Lecture 6: Game Playing

- Why games?
- Overview of state-of-art
- Minimax search
- Evaluation functions
- α-β pruning

Game Playing

- One of the oldest, most well-studied domains in AI!
- Why?
  - People like them! And are good at playing them
  - Often viewed as an indicator of intelligence
    - State space is very large and complicated
    - Sometimes there is stochasticity and imperfect information
  - There is a clear, clean description of the environment
  - Easy to evaluate performance!
- Samuel's checkers player - first notable success

"Games are to AI as grand prix racing is to automobile design"

Types of games

- *Perfect vs. imperfect information*
  - Perfect: See the exact state of the game
    E.g., chess, backgammon, checkers, go, othello
  - Imperfect: Information is hidden
    E.g., Scrabble, bridge, multi-player games
- *Deterministic vs. stochastic*
  - Deterministic; Change in state is fully determined by player move
    E.g. chess
  - Stochastic: Change in state is partially determined by chance
    E.g. backgammon, poker

Human or Computer: Who is Better?

- Checkers:
  - 1994: Chinook (UofA) beat human world champion Marion Tinsley ending 42-year reign (during which he lost only 3 games!)
- Chess:
  - 1997: Deep Blue (IBM) beat world champion Gary Kasparov
  - 2002: Fritz drew with world champion Vladimir Kramnik
- Othello:
  - 1997: Logistelo (NEC Research) beat world champion Takeshi Murakami
  - Today: human champions refuse to play best computer programs (because computers are too good)
- Go:
  - $1,000,000 prize available
  - Master-level play achieved in the last two years

Human or Computer: Who Is Better?

- Scrabble
  - 1998: Maven (UofA) beats world champion Adam Logan 9-5
  - Knowing the whole dictionary helps a lot!
- Bridge:
  - 1988: Ginsberg's program places 12th in world championships
  - Coordination with partner still very difficult
- Poker:
  - 2008: Polaris (UofA) beats some of the best on-line human players
  - Still very difficult to adapt to changing opponents
- Commercial, multi-player games
  - Very hard problems, progress slowly being made
  - Real-time, opponents change, dynamic, cannot see everything, ...
  - Goal is often not to beat human players, but to provide "interesting" opponents

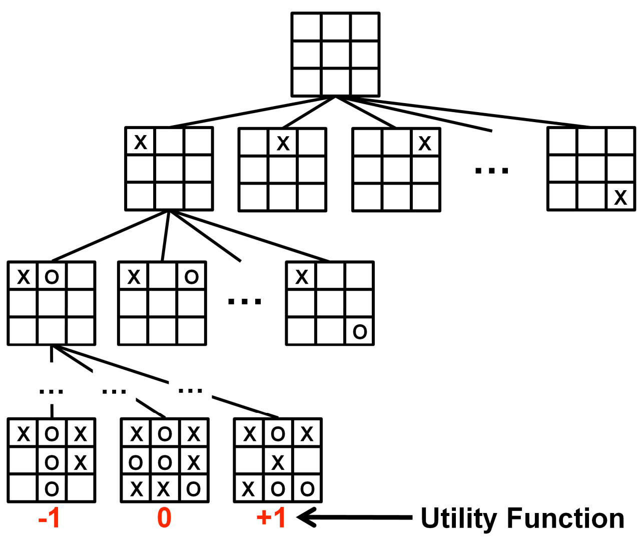
Game Playing as Search

- Consider two-player, perfect information, deterministic games.
- Can we formulate them as search problems?
  - State: state of the board
  - Operators: legal moves
  - Goal: states in which the game is won/lost/drawn
  - Cost:
    - Simple utility: +1 for winning, -1 for losing, 0 for a draw
    - More complex cases: points won, money, ...
  - We want to *find a strategy* (i.e. a way of picking moves) that *maximizes utility*

