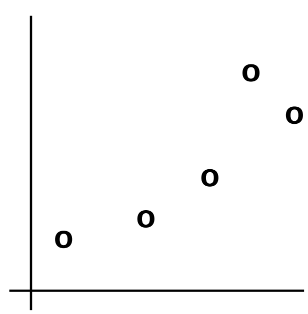
Lecture 2: Concept Learning

- Supervised learning
- Concept learning task
- Version spaces
- The need for bias

Supervised (Inductive) Learning

Assume somebody gives us **labeled examples** of the form $\langle v_1\, v_2\, \ldots v_n, o\rangle$, where:

- v_i are values for *input variables* or *attributes* or *features*
- o is the *output value*



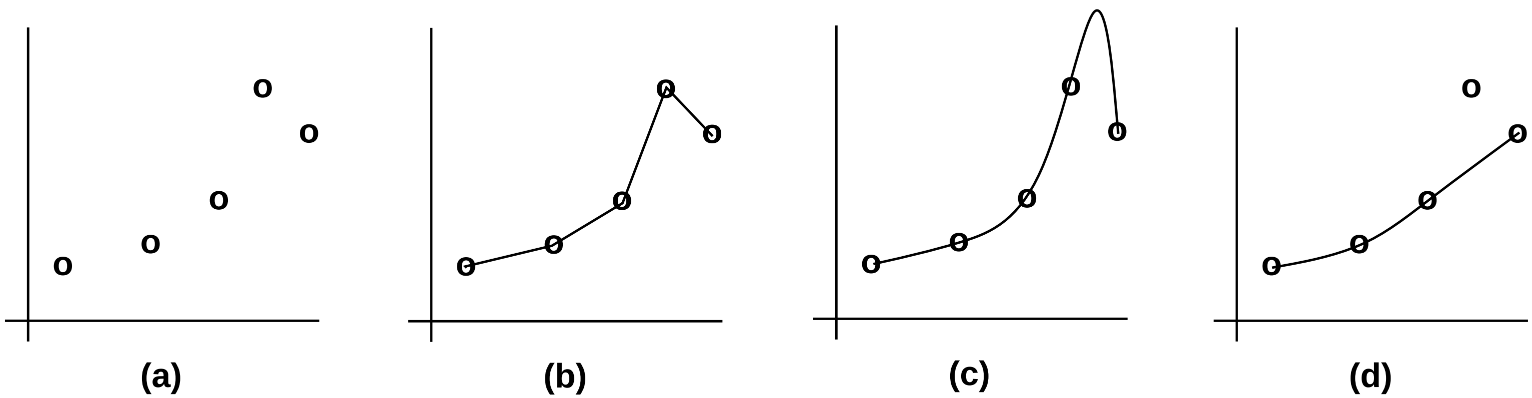
(a)

Try to find a *target function* $f : V_1 \times V_2 \times \cdots \times V_n \rightarrow O$, which maps the input variables into the output domain that fits the examples

Kinds of Supervised Learning

- If the output domain is Boolean, this problem is called *concept learning*
- If the output domain is discrete, this problem is called *classification*
- If the output is continuous, the problem is known as *regression* of *function approximation*.

The Big Question



There are an infinite number of functions that can fit the training points!

How do we find a *good* one?

I.e. one that will give approximately correct values for *unseen* data.

Choosing a Target Function Representation

- The learner usually considers a *class of hypothesis* E.g. decision trees, artificial neural networks, linear combinations of 3rd-degree polynomials, linear combinations of the input etc. etc
- Learning methods *search* through the class of hypothesis to find a good one.
- If there are several hypothesis that fit the data about the same, *choose the simplest one* - **Occam's razor**

Issues:

- What should the hypothesis class be?
- What search method should we use?
- What is a "good" hypothesis?

Concept Learning

Inferring a Boolean function from labeled training examples

Example: “user profile” for web browsing

Domain	Platform	Browser	Day	Screen	Continent	Click?
edu	Mac	Net3	Mon	XVGA	America	Yes
com	Mac	NetCom	Tue	XVGA	America	Yes
com	PC	IE	Sat	VGA	Europe	No
org	Unix	Net2	Wed	XVGA	America	Yes

What is the general concept?

Assumption of the day: the training examples are perfect (no noise)

Representing Hypotheses

Many possible representations!

