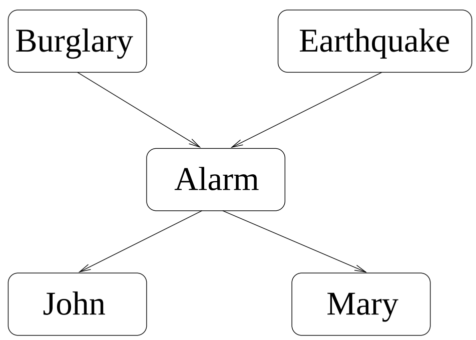
Probabilistic Reasoning in AI - Problem set 2

Due Wednesday, January 30, 2002, in class
This assignment is worth 10% of the homework grade

1. [30 points] One of the questions that came up in class referred to what happens if we perform arc-reversal in a Bayes net. This involves transforming a Bayes net G which contains the arc $X \rightarrow Y$ into another Bayes net G' which contains the reversed arc, $Y \rightarrow X$. When we do this transformation, we want B' to represent the same distribution as B. Therefore, B' will need to be an I-map of the original distribution.

   (a) [5 points] Consider the following network structure:

   Suppose we want to reverse the arc $Burglary \rightarrow Alarm$. What additional minimal modifications to the structure of the network are necessary to ensure that the new network is an I-map of the original one? You do not need to prove that the modifications are minimal.

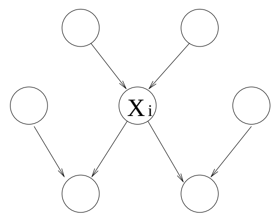
   (b) [15 points] Now consider a general Bayes net G. Assume for simplicity that the arc $X \rightarrow Y$ is the only directed path form X to Y. If you reverse the arc, what additional minimal modifications to G are needed in order to ensure that the new network G' is an I-map of the original distribution? Hint: Consider the algorithm for constructing a minimal I-map that we discussed in class.

   (c) [10 points] Suppose that you use the above method to transform G into G', which has an arc $Y \rightarrow X$. Now suppose you want to reverse the arc back to its original direction, $X \rightarrow Y$, by using your method to transform G' into G''. Are you guaranteed that the final network structure is equivalent to the original network structure ($G = G''$)?

2. [25 points]

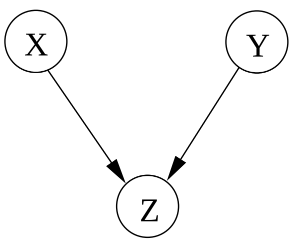
   (a) [8 points] Consider again the alarm network described in the previous problem. Suppose that you want to eliminate the $Alarm$ node. Construct a Bayes net structure with nodes $Burglary$, $Earthquake$, $John$, $Mary$ which is a minimal I-map for the marginal distribution over these variables (as defined by the previous network). Be sure to get all the dependencies that remain from the original network.

   (b) [17 points] Generalize this procedure into a node-elimination algorithm. That is, define an algorithm that takes a Bayes net structure G and a random variable X_i, and turns it into a structure G' such that X_i is not in G', and G' is a minimal I-map of the distribution defined over the remaining variables $X_1 \ldots X_{i-1}, X_{i+1}, \ldots X_n$. Hint: the tricky case is analogous to the example:



3. [30 points] We discussed in class the explaining away property, which occurs when evidence that establishes the cause of an event reduces the likelihood of other possible causes. This problem explores the property in the context a "noisy-or-like" network.

   (a) [20 points] Consider the following network:



   Assume that X, Y and Z are binary random variables and that the CPT for Z is:

Z	$X=0, Y=0$	$X=0, Y=1$	$X=1, Y=0$	$X=1, Y=1$
$Z=1$	0	y	x	xy
$Z=0$	1	$1-y$	$1-x$	$1-xy$

   Show that $P(X=1|Z=1) \geq P(X=1|Y=1, Z=1)$. Hint: the solution fits in half a page.

   (b) [10 points] Show different CPTs for the same network structure, such that the opposite effect occurs: Both causes increase the probability of the effect, i.e. $P(Z=1|X=1) > P(Z=1)$ and $P(Z=1|Y=1) > P(Z=1)$, but each cause also increases the probability of the other: $P(X=1|Z=1) < P(X=1|Y=1, Z=1)$, and $P(Y=1|Z=1) < P(Y=1|X=1, Z=1)$.

4. [15 points] For an undirected graph G, we say that X and Y are separated given evidence Z whenever all paths between variables in X and variables in Y are blocked by some variables in Z. Prove that if X and Y are separated given Z in the moral graph of a Bayes net G, the X and Y are d-separated in G, given Z.