DATA STRUCTURES

COMP 321 – McGill University

These slides are mainly compiled from the following resources.

- Professor Jaehyun Park' slides CS 97SI
- Top-coder tutorials.
- Programming Challenges book.

Data Structure

- A way to store and organize data in order to support efficient insertions, queries, searches, updates, and deletions.

Data Structure

- Basic data structures (built-in libraries).
  - Linear DS.
  - Non-Linear DS.
- Data structures (Own libraries).
  - Graphs.
  - Union-Find Structures.
  - Segment Tree.

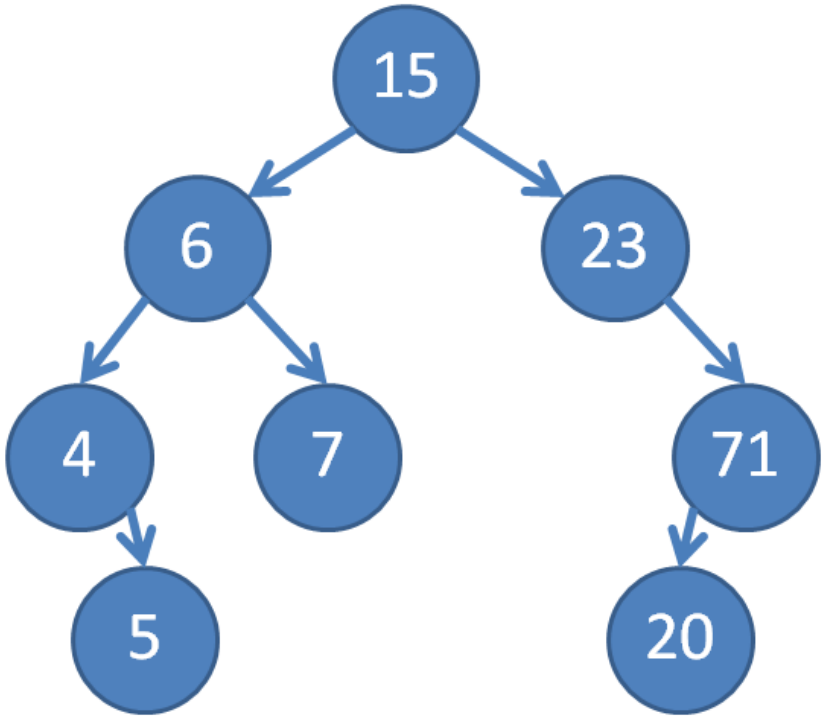
Data Structures

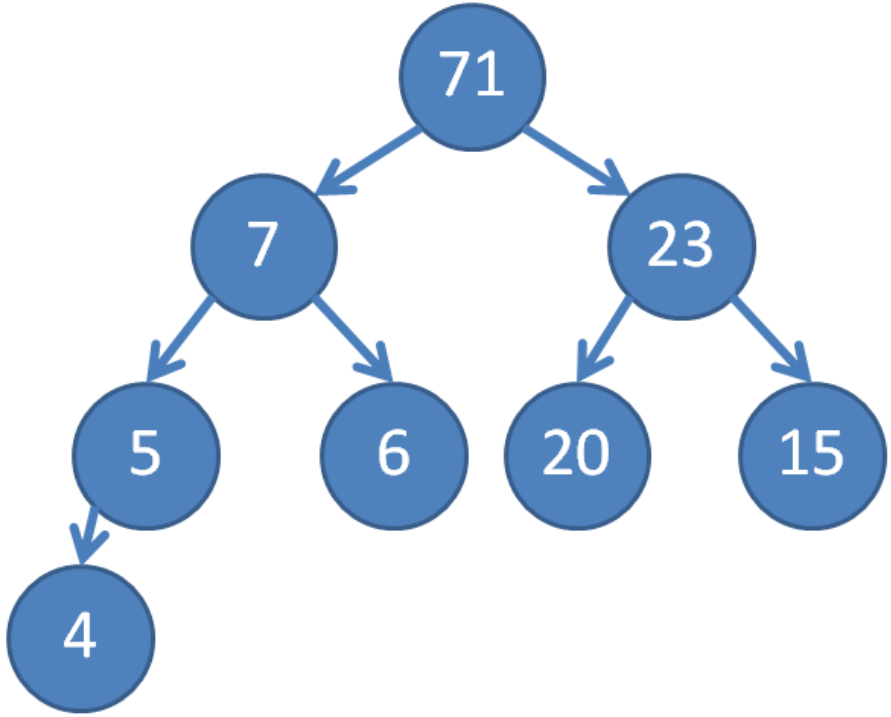
- Basic data structures (built-in libraries).
  - Linear DS (ordering the elements sequentially).
    - Static Array (Array in C/C++ and in Java).
    - Resizeable array (C++ STL<vector> and Java ArrayList).
    - Linked List: (C++ STL<list> and Java LinkedList).
    - Stack (C++ STL<stack> and Java Stack).
    - Queue (C++ STL <queue> and Java Queue).

