80-110 Nature of Mathematical Reasoning

Spring 2002

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— Handout #13 —

Natural Deduction rules

This particular way of writing formulas and proofs is due to Gerhard Gentzen (1909–45). The following rules will be discussed in class:

¬–Intro

$$\frac{\begin{array}{c}[A]\\ \vdots \\ \bot\end{array}}{\neg A}$$

¬–Elim

$$\frac{A \quad \neg A}{\bot}$$

& –Intro

$$\frac{A \quad B}{A \,\&\, B}$$

& –Elim

$$\frac{A \,\&\, B}{A} \qquad \frac{A \,\&\, B}{B}$$

∨ –Intro

$$\frac{A}{A \vee B} \qquad \frac{B}{A \vee B}$$

∨ –Elim

$$\frac{A \vee B \quad \begin{array}{c}[A]\\ \vdots \\ C\end{array} \quad \begin{array}{c}[B]\\ \vdots \\ C\end{array}}{C}$$

→–Intro

$$\frac{\begin{array}{c}[A]\\ \vdots \\ B\end{array}}{A \to B}$$

→–Elim

$$\frac{A \qquad A \to B}{B}$$

ex falso quodlibet

$$\frac{\bot}{A}$$

RAA

$$\frac{\begin{array}{c}[\neg A]\\ \vdots \\ \bot\end{array}}{A}$$

Example

Theorem 1. *From premises $A \vee C$, $A \to B$, and $C \to D$, the conclusion $B \vee D$ follows (argument form of 'dilemma').*

Proof. In Natural Deduction:

$$\frac{A \vee C \qquad \dfrac{\dfrac{[A] \quad A \to B}{B}}{B \vee D} \qquad \dfrac{\dfrac{[C] \quad C \to D}{D}}{B \vee D}}{B \vee D}$$

□