Midterm for COMP-250: Introduction to computer science

February 15, 2006

Read all questions first. Please make sure to **write your name** and ID number on the exam booklet! Write all the answers **on the booklet**.

1. [10 points] **Induction**

   Prove by induction that $(1 + 2 + \ldots + n)^2 = 1^3 + 2^3 + \ldots n^3$

2. [10 points] **Big-Oh**

   Does $3\sqrt{n} \in O(\log_2 n)$? Justify your answer.

3. [20 points] For the following piece of code, say what Big-Oh is (assuming that it is called on an array of n elements. Justify your answer

   Algorithm foomain(a)
   Inputs: an array a of n integers
   Output: and integer
   return foo(a,1,n)

   Algorithm foo(a, l, r)
   $n \leftarrow r - l$
   if $(n = 0)$ return $a[l]$
   else if $a[\lfloor n/10 \rfloor] < 10$ **return** foo(a, l, $l + \lfloor n/10 \rfloor - 1$) **else return** foo($a$, $l + \lfloor n/10 \rfloor + 1$, r)

4. [25 points] **Chocolate bars**

   For Valentine's day you got a chocolate bar made of $n*m$ square pieces. You decide to split it among your $n * m$ friends. In one operation, you are allowed to cut any piece that has been created so far, along one of

its vertical or horizontal lines. You cannot cut more than one piece at a time.

(a) [5 points] Write an algorithm for breaking the chocolate into pieces (how geeky can you get?)

(b) [10 points] How many cuts does your algorithm make? Please give an exact formula in terms of the size of the initial bar.

(c) [10 points] Prove that your method is optimal (you cannot solve the problem correctly in fewer cuts).

5. [35 points] **Find the missing number**

An array a indexed from 1 to n contains all the integers between 0 and n except one. You are allowed to do basic operations of the form ask($a[i]$,j). This will return the value of the jth bit in $a[i]$. This operation takes constant time. Recall that a number n can be encoded in $\log_2 n$ bits.

(a) [15 points] Write down a recursive algorithm that identifies the missing number. Make this as efficient as you can

(b) [10 points] Prove that your algorithm is correct

(c) [10 points] Derive the big-Oh for your algorithm.