Data Structures

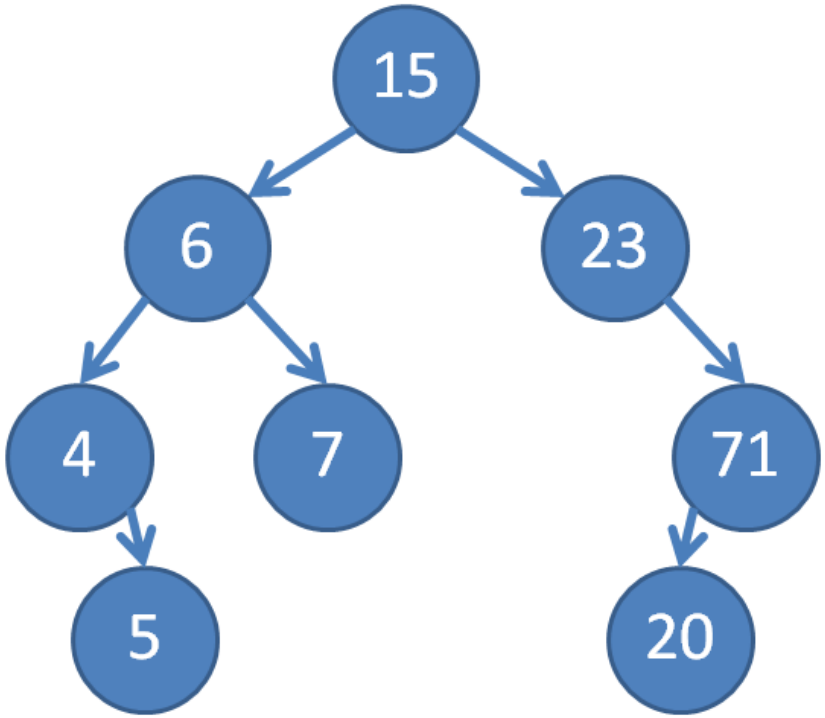
- Basic data structures (built-in libraries).
  - Non-Linear DS.
    - Balanced Binary Search Tree (C++ STL <map>/<set> and in Java TreeMap/TreeSet).
      - AVL and Red-Black Trees = Balanced BST
      - <map> stores (key -> data) VS <set> only stores the key
    - Heap(C++ STL<queue>:priority_queue and Java PriorityQueue).
      - BST complete.
      - Heap property VS BST property.
    - Hash Table (Java HashMap/HashSet/HashTable).
      - Non synchronized vs synchronized.
      - Null vs non-nulls
      - Predictable iteration (using LinkedHashMap) vs non predictable.

Question for you.

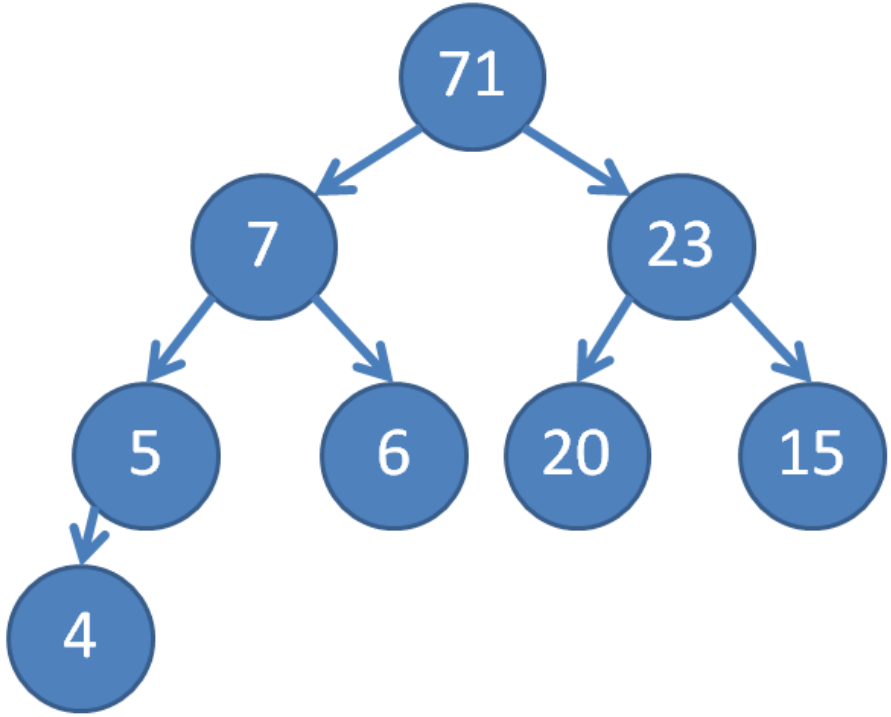




Question for you.



BST



HEAP

Do you recognize the problem?

Deciding the Order of the Tasks

- Returns the newest task (stack)
- Returns the oldest task (queue)
- Returns the most urgent task (priority queue)
- Returns the easiest task (priority queue)

STACK

- Last in, first out (Last In First Out)
- Stacks model piles of objects (such as dinner plates)
- Supports three constant-time operations
  - Push(x): inserts x into the stack
  - Pop(): removes the newest item
  - Top(): returns the newest item
- Very easy to implement using an array

STACK

- Have a large enough array s[] and a counter k, which starts at zero
  - Push(x) : set s[k] = x and increment k by 1
  - Pop() : decrement k by 1
  - Top() : returns s[k - 1] (error if k is zero)
- C++ and Java have implementations of stack
  - stack (C++), Stack (Java)

STACK

- Useful for:
  - Processing nested formulas
  - Depth-first graph traversal
  - Data storage in recursive algorithms

QUEUE

- First in, first out (FIFO)
- Supports three constant-time operations
  - Enqueue(x) : inserts x into the queue
  - Dequeue() : removes the oldest item
  - Front() : returns the oldest item
- Implementation is similar to that of stack

QUEUE

- Assume that you know the total number of elements that enter the queue
  - ... which allows you to use an array for implementation
  - … If not, you can use linked lists or double linked lists
- Maintain two indices head and tail
  - Dequeue() increments head
  - Enqueue() increments tail
  - Use the value of tail - head to check emptiness
- You can use queue (C++) and Queue (Java)

QUEUE

- Useful for
  - implementing buffers
  - simulating waiting lists
  - shuffling cards

