Lecture 9: First-order Logic and Planning

- First-order logic
- Inference in first-order logic
- Expressing planning problems: PDDL and STRIPS language and PDDL
- State-space planning
 - Forward planners
 - Goal regression
- Plan-space planning
- What to do if plans fail

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Recall: Propositional Logic

- The good: Propositional logic is very simple! Just facts (literals), usual logical connectives, inference is simple
- The bad: Propositional logic is very simple!
 - We cannot express things in a compact way
 - Knowledge base may need to have many similar facts and sentences
- E.g. in the wumpus world, we want to be able to express how a move works *for all squares* in one sentence

First-Order Logic (FOL)

• A key element of FOL are *predicates*, which are used to describe objects, properties, and relationships between objects

E.g. On(x,y)

- A *quantified statement* is a statement that applies to a class of objects
 E.g. ∀x On(x, Table) → Fruit(x)
 - This means that there is only fruit on the table
 - The first element is called a *quantifier*, x is a *variable* and *Table* is a *constant*
 - On is a predicate
- The use of *quantifiers* allows FOL to handle *infinite domains*, while propositional logic can only handle finite domains.

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Syntax of FOL: Basic elements

Constants	Wumpus, 2, CS424,
Predicates	<i>At</i> , >,
Functions	log, exp,
Variables	х, у,
Connectives	$\land \ \lor \ \neg \ \rightarrow \leftrightarrow$
Equality	=
Quantifiers	$\forall \exists$

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Atomic sentences

Atomic sentence = $predicate(term_1,...,term_n)$ or $term_1 = term_2$

> Term = $function(term_1,...,term_n)$ or constant or variable

E.g., At(Wumpus,2,1) is an atomic sentence with one predicate

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Complex sentences

Complex sentences are made from atomic sentences using connectives

 $\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \to S_2, \quad S_1 \leftrightarrow S_2$

E.g. $At(Wumpus,2,1) \rightarrow \neg At(Wumpus,1,2)$ >(1,2) $\lor \le (1,2)$ >(1,2) $\land \neg >(1,2)$

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Existential Quantification

- Syntax: ∃*variables sentence*
- Someone taking AI is smart:
 ∃x Taking(x,AI) ∧ Smart(x)
- Semantics: ∃x S is equivalent to the *disjunction of instantiations* of S (*Taking(Ann,AI)* ∧ *Smart(Ann)*)
 - \lor (Taking(John,AI) \land Smart(John))
 - \vee .
- Typically, \land is the main connective with \exists .

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Example

- What does this mean:
 ∃x Taking(x,AI) → Smart(x)
- Common mistake: using → as the main connective with ∃:
 ∃x Taking(x,AI) → Smart(x) is true if there is anyone who is not taking AI!

 Properties of quantifiers ∀x∀y is the same as ∀y∀x ∃x∃y is the same as ∃y∃x ∃x∀y is not the same as ∀y∃x
 ∀x∀y is the same as ∀y∀x ∃x∃y is the same as ∃y∃x ∃x∀y is not the same as ∀y∃x
$\exists x \forall y Loves(x, y)$ "There is a person who loves everyone in the world" $\forall y \exists x Loves(x, y)$ "Everyone in the world is loved by at least one person"
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Example Let x and y be real numbers. • $\forall x \exists y \ x > y$ • $\exists x \forall y \ x > y$ What does each sentence mean? Are they valid, satisfiable or unsatisfiable?

