1. True or False? (Prove or Disprove): $n\sqrt{n} = O \left( \frac{n^2}{\log n} \right)$.
2. Consider the text in the quotation marks "aaaabbbced". Draw the Huffman-tree for this text. Label each node with the corresponding frequency. For each leaf, specify the code for the corresponding letter. (You do not need to explain how you have obtained the tree, nor write the bit sequence of the final coded string).
3. Consider a positive integer $m$, and $n$ intervals $[s_1, f_1], \ldots, [s_n, f_n]$ where each interval $[s_i, f_i]$ has integer endpoints that satisfy $0 \leq s_i < f_i \leq m$. We want to select the smallest possible number of intervals such that their union covers all of the interval $[0, m]$, or declare that this is not possible. For example if $m = 4$, and there are only two intervals $[0, 1]$ and $[2, 4]$, then it is not possible to cover $[0, 4]$. Indeed even selecting both intervals will not cover the points between 1 and 2. However if we had three intervals $[0, 1], [0, 3], [2, 4]$, then we could cover $[0, 4]$ with two intervals $[0, 3]$ and $[2, 4]$, and thus the answer, in this case, would be 2.

Design and analyze a polynomial-time algorithm that given the integers $m, s_1, \ldots, s_n, f_1, \ldots, f_n$, finds the smallest number of intervals that cover $[0, m]$. 
4. Consider a directed graph that contains no directed cycles. Design a polynomial-time algorithm that decides whether this graph has a directed path that visits all the vertices. Recall that a path is not allowed to visit any vertex more than once.
5. Consider a binary tree with $n$ nodes, where to every node $i$, an integer value $v_i \geq 0$ is assigned. We want to select a set of nodes with maximum total value such that no vertex in the set is a parent of another vertex in the set. Design and analyze a $O(n)$ algorithm for this task.
6. You are given an $n \times n$ matrix of distinct integers, sorted in increasing order along every row (left to right) and along every column (top to bottom). You are also given a particular integer $x$ that is present in the matrix. Devise and analyze a $o(n^2)$ divide-and-conquer algorithm that, for any such matrix and any such integer, locates the row and column where this element is to be found. You can assume that $n = 2^m$ for a positive integer $m$. 
7. Let \( \{u_1, \ldots, u_n\} \) and \( \{v_1, \ldots, v_n\} \) be the vertices of the two parts of an undirected bipartite graph. A pair of edges \( u_i v_j \) and \( u_i' v_j' \) is called crossing if \( i < i' \) but \( j > j' \).

Design and analyze an algorithm that finds the largest matching in which no pair of edges are crossing.
8. Given a positive number $n$, let $P(n)$ denote the number of different ways that it can be partitioned as a sum of (unordered) positive integers. For example $P(4) = 5$ as it can be written as $4, 3 + 1, 2 + 2, 2 + 1 + 1$, and $1 + 1 + 1 + 1$. Design and analyze a $O(n^3)$ algorithm that computes $P(n)$ for a given number $n$. 