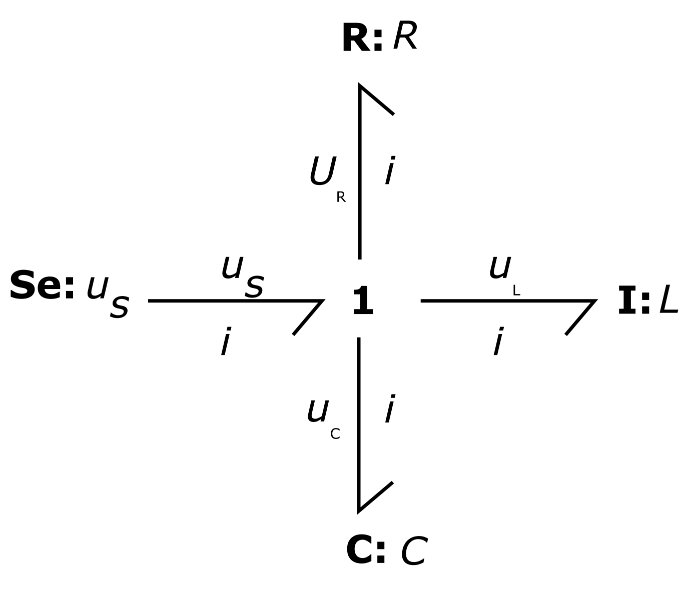
Power Ports



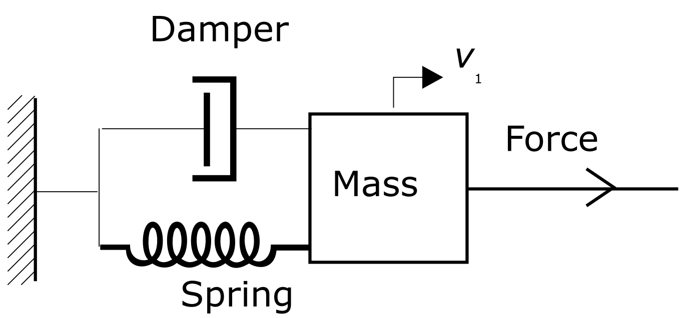
Bond Graph, electrical symbols



Bond Graph, standard



Mechanical Example: Mass Spring



Equations

$$P = Fv \;\; (P = T\omega)$$

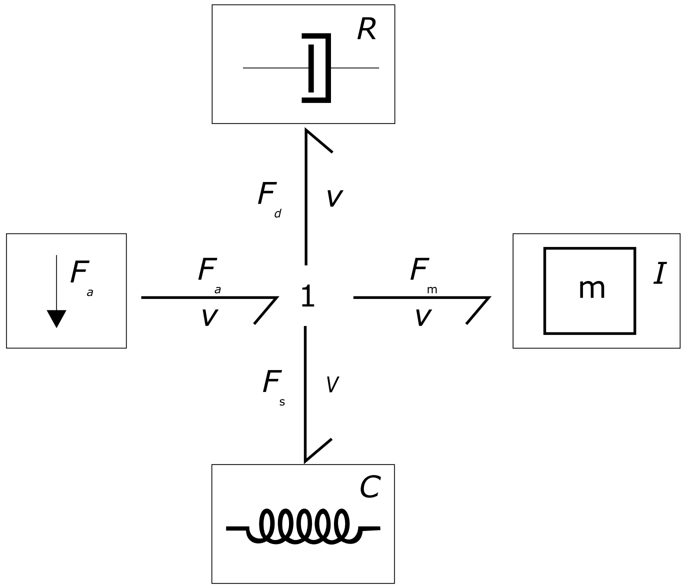
$$F_d = av$$

$$F_S = K_S \int v dt = \frac{1}{C_S} \int v dt$$

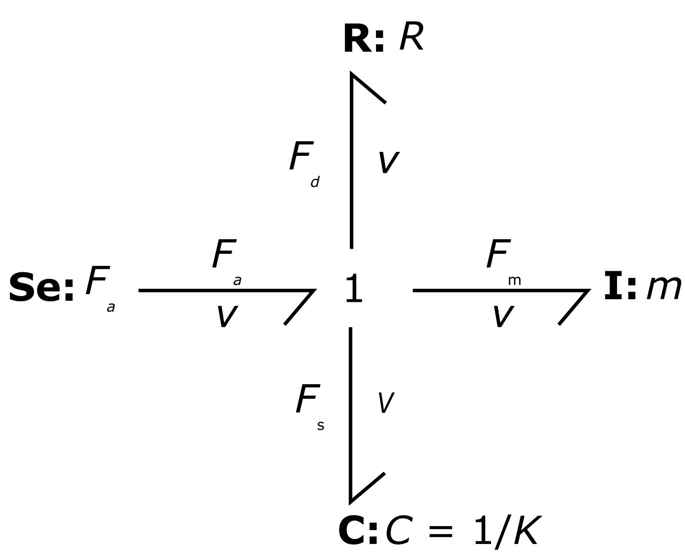
$$F_m = m\frac{dv}{dt} \; or \; v = \frac{1}{m} \int F_m dt$$

$$F_a = force$$

Bond Graph, mechanical symbols



Bond Graph, standard



Power (e, f) and Energy (p, q) variables

	Effort	Flow	Momentum	Displacement
	e	f	p	q
Electrical	Voltage	Current	Flux	Charge
	u [V]	i [A]	Φ [V s]	q [A s]
Translational	Force	Velocity	Momentum	Displacement
	F [N]	v [m s^{-1}]	$\mathcal{I}$ [N s]	x [m]
Rotational	Torque	Angular Velocity	Twist	Angle
	T [N m]	ω [rad s^{-1}]	$\mathcal{T}$ [N m s]	ϕ [rad]
Hydraulic	Pressure	Volume Flow	Pressure Momentum	Volume
	p [N m^{-2}]	q [m^3 s^{-1}]	Γ [N m^{-2} s]	V [m^3]