Game Search Challenge

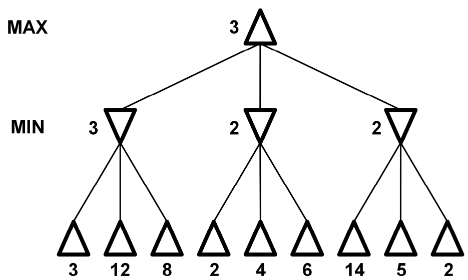
- Not quite the same as simple searching
- There is a malicious opponent!
  - It is trying to make things good for itself, and bad for us
  - We have to *simulate the opponent's decision*
- Main idea: utility from a single agent's perspective
  - Define a *max player* (who wants to maximize its utility)
  - And a *min player* (who wants to minimize it).

Example: Tic-Tac-Toe



Minimax Search

- Expand a complete search tree, until terminal states have been reached and their utilities can be computed
- Go back up from the leaves towards the current state of the game
  - At each min node, back up the worst value among children
  - At each max node, back up the best value among children



MINIMAX(Node n) =

$$\begin{cases} \text{UTILITY}(n) & \text{if n is a terminal state} \\ \max_{s \in \text{Succ}(n)} \text{MINIMAX}(s) & \text{if n is a MAX node} \\ \min_{s \in \text{Succ}(n)} \text{MINIMAX}(s) & \text{if n is a MIN node} \end{cases}$$

Minimax Algorithm

Operator MinimaxDecision ()

1. For each legal operator o:
   (a) Apply the operator o and obtain the new game state s
   (b) Value$[o]$ = MinimaxValue(s)
2. Return the operator with the highest value Value$[o]$

double MinimaxValue (s)

1. if isTerminal(s) return Utility(s);
2. For each state $s' \in$ Successors(s), Value(s') = MinimaxValue(s')
3. If Max is to move in s, return $\max_{s'}$ Value(s')
4. If Min is to move in s, return $\min_{s'}$ Value(s')

Properties of Minimax Search

- Complete if the game tree is finite
- *Optimal against an optimal opponent*
  Otherwise, we do not know!
- Time complexity $O(b^m)$
- Space complexity $O(bm)$ (because search goes depth-first, and at each of the m levels we keep b candidate moves)
- Why not use minimax to solve chess for example?
  For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games, so an *exact solution is impossible*

Coping with Resource Limitations

- Suppose we have 100 seconds to make a move, and we can search 10^4 nodes per second
  - That means we have to limit the search to 10^6 nodes before choosing a move.
- Standard approach:
  - Use a cutoff test (e.g. based on depth limit)
  - Use an *evaluation function* (akin to a heuristic) to estimate the value of nodes where we cut off the search
- This resembles real-time search

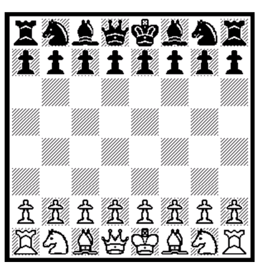
Evaluation Functions

- An *evaluation function* $v(s)$ represents the "goodness" of a board state s (i.e. the chance of winning from that position)
- If the features of the board can be judged independently, then a good choice is a *weighted linear function*:

$$v(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s)$$

  where s is the board state
- This can be given by the designer or learned from experience

Example: Chess



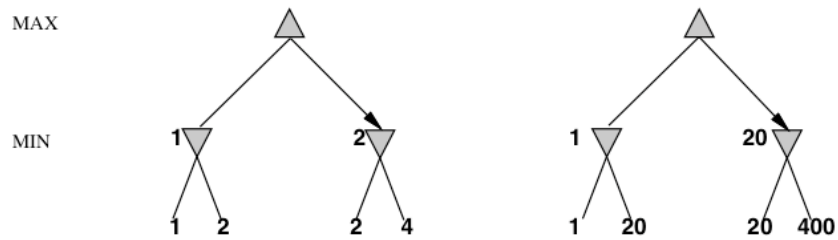
Assign a value to each piece: Pawn = 1; Knight =3; Bishop =3; etc. Score of a position is the sum of the values of all my pieces minus all opponent's pieces.

