80-211 Arguments and Inquiry
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Natural Deduction rules for propositional logic

This particular way of writing inference rules is due to Gerhard Gentzen (1909–45).

$\neg$–Intro
$$\dfrac{\begin{matrix}[A]\\ \vdots\\ \bot\end{matrix}}{\neg A}$$

$\neg$–Elim
$$\dfrac{A \quad \neg A}{\bot}$$

& –Intro
$$\dfrac{A \quad B}{A \,\&\, B}$$

& –Elim
$$\dfrac{A \,\&\, B}{A} \qquad \dfrac{A \,\&\, B}{B}$$

$\vee$ –Intro
$$\dfrac{A}{A \vee B} \qquad \dfrac{B}{A \vee B}$$

$\vee$ –Elim
$$\dfrac{A \vee B \quad \begin{matrix}[A]\\ \vdots\\ C\end{matrix} \quad \begin{matrix}[B]\\ \vdots\\ C\end{matrix}}{C}$$

$\rightarrow$–Intro
$$\dfrac{\begin{matrix}[A]\\ \vdots\\ B\end{matrix}}{A \rightarrow B}$$

$\rightarrow$–Elim
$$\dfrac{A \quad A \rightarrow B}{B}$$

ex falso quodlibet
$$\dfrac{\bot}{A}$$

RAA
$$\dfrac{\begin{matrix}[\neg A]\\ \vdots\\ \bot\end{matrix}}{A}$$

Example

Theorem 1 ("Dilemma")**.** $A \vee C,\ A \rightarrow B,\ C \rightarrow D \vdash B \vee D\,.$

Proof. In Natural Deduction:

$$\dfrac{A \vee C \qquad \dfrac{\dfrac{[A] \quad A \rightarrow B}{B}}{B \vee D} \qquad \dfrac{\dfrac{[C] \quad C \rightarrow D}{D}}{B \vee D}}{B \vee D}$$

□