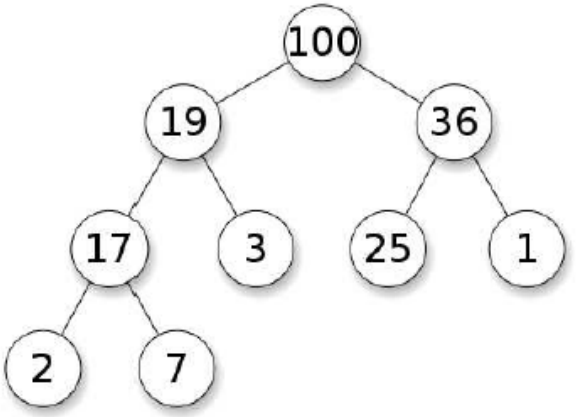
PRIORITY QUEUE

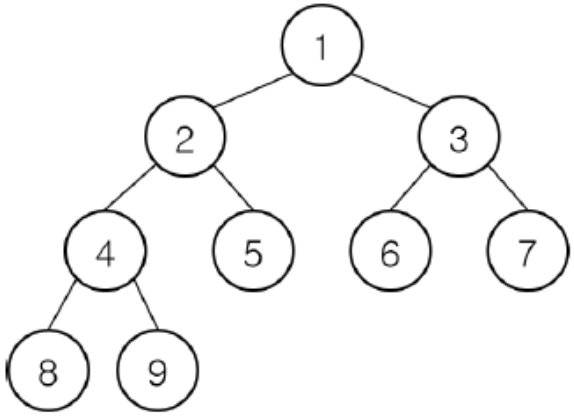
- Each element in a PQ has a priority value
- Three operations:
  - Insert(x, p) : inserts x into the PQ, whose priority is p
  - RemoveTop() : removes the element with the highest priority
  - Top() : returns the element with the highest priority
- All operations can be done quickly if implemented using a heap (if not use a sorted array)
- priority_queue (C++), PriorityQueue (Java)
- Useful for
  - Maintaining schedules / calendars
  - Simulating events
  - Sweepline geometric algorithms

HEAP

- Complete binary tree with the heap property:
  - The value of a node ≥ values of its children
  - What is the difference between full vs complete?
- The root node has the maximum value
  - Constant-time top() operation
- Inserting/removing a node can be done in O(log n) time without breaking the heap property
  - May need rearrangement of some nodes

HEAP





- Start from the root, number the nodes 1, 2, . . . from left to right
- Given a node k easy to compute the indices of its parent and children
  - Parent node: floor(k/2)
  - Children: 2k, 2k + 1

Heap – Inserting a Node

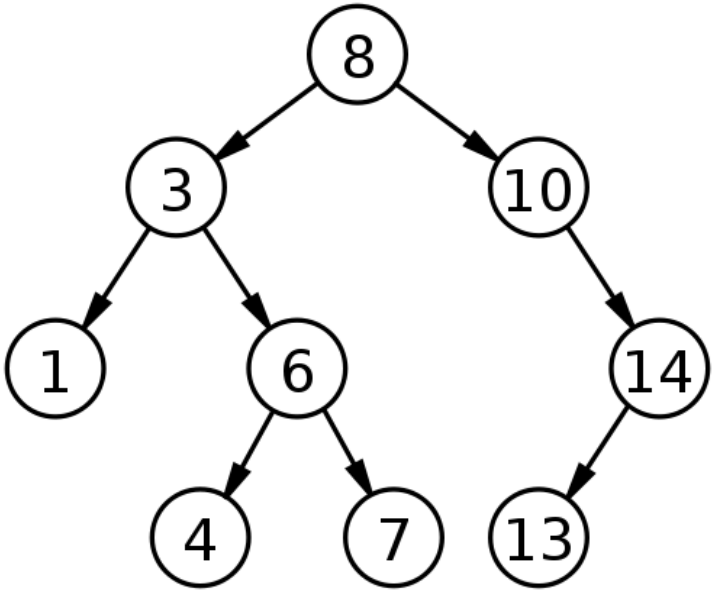
- 1. Make a new node in the last level, as far left as possible
  - If the last level is full, make a new one
- 2. If the new node breaks the heap property, swap with its parent node
  - The new node moves up the tree, which may introduce another conflict
- Repeat 2 until all conflicts are resolved
- Running time = tree height = O(log n)

Heap – Deleting a Node

- 1. Remove the root, and bring the last node (rightmost node in the last level) to the root
- 2. If the root breaks the heap property, look at its children and swap it with the larger one
  - Swapping can introduce another conflict
- 3 Repeat 2 until all conflicts are resolved
- Running time = O(log n)

BINARY SEARCH TREE (BST)

- The idea behind is that each node has, at most, two children
- A binary tree with the following property: for each node v,
  - value of v ≥ values in v 's left subtree
  - value of v < values in v 's right subtree



BST

- Supports three operations
  - Insert(x) : inserts a node with value x
  - Delete(x) : deletes a node with value x , if there is any
  - Find(x) : returns the node with value x , if there is any
- Many extensions are possible
  - Count(x) : counts the number of nodes with value less than or equal to x
  - GetNext(x) : returns the smallest node with value ≥ x

BST

- Simple implementation cannot guarantee efficiency
  - In worst case, tree height becomes n (which makes BST useless)
- Guaranteeing O(log n) running time per operation requires balancing of the tree (hard to implement).
  - For example AVL and Red-Black trees (We will skip the details of these balanced trees, but you should review it.).
  - What does balanced mean??
- Use the standard library implementations
  - set, map (C++)
  - TreeSet, TreeMap (Java)

