

**COMP/MATH 553 Algorithmic  
Game Theory  
Lecture 8: Combinatorial Auctions  
& Spectrum Auctions**

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## An overview of today's class

- 
- Vickrey-Clarke-Groves Mechanism*
  - Combinatorial Auctions*
  - Case Study: Spectrum Auctions*

# The VCG Mechanism



[The Vickrey-Clarke-Groves (VCG) Mechanism] In every general mechanism design environment, there is a DSIC mechanism that maximizes the social welfare. In particular the allocation rule is

$$x(b) = \operatorname{argmax}_w \sum_i b_i(w) \quad (1);$$

and the payment rule is

$$p_i(b) = \max_w \sum_{j \neq i} b_j(w) - \sum_{j \neq i} b_j(w^*) \quad (2),$$

where  $w^* = \operatorname{argmax}_w \sum_i b_i(w)$  is the outcome chosen in (1).

# Discussion of the VCG mechanism

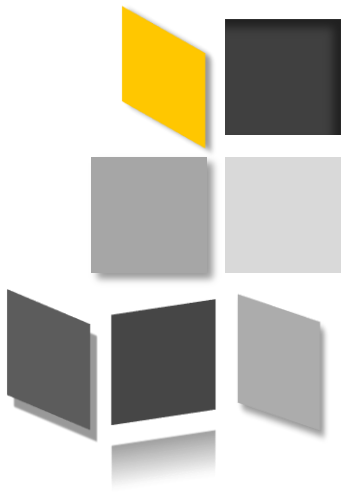


- ❑ *DSIC* mechanism that *optimizes social welfare* for *any* mechanism design problem !
  
- ❑ However, sometimes *impractical*.
  
- ❑ How do you find the allocation that maximizes social welfare. If  $\Omega$  is really large, what do you do?
  - m items, n bidders, each bidder wants only one item.
  - m items, n bidders, each bidder is single-minded (only like a particular set of items).
  - m items, n bidders, each bidder can take any set of items.

# Discussion of the VCG mechanism



- ❑ Sometimes Computational intractable.
- ❑ If you use approximation alg., the mechanism is no longer DSIC.
- ❑ Serves as a useful benchmark for more practical approaches!



# Combinatorial Auctions

# Combinatorial Auctions



## ❑ *Important* in practice

- spectrum auctions
- allocating take-off and landing slots at airports

## ❑ *Notoriously hard* in both theory and practice

- In theory, many impossibility results for what can be done with reasonable communication and computation
- In practice, badly designed combinatorial auctions with serious consequences

# Combinatorial Auctions (set-up)



- ❑  $n$  bidders. For example, Bell, Rogers, Telus and several regional providers.
- ❑ There is a set  $M$  of  $m$  non-identical items. For example, a license for broadcasting at a certain frequency in a given region.
- ❑ An outcome is a  $n$ -dimensional vector  $(S_1, S_2, \dots, S_n)$ , with  $S_i$  denoting the set of items allocated to bidder  $i$  (her bundle). All  $S_i$ 's are *disjoint*!
- ❑ There are  $(n+1)^m$  outcomes!!!



# Combinatorial Auctions (set-up)



- ❑ Each bidder could value every different outcome differently, but we simplify it a bit here.
  
- ❑  $i$  has a private value  $v_i(S)$  for each subset  $S$  of  $M$ . Each bidder needs  $2^m$  numbers to specify her valuation.
  - $v_i(\emptyset) = 0$
  - $v_i(S) \leq v_i(T)$ , if  $S$  is a subset of  $T$ . (free disposal)
  - Could make other assumptions on the valuation function. Usually simplifies the auction design problem. Talk about it later.
  
- ❑ The welfare of an outcome  $(S_1, S_2, \dots, S_n)$  is  $\sum_i v_i(S_i)$ .

# Challenges of Combinatorial Auctions



- ❑ How do you optimize social welfare in combinatorial auctions?
  
- ❑ **VCG!**
  
- ❑ Unfortunately, several impediments to implementing VCG.
  
- ❑ *Challenge 1 -- Preference elicitation:* Is direct-revelation sealed-bid auction a good idea?
  
- ❑ No! Each bidder has  $2^m$  numbers to specify. When  $m=20$ , means 1 million numbers for every bidder.

# Indirect Mechanisms



- ❑ Ascending English Auction.
- ❑ The one you see in movies!
- ❑ Many variants, the Japanese variant is easy to argue about.
- ❑ The auction begins at some opening price, which is publicly displayed and increases at a steady rate. Each bidder either chooses “in” or “out,” and once a bidder drops out it cannot return. The winner is the last bidder in, and the sale price is the price at which the second-to-last bidder dropped out.
- ❑ Each bidder has a **dominant** strategy: stay till the price is higher than her value.
- ❑ Apply **revelation principle** on this auction, you get Vickrey auction.

# Indirect Mechanisms



- ❑ We'll discuss the auction formats used in practice for the spectrum auctions.
- ❑ Main question: can indirect mechanism achieve **non-trivial welfare guarantees**?
- ❑ A lot of work has been done on this front.
- ❑ Short answer: depends on **the bidders' valuation functions**.
- ❑ For simple valuations, “yes”; for complex valuations, “no”.

# Challenges of Combinatorial Auctions



- ❑ Challenge 2: Is welfare maximization tractable?
- ❑ Not always. E.g. Maximizing welfare for Single-minded bidders is NP-Hard.
- ❑ Doesn't matter what auction format is used.
- ❑ This is hard to check in practice either.