Consider hypotheses h formed as a conjunction of constraints on attributes

Each constraint can be

- a specific value (e.g., *Domain* = *com*)
- don't care (e.g., *Domain* = *?*)
- no value allowed (e.g.,*Domain*=$\emptyset$)

For example,

Domain	*Platform*	*Browser*	*Day*	*Screen*	*Continent*
$\langle$ *com*	*?*	*?*	*Sun*	*?*	*America* $\rangle$

Prototypical Concept Learning Task

Given:

- Instances X: Possible days, each described by the attributes:
  - *Domain* $\in$ {*com, edu, org*}
  - *Platform* $\in$ {*Mac, PC, Unix*}
  - *Browser* $\in$ {*Net2, Net3, NetCom, IE*}
  - *Day* $\in$ { *Monday, ... Sunday* }
  - *Screen* $\in$ {*VGA, XVGA*}
  - *Continent* $\in$ { *America, Europe, Africa, Asia, Australia* }

  How many possible instances are there?
- Target function (or concept) c: *Click*: $X \rightarrow \{0, 1\}$

  How many concepts are possible?

Given (continued)

- Hypotheses H: Conjunctions of literals.

  E.g. $\langle$*?, Cold, High, ?, ?, ?* $\rangle$

  How many syntactically distinct hypotheses are there?

  How many are semantically distinct?

- Training examples D: Positive and negative examples of the target function

$$\langle x_1, c(x_1)\rangle, \ldots \langle x_m, c(x_m)\rangle$$

Determine:

A hypothesis h in H such that $h(x) = c(x)$ for all x in X.

The Inductive Learning Hypothesis

Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.

Why would this be true?

Sampling: statistical theory for inferring population parameters from samples

Concept Learning as Search in Hypothesis Space

The hypotheses can be partially ordered under the *more-general-than-or-equal-to* relation ($\geq_g$)

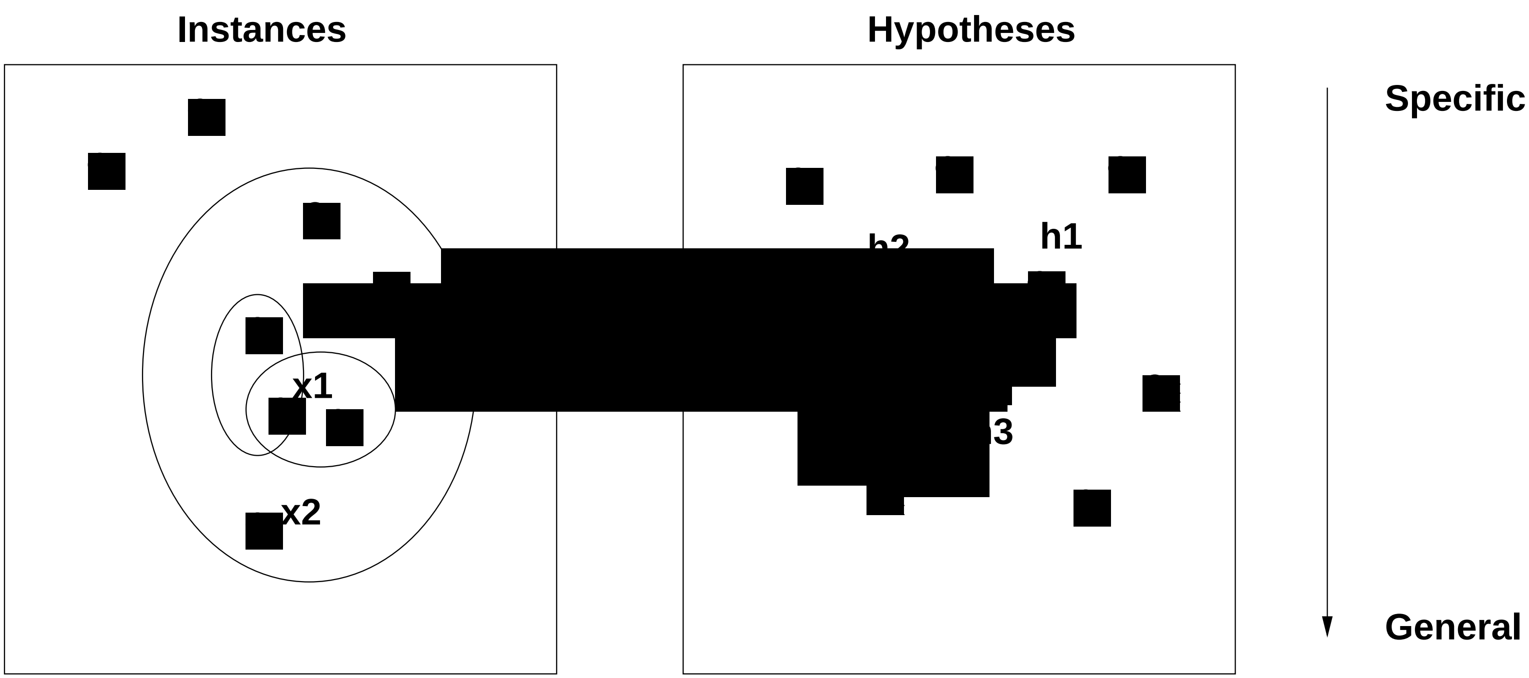
Definition: $h_1 \geq_g h_2$ iff $\forall x \in X, h_2(x) = 1 \rightarrow h_1(x) = 1$