Abellard-Eloise Games

- Abellard handles the universal quantifiers \forall
- Eloise handles the existential quantifiers \exists
- Abellard is trying to choose values to make the sentence inside the quantifiers false
- Eloise is trying to make the statement inside the quantifiers true
- They take turns as specified by the order of the quantifiers, left to right
- The sentence is valid if and only if Eloise has a winning strategy

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Quantifier Duality

Each quantifier can be expressed using the other quantifier and negation:

- $\forall x \ Likes(x, lceCream)$ is equivalent to $\neg \exists x \neg Likes(x, lceCream)$
- $\exists x \ Likes(x, Broccoli)$ is equivalent to $\neg \forall x \neg Likes(x, Broccoli)$

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Fun with Sentences

• Brothers are siblings

 $\forall x \forall y Brother(x, y) \rightarrow Sibling(x, y)$

• "Sibling" is reflexive

 $\forall x \forall y Sibling(x, y) \leftrightarrow Sibling(y, x)$

• One's mother is one's female parent

 $\forall x \forall y Mother(x, y) \leftrightarrow (Female(x) \land Parent(x, y))$

• A first cousin is a child of a parent's sibling

 $\forall x \forall y FirstCousin(x, y) \leftrightarrow \exists p \exists psParent(p, x) \land$ $Sibling(ps, p) \land Parent(ps, y)$

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• $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

Equality

• Example:

Obj1 = Obj2 is satisfiable 2 = 2 is valid

• Example: definition of the sibling predicate:

 $\begin{aligned} \forall x \forall y Sibling(x, y) &\leftrightarrow [\neg (x = y) \land \\ \exists m \exists f \neg (m = f) \land Parent(m, x) \land Parent(f, x) \land \\ Parent(m, y) \land Parent(f, y)] \end{aligned}$

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Proofs

The proof process can be viewed as a *search* in which the operators are *inference rules*:

• Modus Ponens (MP)

$$\frac{\alpha, \quad \alpha \to \beta}{\beta} \qquad \frac{\mathsf{Takes}(\mathsf{Joe}, \mathsf{AI}) \quad \mathsf{Takes}(\mathsf{Joe}, \mathsf{AI}) \to \mathsf{Cool}(\mathsf{Joe})}{\mathsf{Cool}(\mathsf{Joe})}$$

• And-Introduction (AI)

$$\frac{\alpha \quad \beta}{\alpha \land \beta} \qquad \frac{Cool(Joe) \quad CSMajor(Joe)}{Cool(Joe) \land CSMajor(Joe)}$$

• Universal Elimination (UE): τ must be a ground term i.e. a term with no variables

$$\frac{\forall x\alpha}{\alpha\{x/\tau\}} \qquad \frac{\forall x \; Takes(x,AI) \to Cool(x)}{Takes(Pat,AI) \to Cool(Pat)}$$

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Example Proof

Bob is a buffalo	1. Buffalo(Bob)
Pat is a pig	2. Pig(Pat)
Buffaloes outrun pigs	3. $\forall x \forall y \text{ Buffalo}(x) \land Pig(y) \rightarrow Faster(x,y)$
AI 1 & 2	4. Buffalo(Bob) \land Pig(Pat)
UE 3, x/Bob, y/Pat	5. $Buffalo(Bob) \land Pig(Pat) \rightarrow Faster(Bob,Pat)$
MP 4 & 5	6. Faster(Bob,Pat)

Search with Primitive Inference Rules

Operators are inference rules

States are sets of sentences

Goal test checks state to see if it contains query sentence

AI, UE, MP is a common inference pattern Problem: branching factor huge, especially for UE Idea: find a substitution that makes the rule premise match some known facts

 \Rightarrow a single, more powerful inference rule

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Unification

• A substitution σ unifies atomic sentences p and q if $p\sigma = q\sigma$

р	q	σ
Knows(John,x)	Knows(John,Jane)	x/Jane
Knows(John,x)	Knows(y,Mary)	y/John,x/Mary
Knows(John,x)	Knows(y,Mother(y))	y/John,x/Mother(John)

- Idea: Unify rule premises with known facts, apply unifier to conclusion
- E.g., if we know q and the rule: $Knows(John,x) \rightarrow Likes(John,x)$, we conclude:
 - Likes(John, Jane)
 - Likes(John, Mary)
 - Likes(John,Mother(John))

