COMP 760 - Winter 2017 - Exercises

General rules:

1. Let A be a random subset of $\{0,1\}^n$ chosen uniformly at random. Find the best δ (in the big O notation) so that with high probability $|\widehat{A}(\emptyset) - \frac{1}{2}| \leq \delta$ and for every $S \neq \emptyset$, we have $|\widehat{A}(S)| \leq \delta$.
2. Construct a $2^n \times 2^n$ matrix A such that $Af = \widehat{f}$.
3. Let $f : \{0,1\}^n \times \{0,1\}^n \to$ R. What are the eigenvalues and eigenvectors of the $2^n \times 2^n$ matrix with entries $f(x \oplus y)$?
4. Construct a Boolean function whose total influence is $O(\log n)$ but all its variables have non-zero Fourier coefficients.
5. 1.1(n), 1.11 of O'Donnell's book.
6. 2.4, 2.18, 2.21, 2.46, 2.55, 2.57 of O'Donnell's book.