BST

- Simple implementation cannot guarantee efficiency
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- Guaranteeing O(log n) running time per operation requires balancing of the tree (hard to implement).
  - For example AVL and Red-Black trees (We will skip the details of these balanced trees, but you should be review it.).
  - What does balanced mean??=> The heights of the two child subtrees of any node differ by at most one.
- Use the standard library implementations
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  - TreeSet, TreeMap (Java)

Question for you

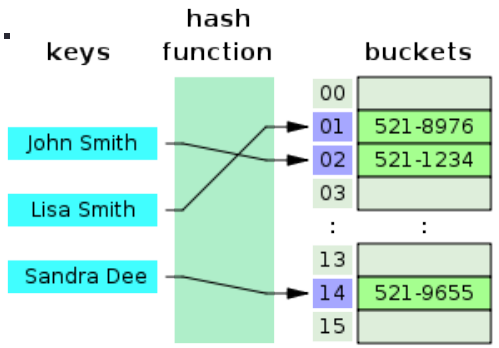
- Why a binary tree is preferable to an array of values that has been sorted?
  - O(?) Finding a given key?

Question for you

- Why a binary tree is preferable to an array of values that has been sorted?
  - O(log n) to find a given key => traversing BST and binary search.
  - Problem is the adding of a new item.

Hash Tables

- A key is used as an index to locate the associated value.
  - Content-based retrieval, unlike position-based retrieval.
  - Hashing is the process of generating a key value.
  - An ideal algorithm must distribute evenly the hash values => the buckets will tend to fill up evenly = fast search.
  - A hash bucket containing more than one value is known as a “collision”.
    - Open addressing => A simple rule to decide where to put a new item when the desired space is already occupied.
    - Chaining => We associate a linked list with each table location.
  - Hash tables are excellent dictionary data structures.



Hash Function

- A function that takes a string and outputs a number
  - A good hash function has few collisions
  - i.e. , If x != y , H(x) != H(y) with high probability
- An easy and powerful hash function is a polynomial mod some prime p.
  - Consider each letter as a number (ASCII value is fine)
  - $H(x_1 \ldots x_k) = x_1 a^{k-1} + x_2 a^{k-2} + \ldots + x_{k-1} a + x_k \pmod{p}$

Data Structures

- Data structures (Own Libraries).
  - Graph.
    - Lets talk about graphs later.
  - Union-Find Disjoint Sets
  - Segment tree.

Union-Find Structure

- Used to store disjoint sets
  - What is a disjoint set?
- Can support two types of operations efficiently
  - Find(x) : returns the “representative” of the set that x belongs
  - Union(x, y) : merges two sets that contain x and y
- Both operations can be done in (essentially) constant time
- Super-short implementation!
- Useful for problems involving partitioning.
  - Ex: keeping track of connected components.
  - Kruskal’s algorithm (minimum spaning tree).

Union-Find Structure

- Used to store disjoint sets
  - What is a disjoint set? => sets whose intersection is the empty set.
- Can support two types of operations efficiently
  - Find(x) : returns the “representative” of the set that x belongs
  - Union(x, y) : merges two sets that contain x and y
- Both operations can be done in (essentially) constant time
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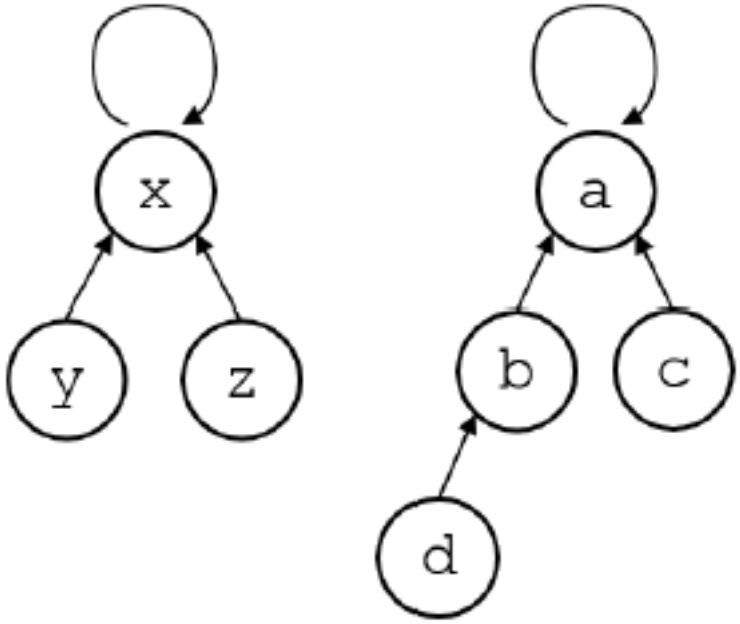
Union-Find Structure

- Main idea: represent each set by a rooted tree
  - Every node maintains a link to its parent
  - A root node is the “representative” of the corresponding set
  - Example: two sets {x, y, z} and {a, b, c, d}



Union-Find Structure

- Find(x): follow the links from x until a node points itself
  - This can take O(n) time but we will make it faster
- Union(x, y): run Find(x) and Find(y) to find corresponding root nodes and direct one to the other.
- If we assume that the links are stored in L[], then

```
int Find(int x) {
    while(x != L[x]) x = L[x];
    return x;
}
void Union(int x, int y) {
    L[Find(x)] = Find(y);
}
```

Union-Find Structure

- In a bad case, the trees can become too deep
  - ... which slows down future operations
- Path compression makes the trees shallower every time Find() is called.
- We don’t care how a tree looks like as long as the root stays the same
  - After Find(x) returns the root, backtrack to x and reroute all the links to the root

Question for you

- How can you implement the operation isSameSet(i,j)?

Question for you

- How can you implement the operation isSameSet(i,j)?
  - simply calls findSet(i) and findSet(j) to check if both refer to the same representative.