Thermodyn.	Temperature	Entropy Flow	—	Entropy
	T [K]	$\frac{dS}{dt}$ [W K^{-1}]		S [J K^{-1}]

Physical Analogy

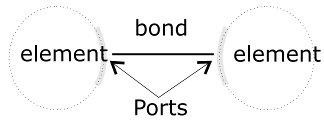
dissipation	reservoir of potential	reservoir of flux
$\frac{\partial P}{\partial x} = - D f$	$\frac{\partial P}{\partial t} = - \frac{1}{Ep} \frac{\partial f}{\partial x}$	$\frac{\partial P}{\partial x} = - Ef \frac{\partial f}{\partial t}$

Bond Graph Foundations

Network-like description of physical systems:

- *conservation of energy*
- *lumped*, idealized physical concepts

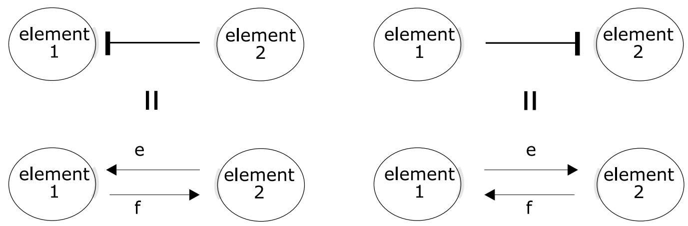
Bonds and Ports



Two interpretations:

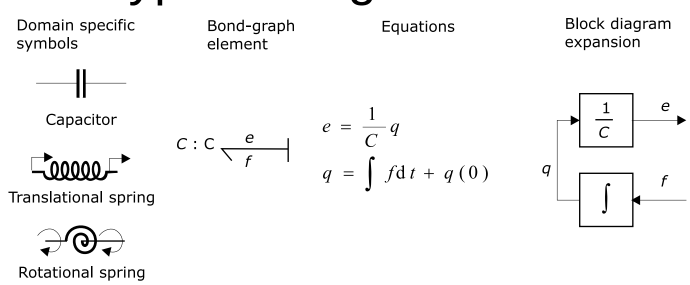
1. Interaction of Energy. (for physical reasoning)
2. Bilateral signal flow. Signals: effort and flow.
   (to derive a simulation model)

Signal Direction (causality)




```
element1.e := element2.e    element2.e := element1.e
element2.f := element1.f    element1.f := element2.f
```