More sophisticated: Linear evaluation function $w_1 f_1(s) + w_2 f_2(s) + \ldots$ where, e.g.

- $w_1 = 9$ with $f_1(s) =$ (nr. white queens) - (nr. black queens) etc.
- $w_2 = 12$ with $f_2(s) =$ nr. of available moves (mobility)
- $w_3 = -12$ $f_3(s)$=nr, available moves for opponents (it is bad for the opponent to have many choices)
- . . .

How Precise Should the Evaluation Function Be?

- Evaluation function is only approximate, and is usually more accurate for positions close to the end of the game
- The move chosen is the same if we apply a *monotonic transformation* to the evaluation function!



- Only the order of the numbers matters: payoffs in deterministic games act as an *ordinal utility function*

Cutting the Search Effort

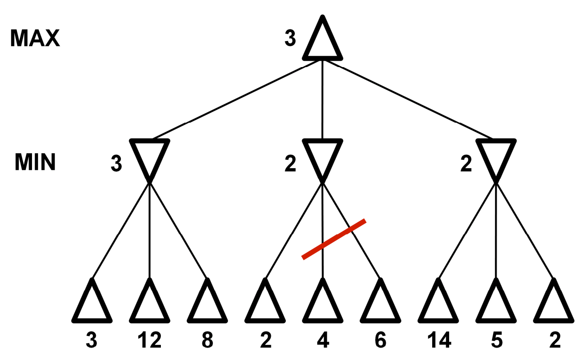
- Evaluation functions help us make a decision without searching until the end of the game
- Imagine a *MinimaxCutoff* algorithm, which is the same as *MinimaxValue*, except it goes to some maximum depth m and uses the evaluation function on those nodes (instead of going to the end of the game and using the correct utility)
- How many moves can we search ahead in chess?

  10^6 nodes with $b = 35$ allows us to search $m = 4$ moves ahead!

Minimax Cutoff in Chess

- 4-ply search gives a pretty bad chess player!
  - 4-ply $\approx$ human novice
  - 8-ply $\approx$ human master, typical PC
  - 12-ply $\approx$ Deep Blue, Kasparov
- Human experts tend to search very few lines of play, but they search them very deeply!
- Main idea: use *pruning*!

α-β pruning example



- Suppose the leftmost subtree has been searched, and Max knows that the value of its move there is 3
- Searching the center tree, Max discovers than Min has a move of value 2, so Min can get a move of value ≤ 2 in this subtree
- But this is worse for Max!
- Max would never take this move, since it has a better alternative, so there is no point in searching this subtree further

α-β pruning

- Standard technique for deterministic, perfect information games
- The idea is similar to α-pruning: if a move estimate looks worse than another choice we already have, discard it
- The algorithm is like minimax, but keeps track of the best leaf value for the Max palyer (α) and the best value for the Min player (β)
- If the best move at a node cannot change, regardless of what we find by searching, then no need to search further!

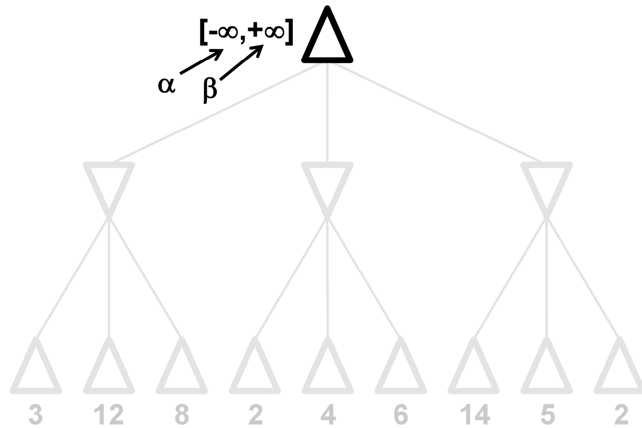
α-β Algorithm

Instead of *MinimaxValue*, we have two functions, *MaxValue* and *MinValue*, which update the two cutoffs differently

