COMP/MATH 553 Algorithmic Game Theory
Lecture 8: Combinatorial Auctions & Spectrum Auctions

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An overview of today's class

- *Vickrey-Clarke-Groves Mechanism*
- *Combinatorial Auctions*
- *Case Study: Spectrum Auctions*

The VCG Mechanism

[The Vickrey-Clarke-Groves (VCG) Mechanism] In every general mechanism design environment, there is a DSIC mechanism that maximizes the social welfare. In particular the allocation rule is

$$x(b) = \operatorname{argmax}_{\omega} \Sigma_i \, b_i(\omega) \qquad (1);$$

and the payment rule is

$$p_i(b) = \max_{\omega} \Sigma_{j \neq i} \, b_j(\omega) - \Sigma_{j \neq i} \, b_j(\omega^*) \qquad (2),$$

where $\omega^* = \operatorname{argmax}_{\omega} \Sigma_i \, b_i(\omega)$ is the outcome chosen in (1).

Discussion of the VCG mechanism

- *DSIC* mechanism that *optimizes social welfare* for *any* mechanism design problem !

- However, sometimes *impractical*.

- How do you find the allocation that maximizes social welfare. If Ω is really large, what do you do?
  - m items, n bidders, each bidder wants only one item.
  - m items, n bidders, each bidder is single-minded (only like a particular set of items).
  - m items, n bidders, each bidder can take any set of items.

Discussion of the VCG mechanism

- Sometimes Computational intractable.

- If you use approximation alg., the mechanism is no longer DSIC.

- Serves as a useful benchmark for more practical approaches!

Combinatorial Auctions

Combinatorial Auctions

- ***Important*** in practice
  - spetrum auctions
  - allocating take-off and landing slots at airports

- ***Notoriously hard*** in both theory and practice
  - In theory, many impossibility results for what can be done with reasonable communication and computation
  - In practice, badly designed combinatorial auctions with serious consequences

Combinatorial Auctions (set-up)

- ***n*** bidders. For example, Bell, Rogers, Telus and several regional providers.

- There is a set ***M*** of ***m*** non-identical items. For example, a license for broadcasting at a certain frequency in a given region.

- An outcome is a n-dimensional vector ($S_1, S_2, \ldots, S_n$), with S_i denoting the set of items allocated to bidder i (her bundle). All S_i's are ***disjoint***!

- There are $(n+1)^m$ outcomes!!!

Combinatorial Auctions (set-up)

- Each bidder could value every different outcome differently, but we simplify it a bit here.

- i has a private value $v_i(S)$ for each subset S of M. Each bidder needs 2^m numbers to specify her valuation.
  - $v_i(\emptyset) = 0$
  - $v_i(S) \leq v_i(T)$, if S is a subset of T. (free disposal)
  - Could make other assumptions on the valuation function. Usually simplifies the auction design problem. Talk about it later.

- The welfare of an outcome $(S_1, S_2, \ldots, S_n)$ is $\Sigma_i\, v_i(S_i)$.

Challenges of Combinatorial Auctions

- How do you optimize social welfare in combinatorial auctions?
- ***VCG!***
- Unfortunately, several impediments to implementing VCG.
- *Challenge 1 -- Preference elicitation:* Is direct-revelation sealed-bid auction a good idea?
- No! Each bidder has $\boldsymbol{2^m}$ numbers to specify. When $\boldsymbol{m=20}$, means 1 million numbers for every bidder.

Indirect Mechanisms

- Ascending English Auction.
- The one you see in movies!
- Many variants, the Japanese variant is easy to argue about.
- The auction begins at some opening price, which is publicly displayed and increases at a steady rate. Each bidder either chooses "in" or "out," and once a bidder drops out it cannot return. The winner is the last bidder in, and the sale price is the price at which the second-to-last bidder dropped out.
- Each bidder has a dominant strategy: stay till the price is higher than her value.
- Apply revelation principle on this auction, you get Vickrey auction.

Indirect Mechanisms

- We'll discuss the auction formats used in practice for the spectrum auctions.
- Main question: can indirect mechanism achieve non-trivial welfare guarantees?
- A lot of work has been done on this front.
- Short answer: depends on the bidders' valuation functions.
- For simple valuations, "yes"; for complex valuations, "no".

Challenges of Combinatorial Auctions

- Challenge 2: Is welfare maximization tractable?
- Not always. E.g. Maximizing welfare for Single-minded bidders is NP-Hard.
- Doesn't matter what auction format is used.
- This is hard to check in practice either.

Challenges of Combinatorial Auctions

- Challenge 3: Even if we can run VCG, it can have bad revenue and incentive properties, despite being DSIC.