Segment Tree

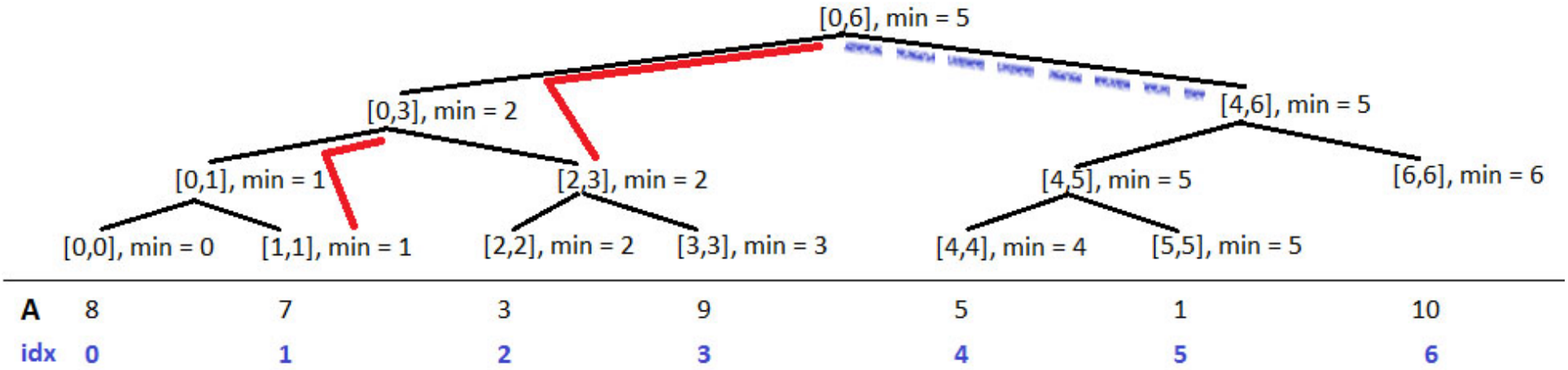
- DS to efficiently answer dynamic range queries.
  - Range Minimum Query (RMQ): finding the index of the minimum element in an array given a range: [i..j].
    - Ex. RMQ(1, 3) = 2, RMQ(3, 4) = 4, RMQ(0, 0) = 0, RMQ(0, 1) = 1, and RMQ(0, 6) = 5.
    - Iterate takes O(n), let make it faster using a binary tree similar to heap, but usually not a complete binary tree (aka segment tree).

```
          Values  =   8   |   7   |   3   |   9   |   5   |   1   |  10
Array A = ---------------------------------------------------------------
          Indices =   0   |   1   |   2   |   3   |   4   |   5   |   6
```

Segment Tree

- Binary tree.
- Each node is associated with some interval of the array.
- Each non-leaf node has two children whose associated intervals are disjoint.
- Each child's interval has approximately half the size of the parent's interval.

Segment Tree



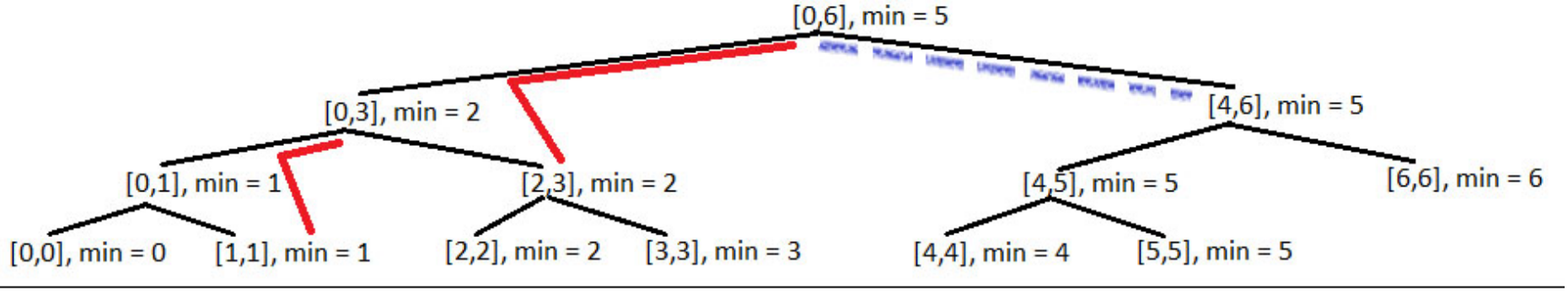
- Root => [0, N – 1] and for each segment [l,r] we split them into [l, (l + r) / 2] and [(l + r) / 2 + 1, r] until l = r.

Question for you

- What is the complexity of built_segment_tree O(?)?
- With segment tree ready, what is the complexity of answering an RMQ?
- Can you give the worst case? RMQ(?,?)