double MaxValue(s, α, β)
1. If cutoff(s) return Evaluation(s)
2. For each s' in Successors(s)
   (a) $\alpha \leftarrow \max(\alpha, \textit{MinValue}(s', \alpha, \beta))$
   (b) If $\alpha \geq \beta$ return β
3. Return α

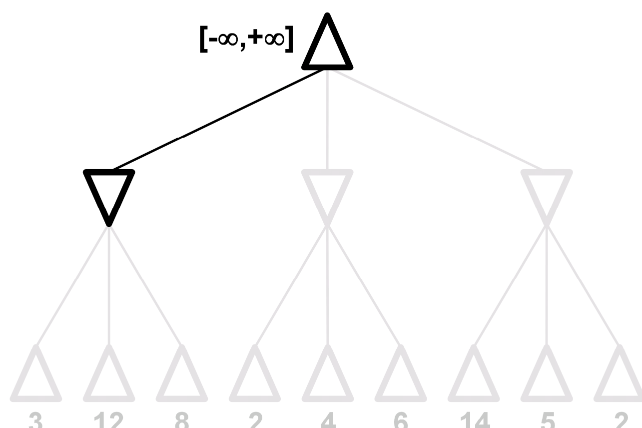
double MinValue(s, α, β)
1. If cutoff(s) return Evaluation(s)
2. For each s' in Successors(s)
   (a) $\beta \leftarrow \min(\beta, \textit{MaxValue}(s', \alpha, \beta))$
   (b) If $\alpha \geq \beta$ return α
3. Return β