Generalized Modus Ponens (GMP)

$$\frac{p_1', p_2', \dots, p_n', (p_1 \land p_2 \land \dots \land p_n \Rightarrow q)}{q\sigma} \text{ where } p_i'\sigma = p_i\sigma \text{ for all } i$$

$$E.g. \ p_1' = Faster(Bob, Pat)$$

$$p_2' = Faster(Pat, Steve)$$

$$p_1 \land p_2 \rightarrow q = Faster(x, y) \land Faster(y, z) \rightarrow Faster(x, z)$$

$$\sigma = x/Bob, y/Pat, z/Steve$$

$$q\sigma = Faster(Bob, Steve)$$

GMP is used with KB of *definite clauses* (*exactly* 1 positive literal):

- A single atomic sentence
- Or a clause of the form: (conjunction of atomic sentences) \Rightarrow (atomic sentence)

All variables assumed universally quantified.

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Completeness in FOL

• Procedure *i* is complete if and only if

 $KB \vdash_i \alpha$ whenever $KB \models \alpha$

• GMP is *complete for KBs of universally quantified definite clauses*, but incomplete for general first-order logic

• Example:

PhD(x)	\rightarrow	HighlyQualified(x)					
$\neg PhD(x)$	\rightarrow	EarlyEarnings(x)	مام الم	مامام	±	infor	
HighlyQualified(x)	\rightarrow	Rich(x)	snould	be	able	το	Inter
EarlyEarnings(x)	\rightarrow	Rich(x)					

Rich(Doina). But the second sentence has two positive literals, so Modus ponens will not work.

Resolution

- Entailment in first-order logic is only *semi-decidable*: we can find a proof of α if KB ⊨ α but cannot always prove that KB ⊭ α
 Cf. Halting Problem (see COMP-330): proof procedure may be about to terminate with success or failure, or may go on for ever
- However, there is a sound an complete inference procedure, called *resolution*

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Resolution

- Resolution is a sound and complete inference method for first-order logic.
- Resolution is a *refutation* procedure: to prove that $KB \models \alpha$, we show that $KB \land \neg \alpha$ is unsatisfiable
- \bullet The knowledge base and $\neg \alpha$ are expressed in universally quantified, conjunctive normal form
- Like in propositional logic, the resolution inference rule combines two clauses to make a new one:
- Inference continues until an empty clause is derived (contradiction)

Resolution inference rule

Basic propositional version:

 $\frac{\alpha \lor \beta, \ \neg \beta \lor \gamma}{\alpha \lor \gamma} \qquad \text{or equivalently} \qquad \frac{\neg \alpha \to \beta, \ \beta \to \gamma}{\neg \alpha \to \gamma}$

Full first-order version:

where $p_j \sigma = \neg q_k \sigma$

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Conjunctive Normal Form (CNF)

- Literal = (possibly negated) atomic sentence, e.g., \neg *Rich(Me)*
- Clause = disjunction of literals, e.g., \neg *Rich(Me)* \lor *Unhappy(Me)*
- The knowledge base is a big conjunction of clauses.
- Example:

 $\neg Rich(x) \lor Unhappy(x)$ Rich(Me)Unhappy(Me)

with $\sigma = \{x/Me\}$

Converting a Knowledge Base to CNF

- 1. Replace $P \rightarrow Q$ by $\neg P \lor Q$
- 2. Move \neg inwards, e.g., $\neg \forall x P$ becomes $\exists x \neg P$
- 3. Standardize variables apart, e.g., $\forall x P \lor \exists x Q$ becomes $\forall x P \lor \exists y Q$
- 4. Move quantifiers left in order, e.g., $\forall x P \lor \exists x Q$ becomes $\forall x \exists y P \lor Q$
- 5. Eliminate \exists by Skolemization (next slide)
- 6. Drop universal quantifiers
- 7. Distribute \land over \lor , e.g., $(P \land Q) \lor R$ becomes $(P \lor R) \land (Q \lor R)$