# Challenges of Combinatorial Auctions

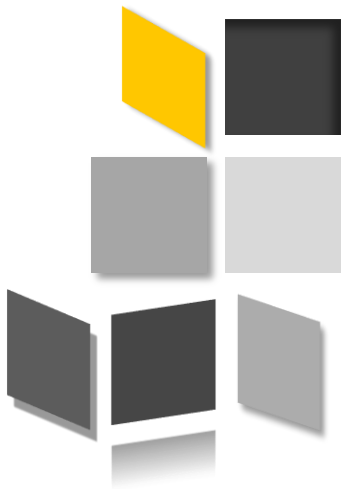


- ❑ Challenge 3: Even if we can run VCG, it can have bad revenue and incentive properties, despite being DSIC.
  
- ❑ Example: 2 bidders and 2 items, A and B.
  - Bidder 1 only wants both items:  $v_1(AB) = 1$  and is 0 otherwise.
  - Bidder 2 wants only item A:  $v_2(AB) = v_2(A) = 1$  and is 0 otherwise.
  - VCG gives both items to 1 and charges him 1.
  - Suppose now there is a third bidder who only wants item B:  $v_3(AB) = v_3(B) = 1$  and is 0 otherwise.
  - VCG gives A to 1 and B to 2, but charges them **0!**
  - Can you see a problem?
  
- ❑ Vulnerable to collusion and false-name bidding. Not a problem for Vickrey.

# Challenges of Combinatorial Auctions



- ❑ Challenge 4: indirect mechanisms are usually iterative, which offers new opportunities for strategic behavior.
  
- ❑ Example: bidders use the low-order digits of their bids to send messages to other bidders.
  - #378 license, spectrum use rights in Rochester, MN
  - US West and Macleod are battling for it.
  - US West retaliate by bidding on many other licenses in which Macleod were the standing high bidder.
  - Macleod won back all these licenses but had to pay a higher price
  - US West set all bids to be multiples of 1000 plus 378!



# Spectrum Auctions



# Selling Items Separately



- ❑ Indirect mechanisms. Have relax both DSIC and welfare maximization.
  
- ❑ Obvious mechanism to try is to sell the items separately, for each, use some single-item auction.
  
- ❑ Main take away is: for *substitutes* this works quite well (if the auction is designed carefully), but not for *complements*.
  - substitutes:  $v(AB) \leq v(A) + v(B)$
  - complements:  $v(AB) > v(A) + v(B)$
  
- ❑ Welfare maximization is computationally *tractable* when the items are substitutes and true valuations are known. But it's still *intractable* for complements.
  
- ❑ In real life the items are a mixture of substitutes and complements. When the problem is “mostly substitutes”, then selling items separately could have good performance.

# Selling Items Separately



- Rookie mistake 1: Run the single-item auctions sequentially, one at a time.
- Imagine the items are identical and you have  $k$  copies.
- DSIC mechanism gives the top  $k$  bidders each a copy of the item and charge them the  $(k+1)$ -th highest bidder's bid.
- What if you run it sequentially? Say  $k=2$ .
- If you are the highest bidder will you bid truthfully for the first item?
- Everyone will do the same reasoning, in the end the outcome is unpredictable.

# Selling Items Separately



- ❑ In March 2000, Switzerland auctioned off 3 blocks of spectrum via a sequence of Vickrey auctions.
- ❑ The first two were identical items , 28 Mhz blocks, and sold for 121 million and 134 million Swiss francs.
- ❑ For the third auction, the item is a larger 56 MHz block. The price was only 55 million.
- ❑ This is clearly far from equilibrium.
- ❑ Not close to optimal welfare and low revenue as well.
- ❑ Lesson learned: holding the single-item auction *simultaneously*, rather than *sequentially*.

# Selling Items Separately



- ❑ Rookie mistake 2: Use sealed-bid single-item auctions.
  
- ❑ Imagine the items are identical and each bidder wants only one of them.
  
- ❑ Two reasonable things to do:
  - (1) pick one item and go for it
  - (2) bid less aggressively on multiple items and hope to get one with a bargain price and not winning too many extra ones.
  
- ❑ But which one to use? Tradeoff between winning too few and winning too many.
  
- ❑ The difficulty of bidding and coordinating gives low welfare and revenue sometimes.
  
- ❑ Assume 3 bidders competing for two identical items, and each wants only one.

# Selling Items Separately



- ❑ In 1990, New Zealand government auctioned off essentially identical licenses for television broadcasting using simultaneous (sealed-bid) Vickrey auctions.
- ❑ The revenue in the 1990 New Zealand auction was only \$36 million, a paltry fraction of the projected \$250 million.
- ❑ On one license, the high bid was \$100,000 while the second-highest bid (and selling price) was \$6! On another, the high bid was \$7 million and the second-highest was \$5,000.
- ❑ The high bids were made public ... Every one can see how much money was left on the table ...
- ❑ They later switched to first-price auction, same problem remains, but at least less evident to the public ...

# Simultaneous Ascending Auctions (SAAs)



- ❑ Over the last 20 years, *simultaneous ascending auctions* (SAAs) form the basis of most spectrum auctions.
- ❑ Conceptually, it's a bunch of single-item English auctions running in parallel in the same room.
- ❑ Each round, each bidder place a new bid on any subset of items that she wants, subject to an *activity rule*.
- ❑ Basically the rule says: the number of items you bid on should decrease over time as prices rise.

# Simultaneous Ascending Auctions (SAAs)



- Big advantage: *price discovery*.
- This allows bidders to do mid-course correction.
- Think about the three bidders two item case.
- Another advantage: value discovery.
- Finding out valuations might be expensive. Only need to determine the value on a need-to-know basis.