E.g.

- $h_1 = \langle edu, Mac, ?, Mon, ?, ? \rangle$
- $h_2 = \langle edu, Mac, IE, Mon, ?, Europe \rangle$

Why is this a partial ordering?

Partial Ordering on Hypothesis Spaces



x1 = <edu,Mac,IE,Mon,VGA,Eur>

x2 = <edu,PC,IE,Mon,VGA,Eur>

h1 = <edu,Mac,?,?,?,Eur>

h2 = <edu,?,IE,?,?,Eur>

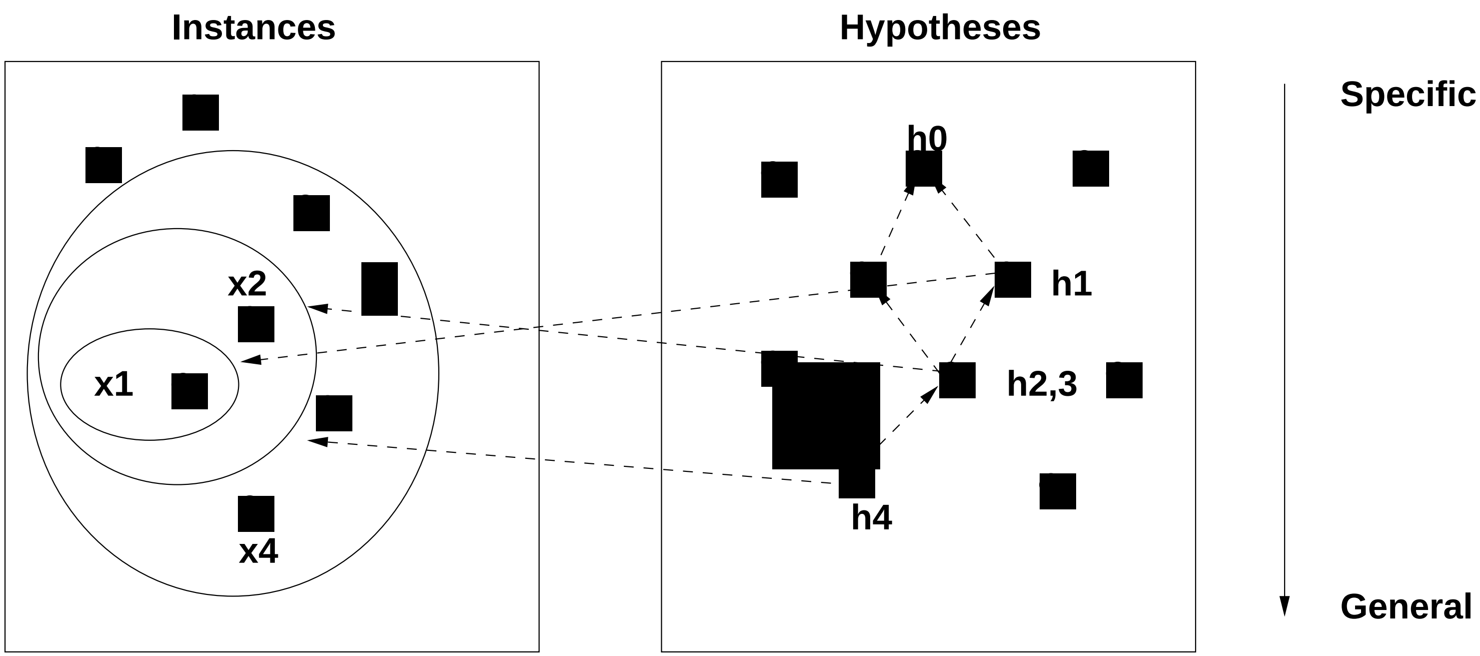
h3 = <edu,?,?,?,?,Eur>

Find-S: Finding a Maximally Specific Hypothesis

1. Initialize h to the most specific hypothesis in H
2. For each positive training instance x, do
   - For each attribute constraint a_i in h, do