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Skolemization

- We want to get rid of existentially quantified variables: ∃*Rich(x)* becomes *Rich(G1)* where *G1* is a new *Skolem constant*.
- Example: $\exists k \frac{d}{dy}(k^y) = k^y$ becomes $\frac{d}{dy}(e^y) = e^y$
- It gets more tricky when \exists is inside \forall
- E.g.: "Everyone has a heart"
 ∀x Person(x) → ∃y Heart(y) ∧ Has(x,y)
 How should we replace y here?
- Incorrect: $\forall x \ Person(x) \rightarrow Heart(H1) \land Has(x,H1)$
- Correct: $\forall x \ Person(x) \rightarrow Heart(H(x)) \land Has(x,H(x))$ where *H* is a new symbol called a *Skolem function*
- Skolem functions have as arguments all enclosing universally quantified variables

Resolution Proof

To prove α :

- 1. Negate it
- 2. Convert to CNF
- 3. Add the result to the knowledge base

4. Infer a contradiction (empty clause)

Example: to prove Rich(Doina), add $\neg Rich(Doina)$ to the CNF KB $\neg PhD(x) \lor HighlyQualified(x)$ $PhD(x) \lor EarlyEarnings(x)$ $\neg HighlyQualified(x) \lor Rich(x)$ $\neg EarlyEarnings(x) \lor Rich(x)$

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Resolution Strategies

Heuristics that impose a sensible order on the resolutions we attempt:

- *Unit resolution*: prefer to perform resolution if one clause is just a literal yields shorter sentences
- Set of support: identify a subset of the KB (hopefully small); every resolution will take a clause from the set and resolve it with another sentence, then add the result to the set of support
 - Can make inference incomplete!
- *Input resolution*: always combine a sentence from the query or *KB* with another sentence
 - Modus ponens is a kind of input resolution
 - Not compete in general

More resolution strategies

- *Linear resolution*: resolve *P* and *Q* if *P* is in the original *KB* or is an ancestor of *Q* in the proof tree.
- *Subsumption*: eliminate all sentences more specific than a sentence already in the *KB*
- *Demodulation and paramodulation*: special extra inference rules to allow treatment of equality

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Applications of First-Order Logic

- Prolog: a logic programming languages
- Production systems
- Semantic nets
- Automated theorem proving
- Planning

STRIPS

- Developed at Stanford in early 1970s (Stanford Research Institute Planning System), for the first "intelligent" robot
- Domain: a set of typed objects; usually represented as propositions
- States are represented as first-order predicates over objects
 - Closed-world assumption: everything not stated is false; the only objects in the world are the ones defined
- Operators/Actions defined in terms of:
 - Preconditions: when can the action be applied?
 - Effects: what happens after the action?

No explicit description of how the action should be executed

• Goals: conjunctions of literals

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STRIPS representations

- States are represented as conjunctions In(Robot,room) ∧ ¬ In(Charger, r) ∧ …
- Goals are represented as conjunctions: (implicit ∃ r) ln(Robot, r) ∧ ln(Charger, r)
- Actions (operators):
 - Name: Go(here, there)
 - Preconditions: expressed as conjunctions At(Robot, here) ∧ Path(here, there)
 - Postconditions (effects): expressed as conjunctions At(Robot, there) ∧ ¬ At(Robot, here)
- Variables can only be instantiated with objects of the correct type

STRIPS Operator Representation

- Operators have a name, preconditions and postconditions or effects
- Preconditions are conjunctions of *positive literals*
- Postconditions/effects are represented in terms of:
 - <u>Add-list</u>: list of propositions that become true after the action
 - <u>Delete-list</u>: list of propositions that become false after the action

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Semantics

- If the precondition is false in a world state, the action does not change anything (since it cannot be applied)
- If the precondition is true:
 - Delete the items in the Delete-list
 - Add the items in the Add-list.

Order of operations is important here!

This is a very restricted language, which means we can do efficient inference.

Example: Buying Action

- Action: *Buy(x)* (where *x* is a good)
- Precondition: At(s), Sells(s,x,p), HaveMoney(p) (where s is a store, p is the price)
- Effect:
 - Add-list: Have(x)
 - Delete-list: HaveMoney(p)
- Note that many important details are abstracted away!
- Additional propositions can be added to show that now the store has the money, the stock has decreased etc.