Question for you

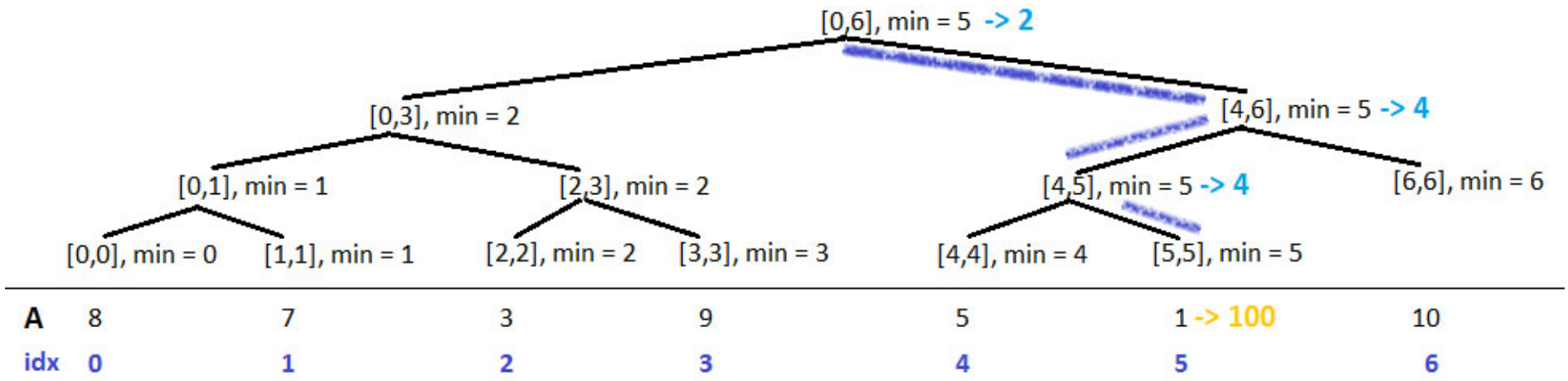
- What is the complexity of built_segment_tree O(n)
  - There are total 2n-1 nodes.
- With segment tree ready, what is the complexity of answering an RMQ => O(log n) (2 root-to-leaf paths)
  - Ex RMQ(4,6) = blue line.
  - Ex RMQ(1,3) = red line.
  - Ex RMQ(3,4) = worst case => one path from [0,6] to [3,3] and another from [0,6] to [4,4].



A	8	7	3	9	5	1	10
idx	0	1	2	3	4	5	6

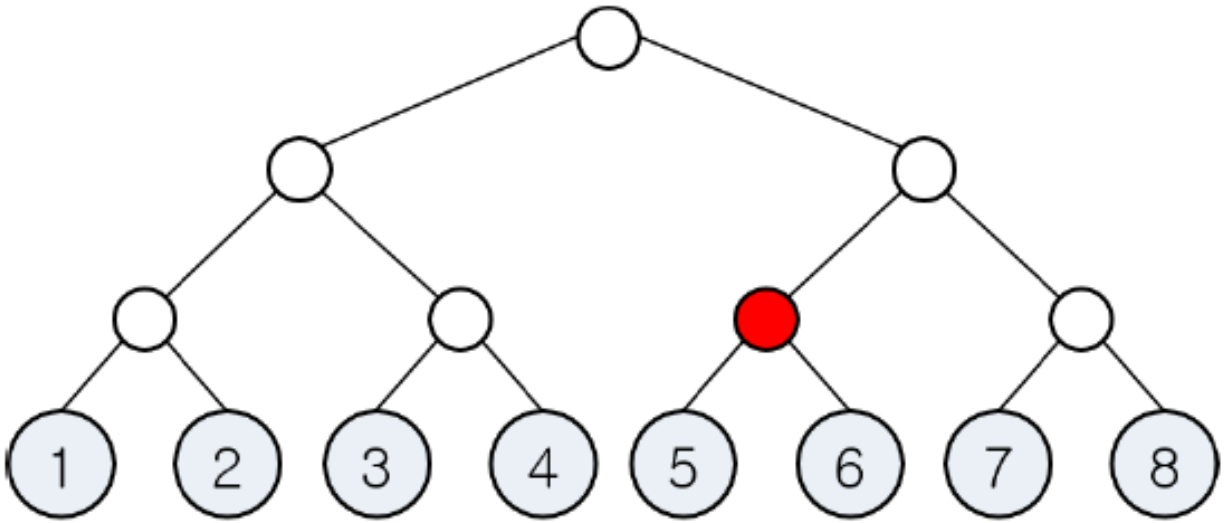
Segment Tree

- If the array A is static, then use a Dynamic Programming solution that requires O(nlogn) pre-processing and O(1) per RMQ.
  - Segment tree becomes useful if array A is frequently updated.
    - Ex. Updating A[5] takes O(logn) vs O(nlogn) required by DP.



Fenwick Tree

- Full binary tree with at least n leaf nodes
  - We will use n = 8 for our example
- kth leaf node stores the value of item k
- Each internal node stores the sum of values of its children
  - e.g. , Red node stores item[5] + item[6]