C-type Storage Elements

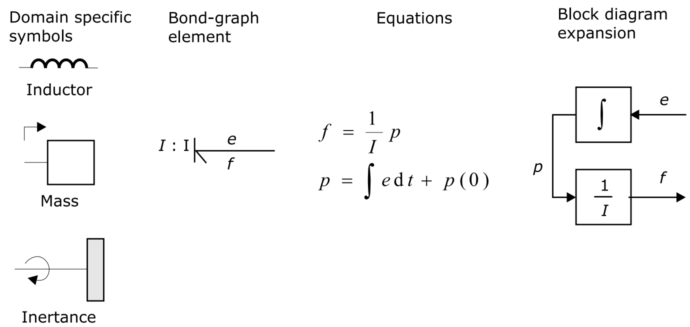


$$\frac{dq}{dt} = f, e = e(q)$$

$$\frac{dq}{dt} = i, \;\; u = \frac{1}{C}q$$

$$\frac{dx}{dt} = v, \;\; F = Kx = \frac{1}{C}x$$

I-type Storage Elements

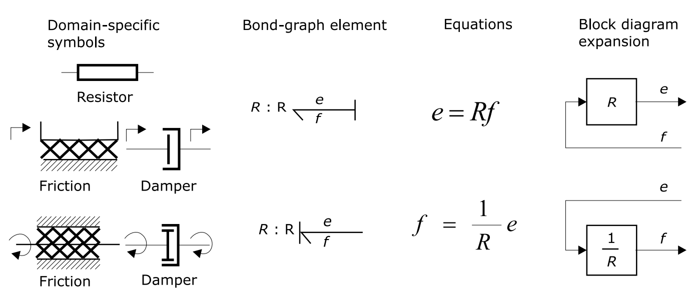


$$\frac{dp}{dt} = e, f = f(p)$$

$$\frac{dI}{dt} = u, \quad i = \frac{1}{L} I$$

$$\frac{dp}{dt} = F, \quad v = \frac{1}{m} p$$

Resistors

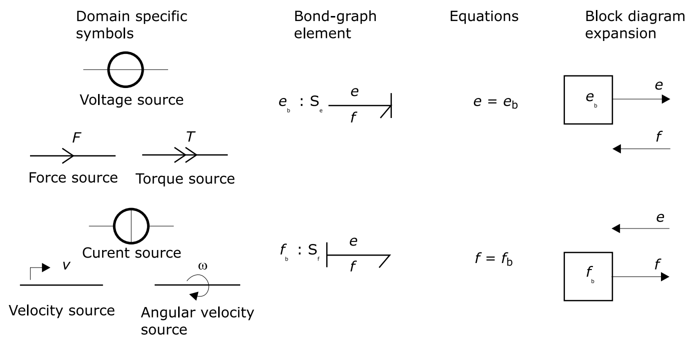


$$e = r(f)$$

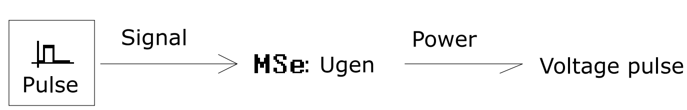
$$u = Ri$$

$$F = Rv$$

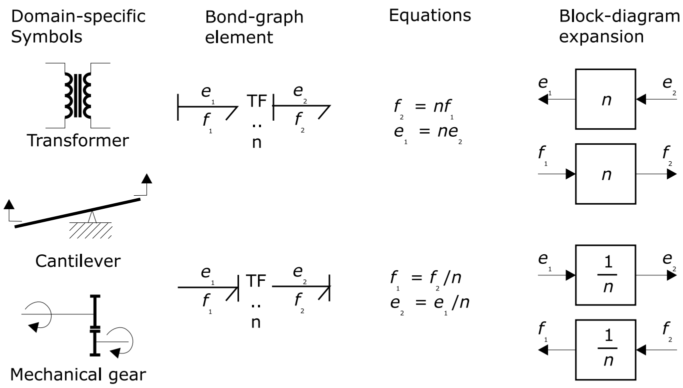