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Example: Move Action

- Action: *Move(object, from, to)*
- Preconditions: At(object, from), Clear(to), Clear(object)
- Effects:
 - Add-list: At(object, to), Clear(from)
 - Delete-list: At(object, from), Clear(to)

Pros and cons of STRIPS

- Pros:
 - Since it is restricted, inference can be done efficiently
 - All operators can be viewed as simple deletions and additions of propositions to the knowledge base
- Cons:
 - Assumes that a small number of propositions will change for each action (otherwise operators are hard to write down, and reasoning becomes expensive)
 - Limited language (preconditions and effects are expressed as conjunctions, implicit quantifiers), so not applicable to all domains of interest.

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 Example: Blocks World

 Imitial state
 Imitial state

 Initial state
 Goal state

 Initial state
 On(B,table) \land On(C,table) \land Clear(A) \land Clear(B) \land Clear(C)

 Goal state
 On(A,table) \land On(B,table) \land On(C,table) \land Clear(A) \land Clear(B) \land Clear(C)

 Goal state
 On(A,table) \land On(B,table) \land On(C,table) \land Clear(A) \land Clear(B) \land Clear(C)

 Goal state
 On(A,table) \land On(B,table) \land On(C,table) \land Clear(A) \land Clear(B) \land Clear(C)

 Action = Move(b,x,y)
 Precondition = On(b,x) \land Clear(b)

 Effect = On(b,Y) \land Clear(b) \land On(b,x) \land \neg Clear(y)

 Action = MoveToTable(b,x)
 Preconditions = On(b,x) \land Clear(b)

 Effect = On(b,Table) \land Clear(x) \land \neg On(b,x)



Plan-Space Planning in the Blocks World

- Start with plan: *Put(A,B)*, *Put(B,C)*
- Plan fails because the precondition of the second action is not satisfied after the first action
- So we can try to add a step, remove a step, or re-order the steps

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State-Space Planners

- *Progression planners* reason from the start state, trying to find the operators that can be applied (match preconditions)
- *Regression planners* reason from the goal state, trying to find the actions that will lead to the goal (match effects or post-conditions)

In both cases, the planners work with <u>sets of states</u> instead of using individual states, like in straightforward search

Progression (Forward) Planning

- 1. Determine all operators that are applicable in the start state
- 2. Ground the operators, by replacing any variables with constants
- 3. Choose an operator to apply
- 4. Determine the new content of the knowledge base, based on the operator description
- 5. Repeat until goal state is reached.

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Example: Supermarket Domain

- In the start state we have At(Home), which allows us to apply operators of the type Go(x,y).
- The operator can be instantiated as Go(Home, HardwareStore), Go(Home, GroceryStore), Go(Home, School), ...
- If we choose to apply *Go(Home, HardwareStore)*, we will delete from the KB *At(Home)* and add *At(HardwareStore)*.
- The new proposition enables new actions, e.g. Buy

Note that in this case there are a lot of possible operators to perform!

Goal Regression

- Introduced in Newell & Simon's General Problem Solver
- Algorithm:
 - 1. Pick an action that satisfies (some of) the goal propositions
 - 2. Make a new goal by:
 - Removing the goal conditions satisfied by the action
 - Adding the preconditions of this action
 - Keeping any unsolved goal propositions
 - 3. Repeat until the goal set is satisfied by the start state

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Example: Supermarket Domain

- In the goal state we have $At(Home) \land Have(Milk) \land Have(Drill)$
- The action *Buy(Milk)* would allow us to achieve *Have(Milk)*
- To apply this action we need to have the precondition At(GroceryStore), so we add it to the set of propositions we want to achieve
- The goal set becomes: *At(Home)* \land *At(GroceryStore)* \land *Have(Drill)*
- Next, we may want to achieve *At(HardwareStore)*

Note that in this case the <u>order</u> in which we try to achieve these propositions matters!