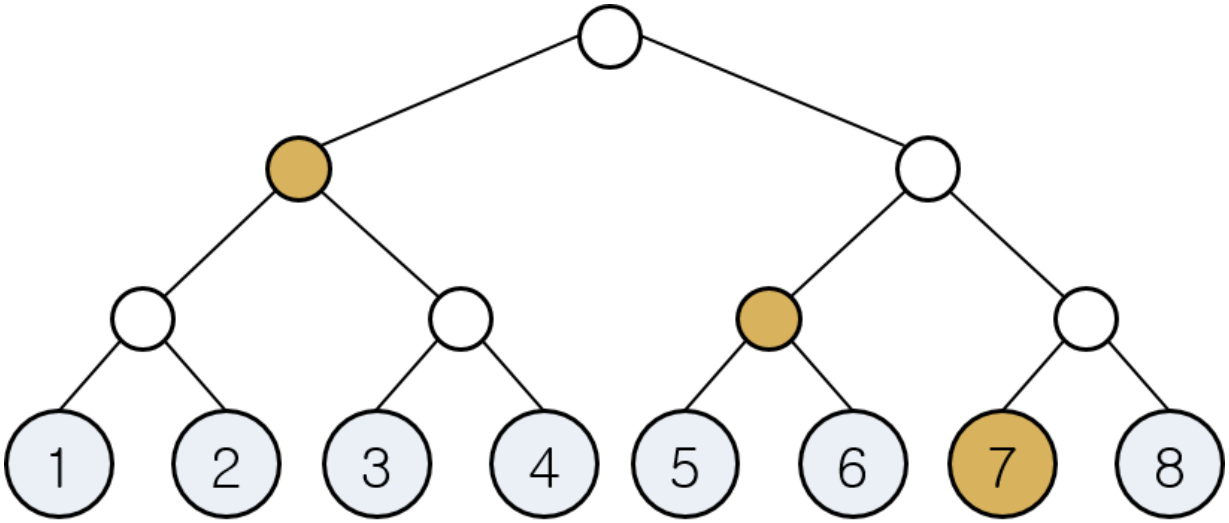


Summing Consecutive Values

- Main idea: choose the minimal set of nodes whose sum gives the desired value
  - at most 1 node is chosen at each level so that the total number of nodes we look at is $\log_2 n$
  - and this can be done in O(log n) time

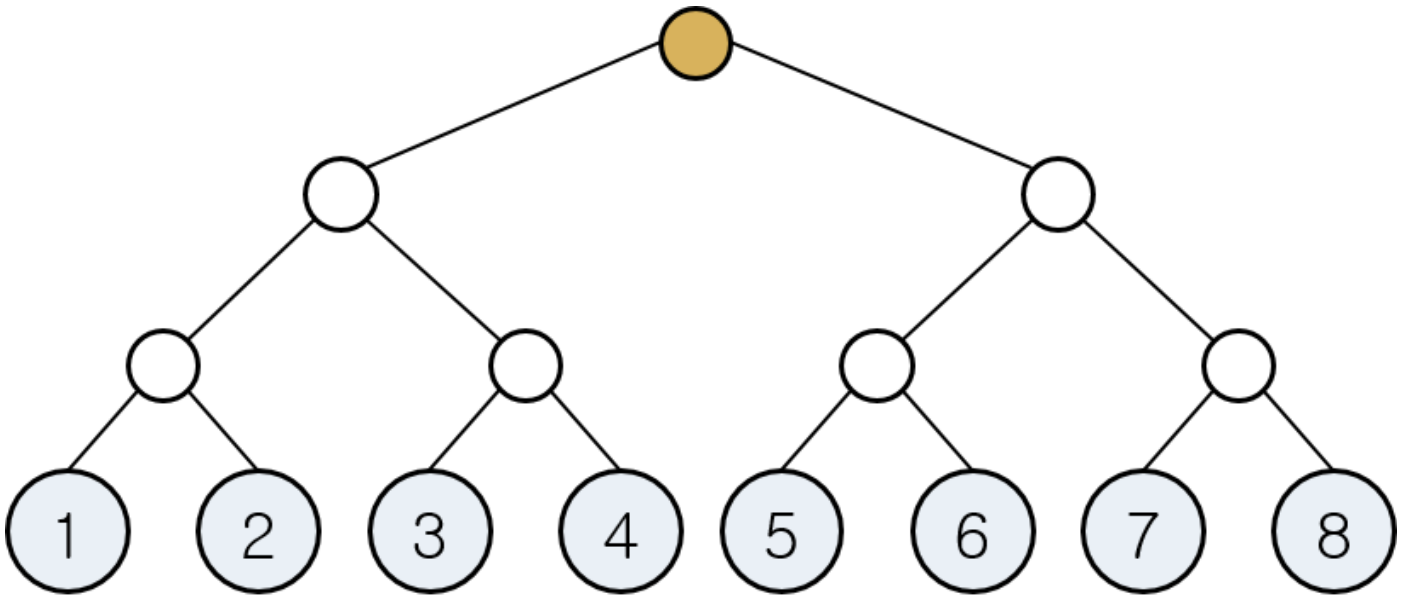
Summing Consecutive Values

- Sum(7) = sum of the values of gold-colored nodes.



