You are allowed to collaborate in solving these questions, but each person should write and submit her own solution.

1. Show that for every function $f : \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}$, we have $U^{\text{pub}}(f) = O(1)$.

2. Show that $\text{PP}^{cc}$ is the class of problems with $\text{disc}(f_n) \geq 2^{-\log^c(n)}$.

3. Show that $\text{PP}^{cc} \supseteq \text{NP}^{cc} \cup \text{CoNP}^{cc}$.

4. Consider a problem $\{f_n\}$ in $\text{BPP}^{cc}$.
   
   (a) Show that there is a randomized protocol $P(x, y, r)$ which uses only $|r| = O(\log n)$ random bits, and still achieves $\Pr[P(x, y, r) \neq f_n(x, y)] \leq \frac{1}{3}$.

   (b) Show that there is a randomized protocol $Q(x, y, r)$ which uses only $m := |r| = O(\log^2 n)$ random bits, and achieves $\Pr[Q(x, y, r) \neq f_n(x, y)] \leq \frac{1}{3m}$.

   (c) Let $m$, and $Q(x, y, r)$ be as above, and we will interpret $r$ as taking values in $\mathbb{F}_2^m$ according to some distribution. Use probabilistic method to prove the following statements:

   • If $f_n(x, y) = 1$ then there exists a choice of $a_1, \ldots, a_m \in \mathbb{F}_2^m$ such that for every $z \in \mathbb{F}_2^m$ there is at least one $i \in \{1, \ldots, m\}$ for which $Q(x, y, a_i + z) = 1$.
   • If $f_n(x, y) = 0$ then for every choice of $a_1, \ldots, a_m \in \mathbb{F}_2^m$, one can find a $z \in \mathbb{F}_2^m$ such that $Q(x, y, a_i + z) = 0$ for all $i \in \{0, \ldots, m\}$.

   (d) Conclude that $\text{BPP}^{cc} \subseteq \Sigma_2^{cc} \cap \Pi_2^{cc}$. 