Sources



Signals



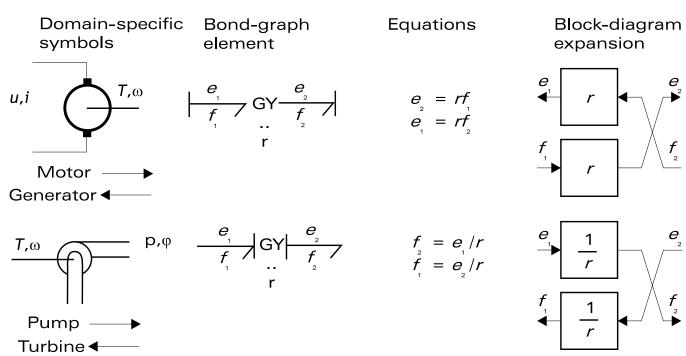
Transformers



$$e1 = ne2$$

$$f2 = nf1$$

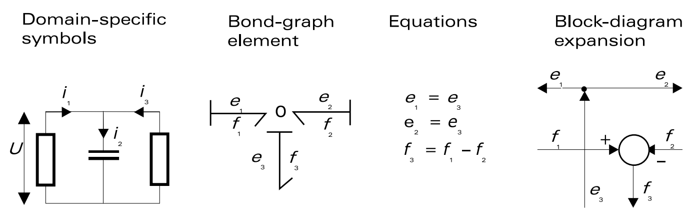
Gyrators



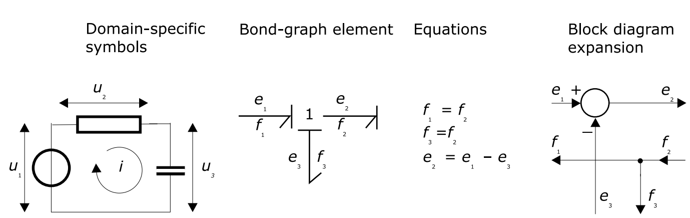
$$e1 = rf2$$

$$e2 = rf1$$

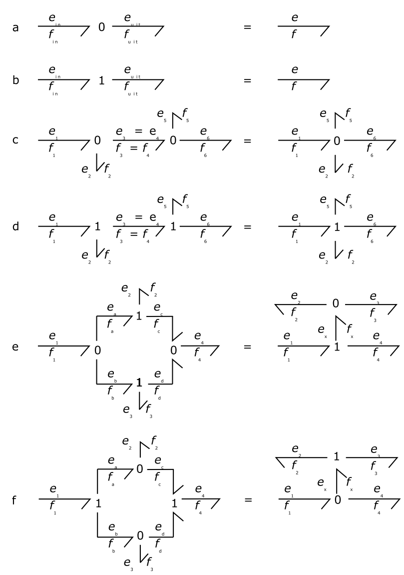
Junctions: 0



Junctions: 1



Simplifications

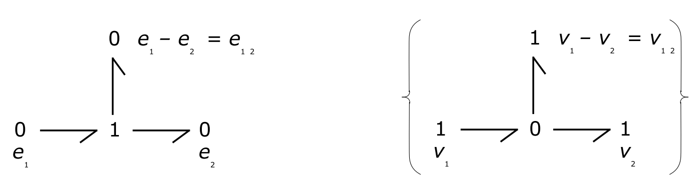


Remarks

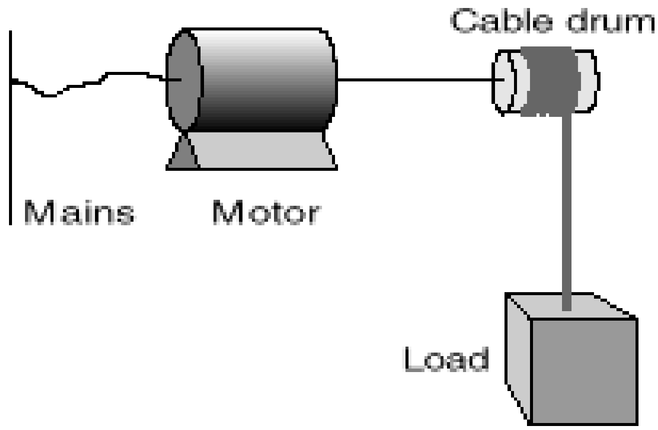
- Positive orientation (energy flow)
  $\rightarrow$ positive (physical) parameters
- Duality (potential/kinetic, electrical/magnetic)