Variations of Goal Regression

- Using a *stack* of goals also called *linear planning* This is *not complete!* I.e. we may not find a plan even if one exists
- Using a set of goals also called non-linear planning
 This is complete, but more expensive (need to decide what to work on next)
- Both versions are *sound*: only legal plans will be found

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Prodigy Planner

- Do both forward search and goal regression at the same time.
- At each step, choose either an operator to apply or goal to regress
- Uses domain-dependent heuristics to guide the search
- General heuristics (e.g. number of propositions satisfied) do not work well in planning, because subgoals interact



Plan-Space Planners

Plan is defined by $\langle A, O, B, L \rangle$:

- A is a set of actions/operators from the problem domain
- O is a set of ordering constraints of the form $a_i < a_j$ The constraint specifies that a_i must come before a_j but does not say exactly when
- B is a set of bindings, of the form $v_i = C$, $v_i \neq C$, $v_i = v_j$ or $v_i \neq v_j$, where v_i, v_j are variables and C is a constant
- L is a set of causal links, which records why a certain ordering has to occur:

 $a_i \rightarrow^c a_j$ means that action a_i achieves effect c which is a precondition of a_j

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Plan Transformations

- Adding *actions*
- Specifying *orderings*
- **Binding** variables

Constraint satisfaction is used along the way to ensure the consistency of orderings

Discussion of Partial-Order Planning

- Advantages:
 - Plan steps may be unordered (plan will be ordered, or linearized, before execution)
 - Handles concurrent plans
 - Least commitment can lead to shorter search times
 - Sound and complete
 - Typically produces the optimal plan
- Disadvantages:
 - Complex plan operators lead to high cost for generating every action
 - Larger search space, because of concurrent actions
 - Hard to determine what is true in a state

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The real world

Things are usually not as expected:

- Incomplete information
 - Unknown preconditions, e.g., *Intact(Spare)*
 - Disjunctive effects, e.g., Inflate(x) causes Inflated(x) according to the knowledge base, but in reality it actually causes $Inflated(x) \lor SlowHiss(x) \lor Burst(x) \lor BrokenPump \lor ...$

• Incorrect information

- Current state incorrect, e.g., spare NOT intact
- Missing/incorrect postconditions in operators
- **Qualification problem**: can never finish listing all the required preconditions and possible conditional outcomes of actions

Solutions

- Conditional (contingency) planning:
 - 1. Plans include observation actions which obtain information
 - 2. Sub-plans are created for each contingency (each possible outcome of the observation actions)

E.g. Check the tire. If it is intact, then we're ok, otherwise there are several possible solutions: inflate, call AAA....

Expensive because it plans for many unlikely cases

• Monitoring/Replanning:

- 1. Assume normal states, outcomes
- 2. Check progress *during execution*, replan if necessary

Unanticipated outcomes may lead to failure (e.g., no AAA card)

In general, some monitoring is unavoidable

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Monitoring

- **Execution monitoring**: "failure" means that the preconditions of the *remaining plan* not met
- Action monitoring: "failure" means that the preconditions of the <u>next action</u> not met (or action itself fails)

In both cases, need to replan

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Replanning

- Simplest: on failure, replan from scratch
- Better: plan to get back on track by reconnecting to best continuation In this case, we can try to reconnect to the plan's next action, or some future action

The latter is typically more expensive in terms of planning computation (lots of possible places to reconnect!) but usually yields better plans (e.g. if it is very hard to achieve the preconditions of the very next action)

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Summary

- Planning is very related to search, but allows the actions/states have more structure
- We typically use logical inference to construct solutions
- State-space vs.plan-space planning
- Least-commitment: we build partial plans, order them only as necessary
- In the real world, it is necessary to consider failure cases replanning
- Hierarchy and abstraction make planning more efficient
- Many varieties of planners that we have not looked at: case-based planners, MDP planners (we will see this later on) etc.