     If x is not covered by h, replace a_i by the next more general constraint that is satisfied by x
3. Output hypothesis h

Hypothesis Space Search by Find-S



x1 = <edu,Mac,Net3,Mon,XVGA,America>, +

x2 = <com,Mac,Net3,Tue,XVGA,America>, +

x3 = <com,PC,IE,Sat,VGA,Eur>, -

x4 = <org,Unix,Net2,Wed,XVGA,America>, +

h0 = <0,0,0,0,0,0>

h1 = <edu,Mac,Net3,Mon,XVGA,America>

h2 = <?,Mac,Net3,?,XVGA,America>

h3 = <?,Mac,Net3,?,XVGA,America>

h4 = <?,?,?,?,XVGA,America>

Complaints about Find-S

- Convergence: cannot tell whether it has learned concept
- Consistency: cannot tell when training data inconsistent
- Picks a maximally specific h (why?)
- Depending on H, there might be several consistent specific hypotheses

Version Spaces

A hypothesis h is **consistent** with a set of training examples D of target concept c if and only if $h(x) = c(x)$ for every training example $\langle x, c(x) \rangle$ in D.

The **version space**, $VS_{H,D}$, with respect to hypothesis space H and training examples D, is the subset of hypotheses from H consistent with all training examples in D.

How can we compute the version space?

- Obvious idea: list-then-eliminate - impractical!
- Candidate elimination (Mitchell)

Representing Version Spaces

Key idea: keep only the *boundary sets*, exploiting the partial ordering of the hypotheses space.

The **General boundary**, G, of version space $VS_{H,D}$ is the set of its maximally general members

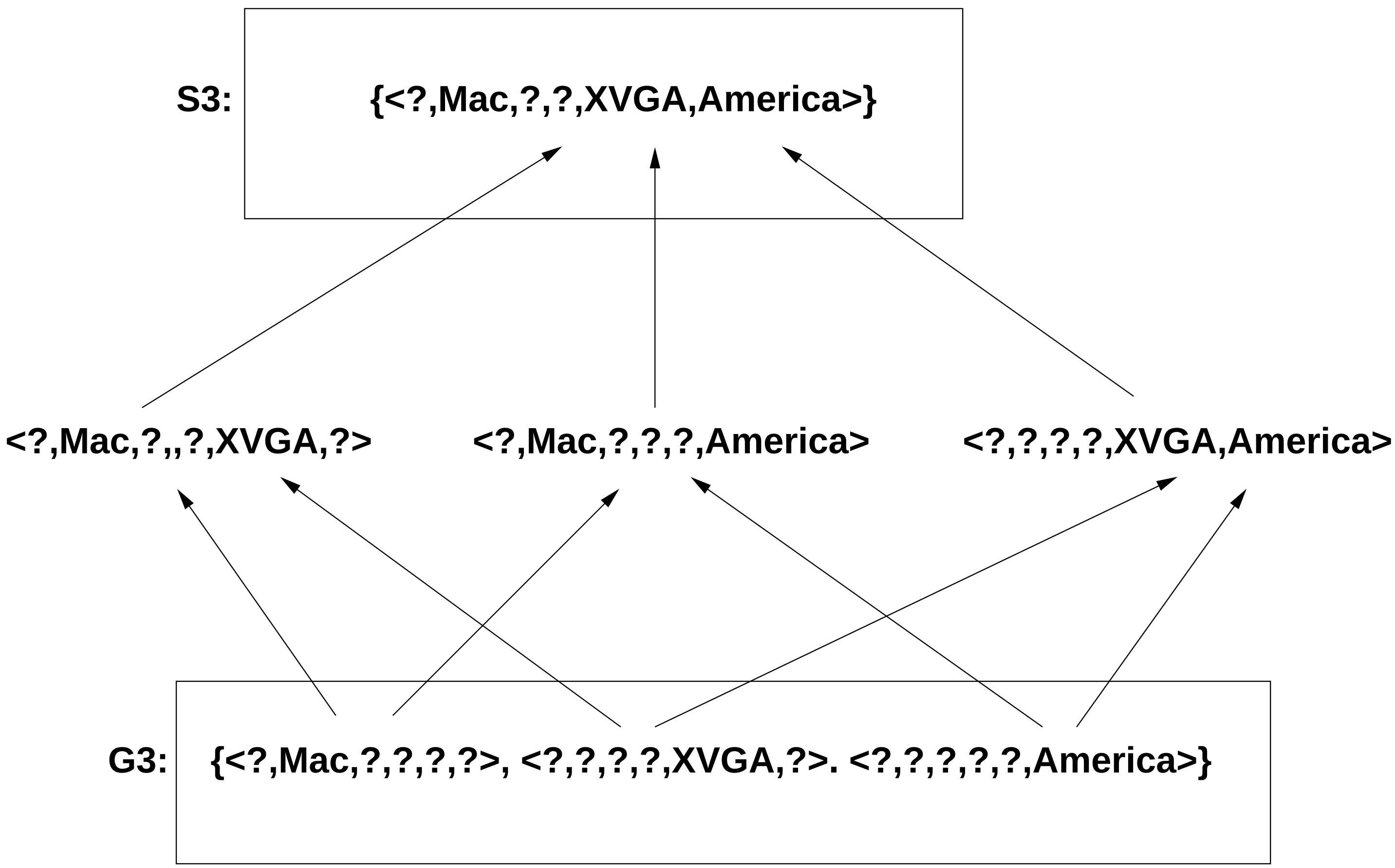
The **Specific boundary**, S, of version space $VS_{H,D}$ is the set of its maximally specific members