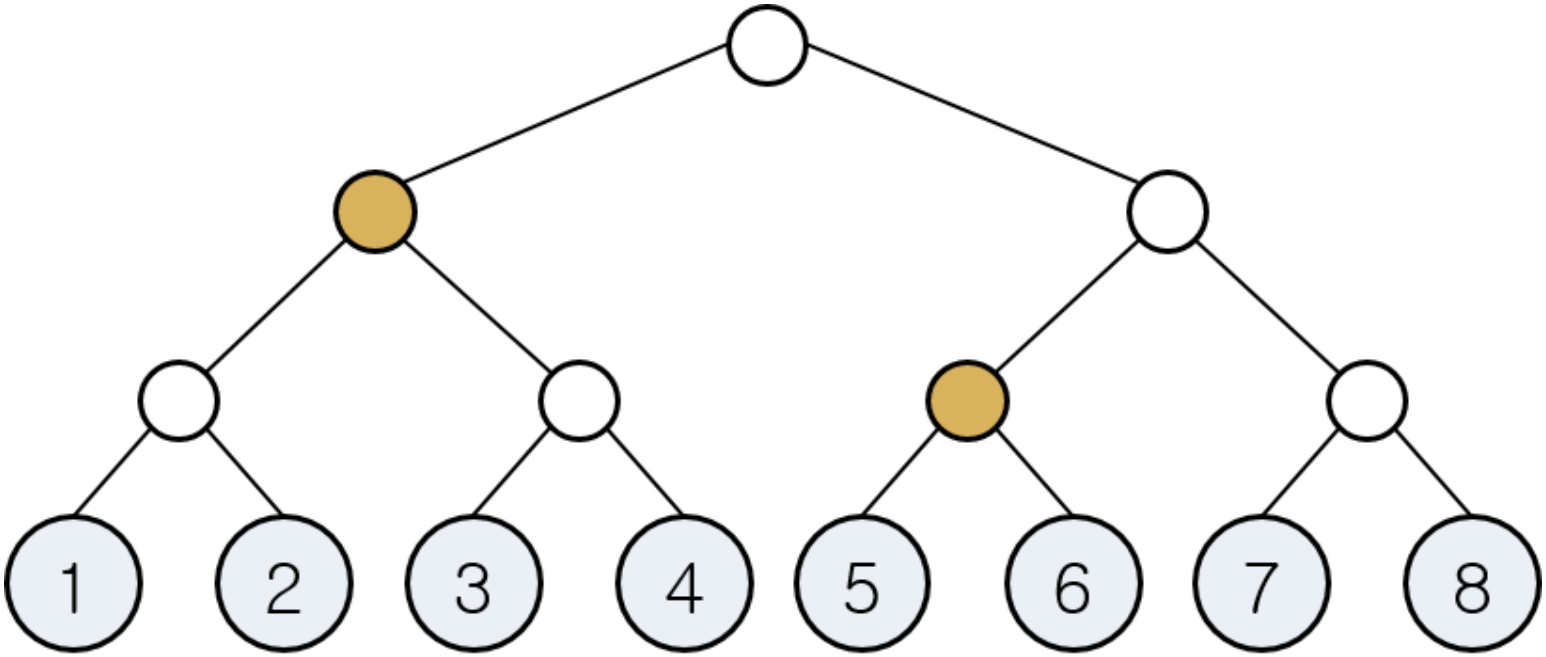
Summing Consecutive Values

- Sum(8) = sum of the values of gold-colored nodes.



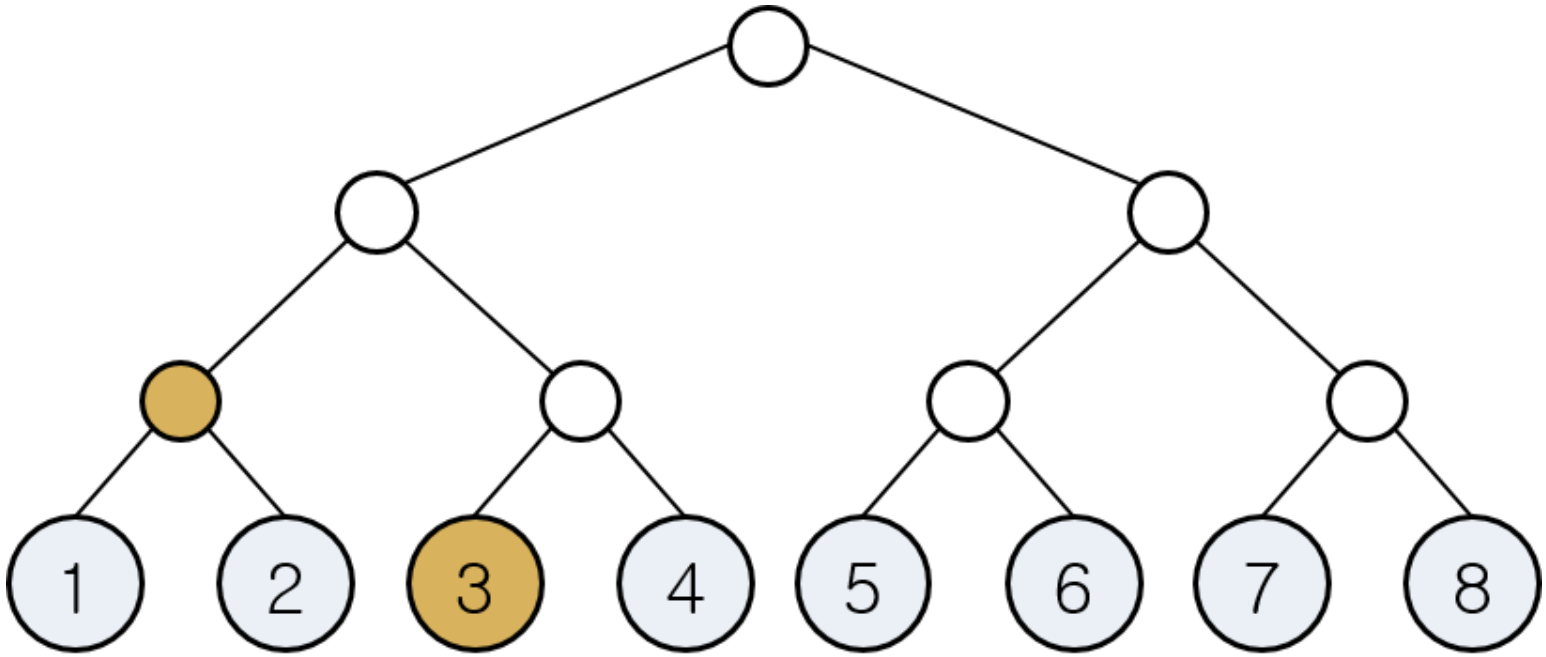
Summing Consecutive Values

- Sum(6) = sum of the values of gold-colored nodes.



Summing Consecutive Values

- Sum(3) = sum of the values of gold-colored nodes.



Summing Consecutive Values

- Say we want to compute Sum(k)
  - Maintain a pointer P which initially points at leaf k
  - Climb the tree using the following procedure:
    - If P is pointing to a left child of some node:
      - Add the value of P
      - Set P to the parent node of P's left neighbor
      - If P has no left neighbor, terminate
    - Otherwise:
      - Set P to the parent node of P
- Use an array to implement

Updating a Value

- Say we want to do Set(k, x) (set the value of leaf k as x)
  - 1. Start at leaf k, change its value to x
  - 2. Go to its parent, and recompute its value
  - 3. Repeat 2 until the root