Example



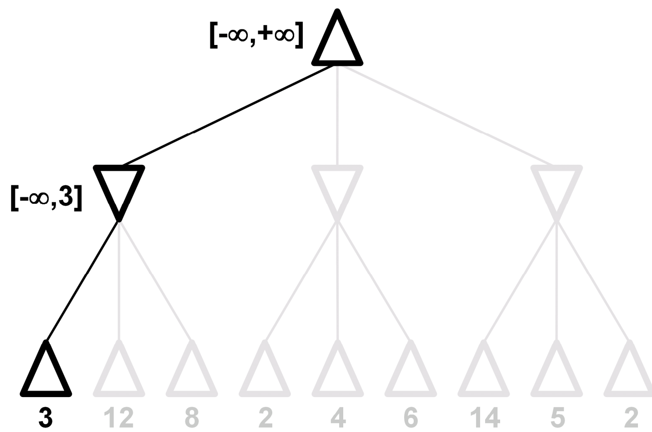
Initialize α and β

Example



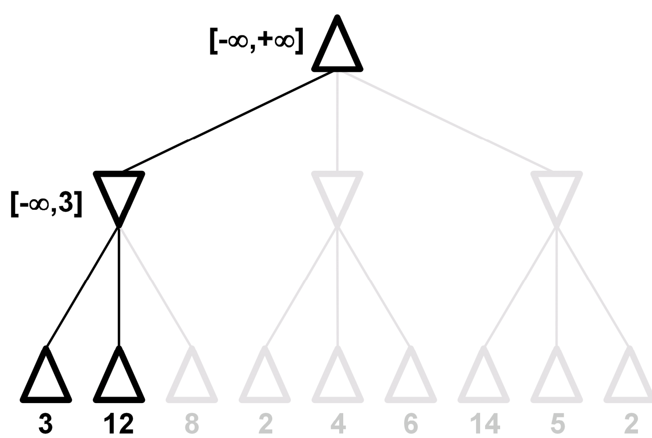
We search the first move for Max

Example



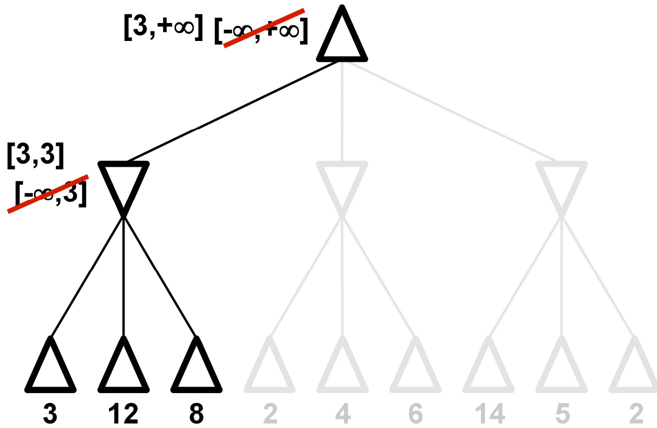
We discovered a move of value 3 for Min, so Max's value for this move will be at most 3

Example



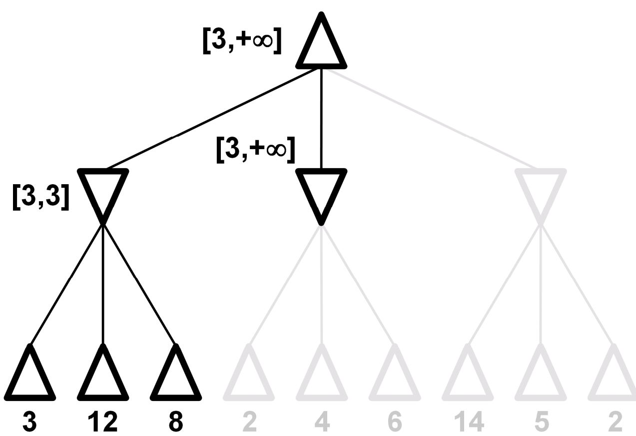
Now we see a move of value 12 for Min, but it already has a better option, so no changes are made

Example



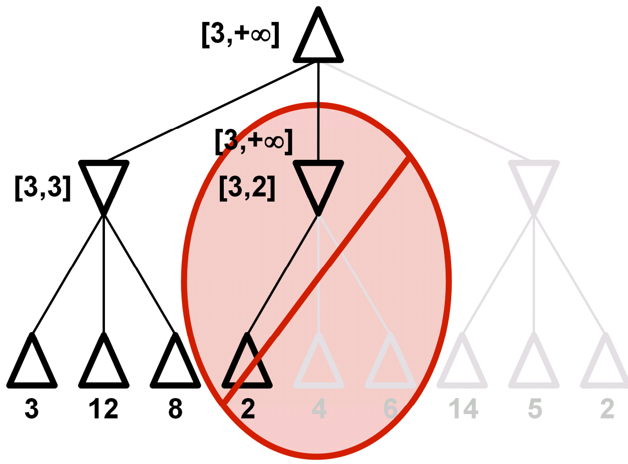
We finished searching all of Min's move on this branch, and figured out that the best Max can hope for is to get a 3 (if Min plays optimally)

Example



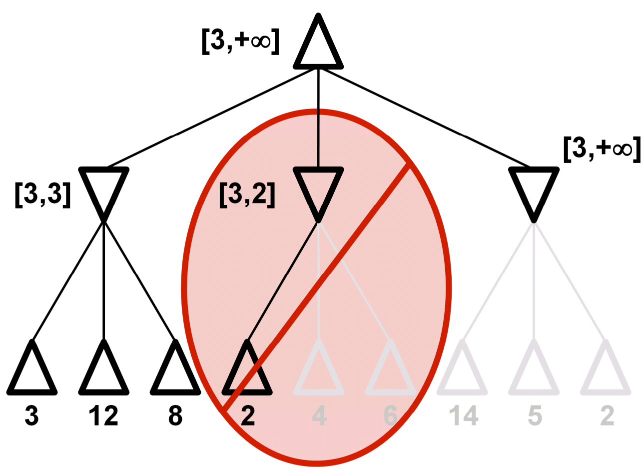
Now we are about to search Max's middle move

