COMP 760 - Winter 2017 - Exercises

General rules:

1. Compute the Fourier coefficients of the AND function $f : \{0,1\}^n \to \{0,1\}$ defined as $f(x) = 1$ if and only if $x = \vec{1}$.

2. Compute the Fourier coefficients of the PARITY function $f : \{0,1\}^n \to \{0,1\}$ defined as $f(x) = x_1 + \ldots + x_n \mod 2$.

3. Compute the Fourier coefficients of the inner product function $f : \{0,1\}^{2n} \to \{-1,1\}$ defined as $f(x) = (-1)^{x_1x_2+x_3x_4+\ldots+x_{2n-1}x_{2n}}$.

4. Let $f : \{0,1\}^n \to \mathbb{R}$ be computed by a decision tree of height k. That is every internal node of the tree is labeled with one of the variables, and the leaves are labeled with real numbers. The value of $f(x)$ is the the label of the leaf obtained by traversing the path from the root to a leaf determined by the value of the variables on the path (Let's say $x_i = 1$ means moving to the left child, and $x_i = 0$ means moving to the right child). Prove that $\widehat{f}(S) = 0$ if $|S| > k$.