Systematic Procedure

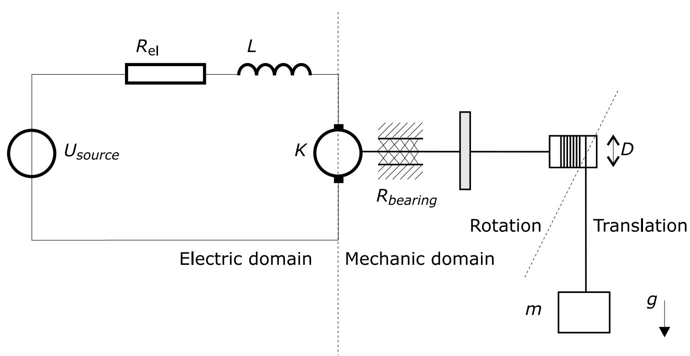
1.
2.
3.
4.
5.
6.
7.



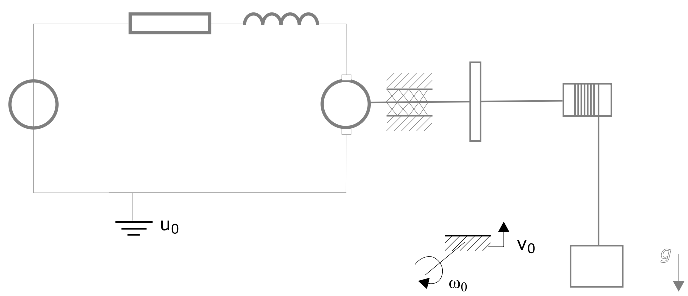
Multi-domain example



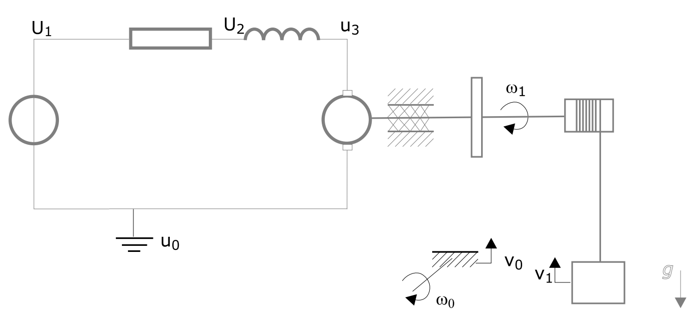
Idealized Physical Model (IPM)



IPM with references



IPM with voltages, velocities, . . .