Example



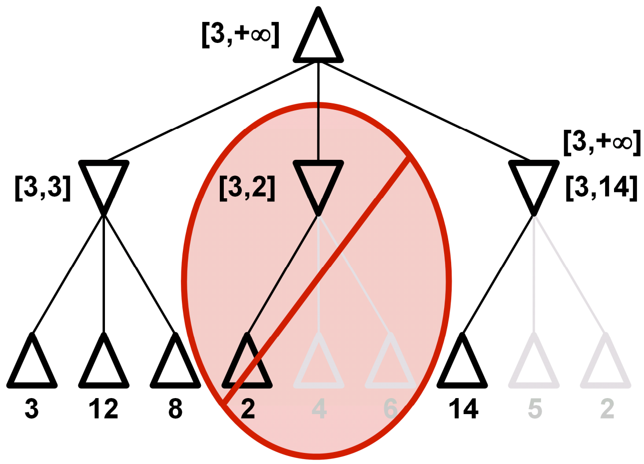
Min has a move of value 2, so Max's value for this branch must be ≤ 2. Hence, Max should never take this move, and no further search can change this decision. The subtree is pruned off

Example



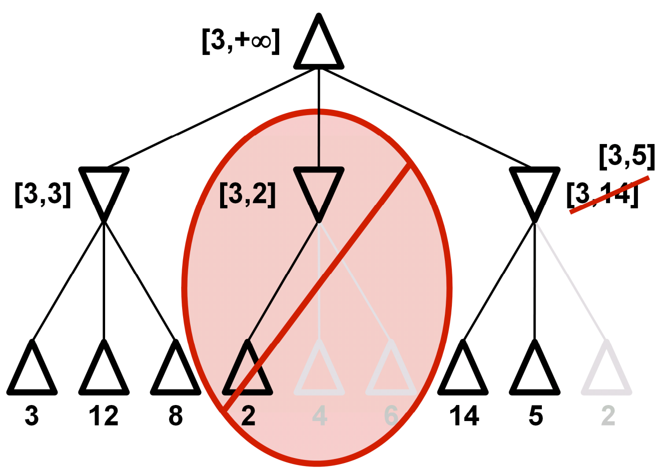
Proceed to Max's rightmost move

Example



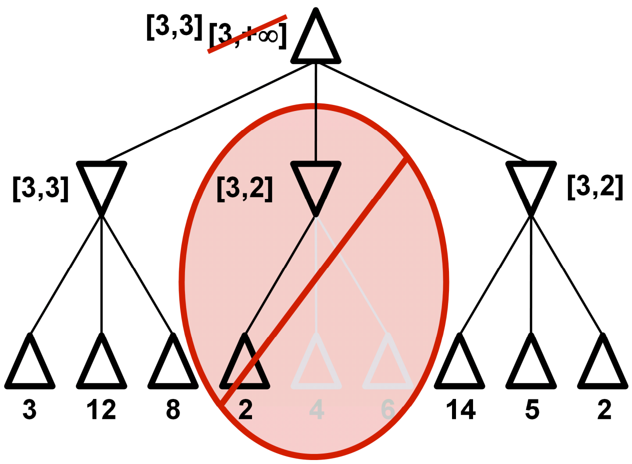
We found a move for Min of value 14, so this move looks like it could be better for Max than its current value of 3