Every member of the version space lies between these boundaries

$$VS_{H,D} = \{h \in H | (\exists s \in S)(\exists g \in G)(g \geq h \geq s)\}$$

where $x \geq y$ means x is more general or equal to y

Example Version Space



Candidate Elimination Algorithm

$G \leftarrow$ maximally general hypotheses in H

$S \leftarrow$ maximally specific hypotheses in H

For each training example d, do

- If d is a positive example
  - Remove from G any hypothesis inconsistent with d
  - For each hypothesis s in S that is not consistent with d
    - Remove s from S
    - Add to S all minimal generalizations h of s such that h is consistent with d, and some member of G is more general than h
    - Remove from S any hypothesis that is more general than another hypothesis in S

- If d is a negative example
  - Remove from S any hypothesis inconsistent with d
  - For each hypothesis g in G that is not consistent with d
    - Remove g from G
    - Add to G all minimal specializations h of g such that h is consistent with d, and some member of S is more specific than h
    - Remove from G any hypothesis that is less general than another hypothesis in G

Example Trace (1)

S0: {<0,0,0,0,0,0>}

G0: {<?,?,?,?,?,?>}

Example Trace (2)

S1: {<edu,Mac,Net3,Mon,XVGA,America>}

G1: {<?,?,?,?,?,?>}

<edu,Mac,Net3,Mon,XVGA,America>, +

Example Trace (3)

S2: {<?,Mac,?,?,XVGA,America>}

G2: {<?,?,?,?,?,?>}

<com,Mac,NetCom,Tue,XVGA,America>, +

Example Trace (4)