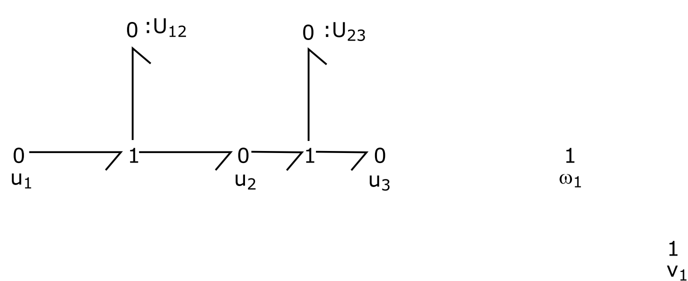


O-junctions

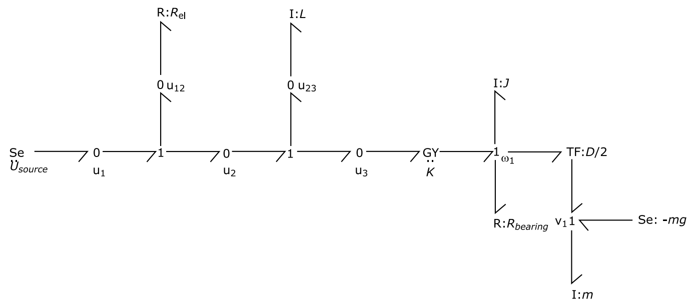
0 u_1 0 u_2 0 u_3 1 ω_1

1 v_1

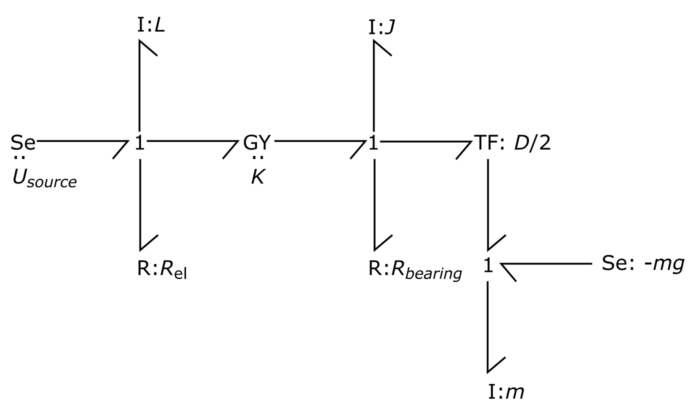
1-junctions (differences)



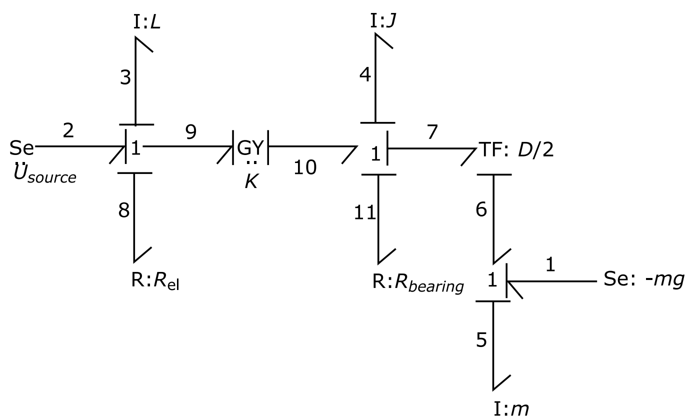
Complete Bond Graph



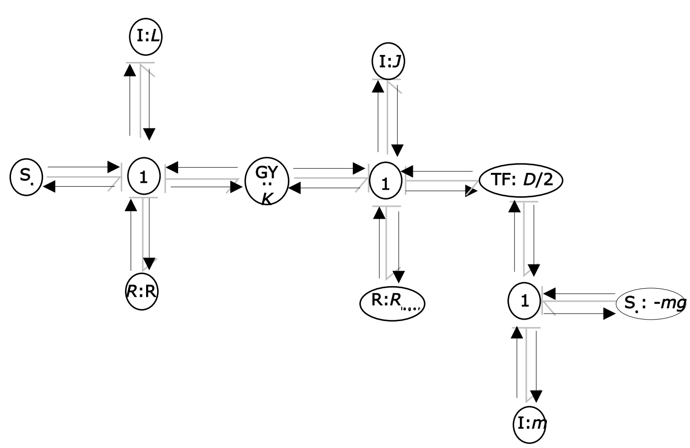
Simplified Bond Graph



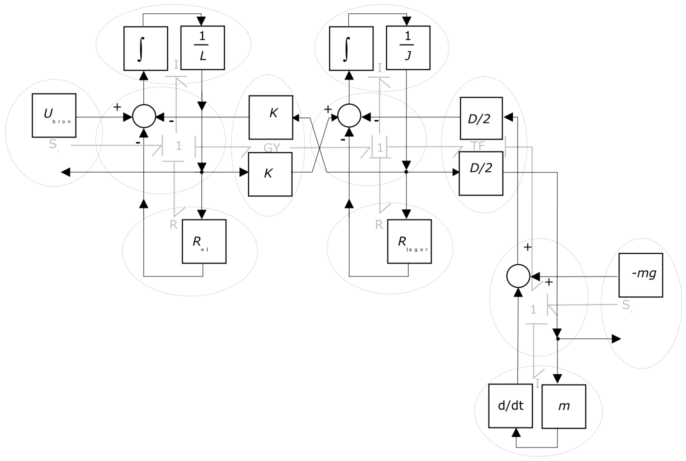
Causal Bond Graph



Block Diagram



Block Diagram



Block Diagram