- Example: 2 bidders and 2 items, A and B.
  - Bidder 1 only wants both items: $v_1(AB) = 1$ and is 0 otherwise.
  - Bidder 2 wants only item A: $v_2(AB) = v_2(A) = 1$ and is 0 otherwise.
  - VCG gives both items to 1 and charges him 1.
  - Suppose now there is a third bidder who only wants item B: $v_3(AB) = v_3(B) = 1$ and is 0 otherwise.
  - VCG gives A to 1 and B to 2, but charges them **0**!
  - Can you see a problem?

- Vulnerable to collusion and false-name bidding. Not a problem for Vickrey.

Challenges of Combinatorial Auctions

- Challenge 4: indirect mechanisms are usually iterative, which offers new opportunities for strategic behavior.

- Example: bidders use the low-order digits of their bids to send messages to other bidders.
  - #378 license, spectrum use rights in Rochester, MN
  - US West and Macleod are battling for it.
  - US West retaliate by bidding on many other licenses in which Macleod were the standing high bidder.
  - Macleod won back all these licenses but had to pay a higher price
  - US West set all bids to be multiples of 1000 plus 378!

Spectrum Auctions

Selling Items Separately

- Indirect mechanisms. Have relax both DSIC and welfare maximization.

- Obvious mechanism to try is to sell the items separately, for each, use some single-item auction.

- Main take away is: for ***substitutes*** this works quite well (if the auction is designed carefully), but not for ***complements***.
  - substitutes: $v(AB) \leq v(A) + v(B)$
  - complements: $v(AB) > v(A) + v(B)$

- Welfare maximization is computationally tractable when the items are substitutes and true valuations are known. But it's still intractable for complements.

- In real life the items are a mixture of substitutes and complements. When the problem is "mostly substitutes", then selling items separately could have good performance.

Selling Items Separately

- Rookie mistake 1: Run the single-item auctions sequentially, one at a time.
- Imagine the items are identical and you have k copies.
- DSIC mechanism gives the top k bidders each a copy of the item and charge them the (k+1)-th highest bidder’s bid.
- What if you run it sequentially? Say k=2.
- If you are the highest bidder will you bid truthfully for the first item?
- Everyone will do the same reasoning, in the end the outcome is unpredictable.

Selling Items Separately

- In March 2000, Switzerland auctioned off 3 blocks of spectrum via a sequence of Vickrey auctions.
- The first two were identical items , 28 Mhz blocks, and sold for 121 million and 134 million Swiss francs.
- For the third auction, the item is a larger 56 MHz block. The price was only 55 million.
- This is clearly far from equilibrium.
- Not close to optimal welfare and low revenue as well.
- Lesson learned: holding the single-item auction ***simultaneously***, rather than ***sequentially***.

Selling Items Separately

- Rookie mistake 2: Use sealed-bid single-item auctions.

- Imagine the items are identical and each bidder wants only one of them.

- Two reasonable things to do:
  (1) pick one item and go for it
  (2) bid less aggressively on multiple items and hope toget one with a bargain price and not winning to many extra ones.

- But which one to use? Tradeoff between winning too few and twinning too many.

- The difficulty of bidding and coordinating gives low welfare and revenue sometimes.

- Assume 3 bidders competing for two identical item, and each wants only one.

Selling Items Separately

- In 1990, New Zealand government auctioned off essentially identical licenses for television broadcasting using simultaneous (sealed-bid) Vickrey auctions.
- The revenue in the 1990 New Zealand auction was only $36 million, a paltry fraction of the projected $250 million.
- On one license, the high bid was $100,000 while the second-highest bid (and selling price) was $6! On another, the high bid was $7 million and the second-highest was $5,000.
- The high bids were made public ... Every one can see how much money was left on the table ...
- They later switched to first-price auction, same problem remains, but at least less evident to the public ...

Simultaneous Ascending Auctions (SAAs)

- Over the last 20 years, ***simultaneous ascending auctions*** (SAAs) form the basis of most spectrum auctions.
- Conceptually, it's a bunch of single-item English auctions running in parallel in the same room.
- Each round, each bidder place a new bid on any subset of items that she wants, subject to an *activity rule*.
- Basically the rule says: the number of items you bid on should decrease over time as prices rise.

Simultaneous Ascending Auctions (SAAs)

- Big advantage: *price discovery.*

- This allows bidders to do mid-course correction.

- Think about the three bidders two item case.

- Another advantage: value discovery.

- Finding out valuations might be expensive. Only need to determine the value on a need-to-know basis.