S3: {<?,Mac,?,?,XVGA,America>}

G3: {<?,Mac,?,?,?,?>, <?,?,?,?,XVGA,?>. <?,?,?,?,?,America>}

<com,PC,IE,Sat,VGA,Eur>, -

Active Learning with Version Spaces

S3: {<?,Mac,?,?,XVGA,America>}

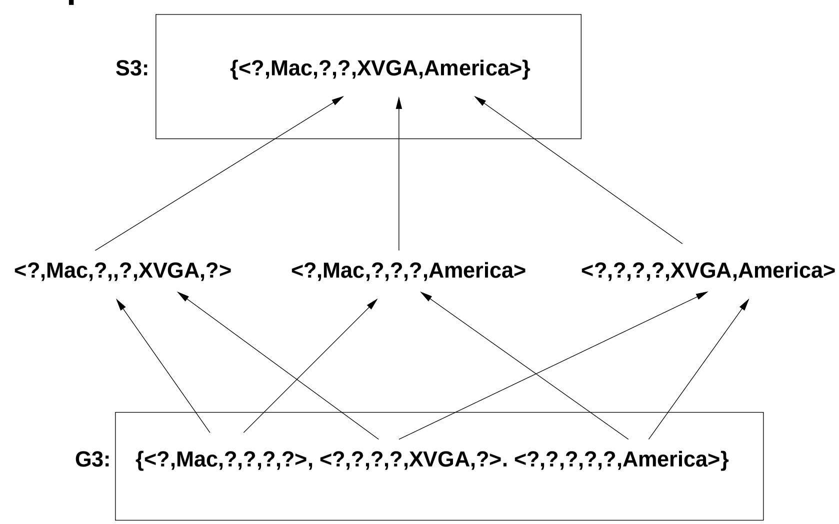
<?,Mac,?,,?,XVGA,?>   <?,Mac,?,?,?,America>   <?,?,?,?,XVGA,America>

G3: {<?,Mac,?,?,?,?>, <?,?,?,?,XVGA,?>. <?,?,?,?,?,America>}

What should be the best new example?

Using Partially Learned Concepts

Given the version space:



How should these be classified?

edu	*Mac*	*IE*	*Fri*	*XVGA*	*America*
com	*PC*	*NetCom*	*Wed*	*VGA*	*Europe*
org	*Unix*	*Net2*	*Wed*	*XVGA*	*America*

Example Trace (5)

S4: {<?,?,?,?,XVGA,America>}

G4: { <?,?,?,?,XVGA,?>. <?,?,?,?,?,America>}

<org,Unix,Net2,Wed,XVGA,America>, +

Example Trace (Final)

S5: {<?,?,?,?,XVGA,?>}

G5: { <?,?,?,?,XVGA,?>}

<com,Unix,Net2,Wed,XVGA,Europe>, +

VS has Exponential Sample Complexity

Let the concept be $A_1 = true$. Let the instances be described by n Boolean attributes, and consider the following sequence of examples:

- $A_1 = true \wedge A_2 = true \wedge \ldots \wedge A_{n-1} = false \wedge A_n = false$
- $A_1 = true \wedge A_2 = true \wedge \ldots \wedge A_{n-1} = false \wedge A_n = true$
- $A_1 = true \wedge A_2 = true \wedge \ldots \wedge A_{n-1} = true \wedge A_n = false$
- ...
- $A_1 = true \wedge A_2 = true \wedge \ldots \wedge A_{n-1} = true \wedge A_n = true$

Bias in Concept Learning

Bias is defined as any criteria (other than consistency with the training data) used to select one hypothesis over another.

Note: Inductive bias $\neq$ *statistical bias!*

Sources of bias

- Hypothesis language - *representational bias*
  E.g. our hypothesis language only allows conjunctions
- Search (generalization) algorithm - *algorithmic bias* E.g. Find-S searches hypotheses from specific to general

An Unbiased Learner

Idea: Choose H that expresses every teachable concept (i.e., H is the power set of X)

In our case: consider $H' =$ disjunctions, conjunctions, negations over previous H.

E.g.,*(Platform = Unix* $\vee$ *Platform = Mac)* $\wedge\neg$ *(Platform = PC)*

Given positive instances $x_1, \ldots x_i$ and negative instances $y_1, \ldots y_j$, what are S, G in this case?

$$\begin{aligned} S &\leftarrow x_1 \vee \ldots \vee x_i \\ G &\leftarrow \neg y_1 \wedge \ldots \neg y_j \end{aligned}$$

Bias-free learning does not allow any generalization beyond the training instances!

Bias Is Necessary!

An unbiased generalization algorithm (e.g. version space candidate elimination) that uses an unbiased hypothesis space (e.g. all Boolean functions) can never go beyond memorizing the training instances

The power of a learning system comes from the appropriateness of its biases!

Where do biases come from?

- Knowledge about the domain
- Knowledge about the source of the training data
- Intended use of the learned concept
- Simplicity and generality (e.g. Occam's razor)

Summary

- Concept learning can be viewed as search through some hypothesis space H
- Version space candidate elimination algorithm takes advantage of the partial ordering of hypotheses (general-to-specific)
- The S and G boundaries characterize learner's uncertainty
- Learner can generate useful queries
- Learner can use partially learned concepts to label new examples
- Inductive leaps are possible *only if the learner is biased*
- *Bias is a constant theme in machine learning!*