Example



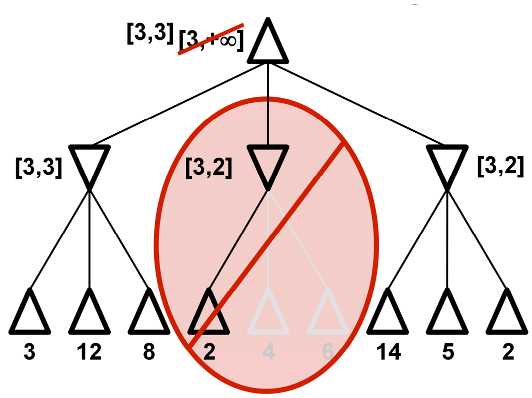
Max could still get a 5 on this move

Example



We finished searching, this last move was not as good as hoped. The optimal play gives Max a value of 3, on the leftmost move

Example: Important lessons



- *Order matters!* On the middle branch, nodes were ordered well and we pruned a lot; on the right branch, the order was bad and there was no proning
- The best moves were *same as returned by Mimimax* (it can be proved that this is always true for an optimal opponent)

Properties of α-β Pruning

- *Pruning does not affect the final result!*
- Good move ordering is key to the effectiveness of pruning
  - With *bad move ordering* complexity is $\approx O(b^m)$ (nothing pruned)
  - With *perfect ordering*, the time complexity is $\approx O(b^{m/2})$ (because we cut off the branching at every other level)
    * Means we double the search depth for the same resources
    * In chess (and other games) this is the difference between a novice and an expert player
  - On the average, $O(b^{3m/4})$ (if we expect to find the max or min after $b/2$ expansions)
  - Randomizing the move ordering can achieve the average case
  - Evaluation function can be used to give a good initial ordering for the nodes
- α-β pruning demonstrates the value of reasoning about which computations are important.

Deep Blue (IBM)

- Specialized chess processor, with special-purpose memory organization
- A very sophisticated evaluation function, with expert features and hand-tuned weights
- Database of opening/closing moves
- Uses a version of α-β pruning with undisclosed improvements, which allow searching some lines up to 40 ply deep.
- Can search over 200 million positions per second!
- Overall, an impressive engineering feat
- Now, several computer programs running on regular hardware are on par with human champions (e.g. Fritz).

Chinook (Schaeffer, U. of Alberta)

- Plain α-β search, performed on standard PCs
- Evaluation function based on expert features of the board
- Opening database
- *Huge endgame database!*

  Chinook has perfect information for all checkers positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.
- Only a few moves in the middle of the game were actually searched!
- They have now done an exhaustive search for checkers, and discovered that optimal play leads to a draw

Logistello (Buro, U. of Alberta)

- Opening book, continuously updated based on the games played ($\approx$ 23000 games)
- α-β search with a *linear evaluation function*:
  - Hand-selected features
  - 1.5 million weights tuned by learning during self-play games
- Thinks during the opponent's time
- Search speed (on a Pentium-Pro 200) $\approx$ 160,000 nodes/sec in the middle game, $\approx$ 480,000 nodes/sec in the endgame
- Search depth $\approx$ 18-23 ply in the middle game
- Win/loss/draw determination at 26-22 empty squares, exact score 1-2 ply later

Drawbacks of α-β

- If the *branching factor is really big*, search depth is still too limited
  E.g. in Go, where branching factor $b \approx 300$
- Optimal play is guaranteed against an optimal opponent if search proceeds to the end of the game
- But the opponent may not be optimal!
- If heuristics are used, this assumption turns into the opponent playing optimally *according to the same heuristic function* as the player
- This is a very big assumption! What to do if the opponent plays very differently?

Summary

- Games are a cool, realistic testbed for AI ideas
- Search is similar to A^* (using heuristics), but one needs to consider that the opponent will try to harm
- It is crucial to decide where to spend the computation effort, and prune unimportant paths
- Computers dominate many classical, perfect-information games, using $\alpha - \beta$ pruning
- However, this may not be good enough in games with very large branching factor (e.g. Go) or imperfect information / stochastic games
- Next time: Monte Carlo tree search