COMP/MATH 553: Algorithmic Game Theory

Lecture 1

Fall 2014

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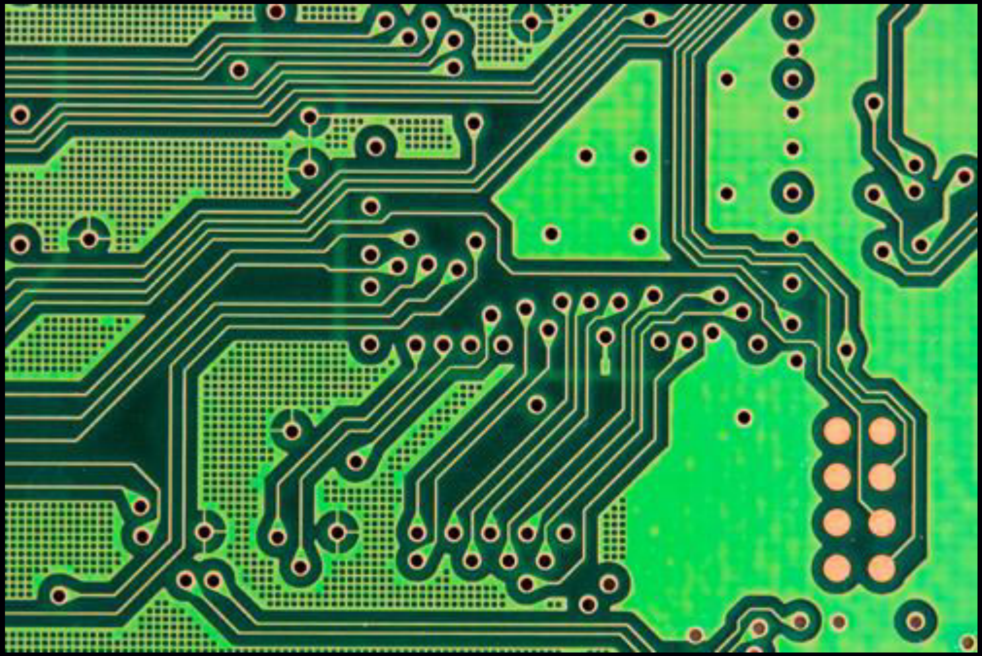
what we won’t study in this class…



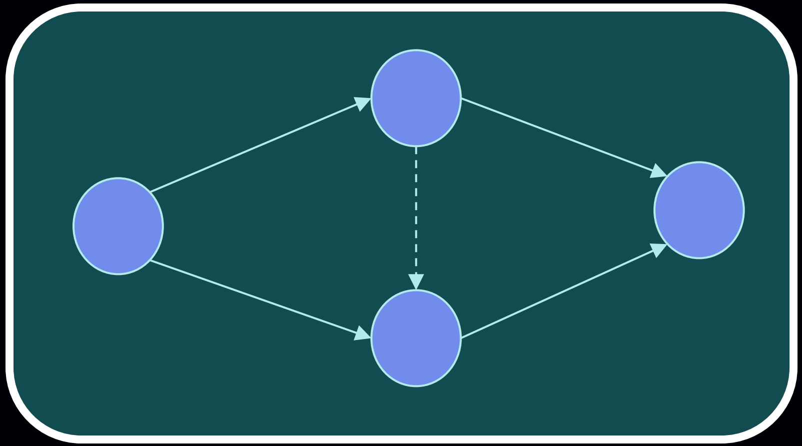
I only mean this as a metaphor of what we usually study in Eng.:

- *central design*
- *cooperative components*
- *rich theory*

what we will study in this class…

Markets

Routing in Networks



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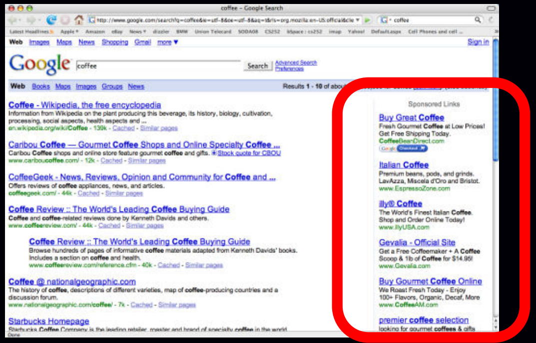
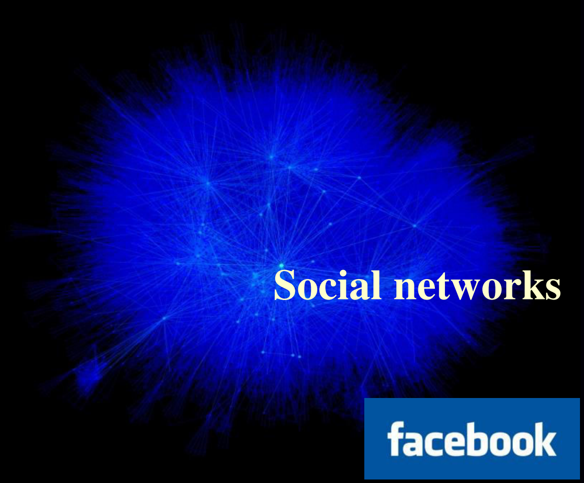
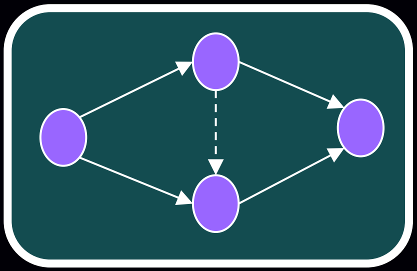


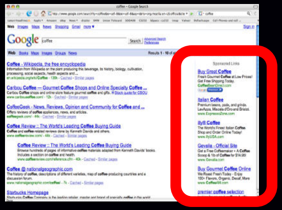
Evolution

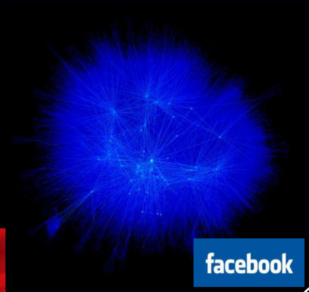


Social networks

Elections





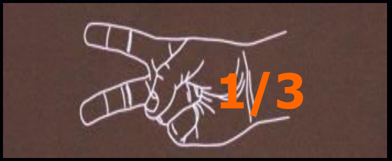
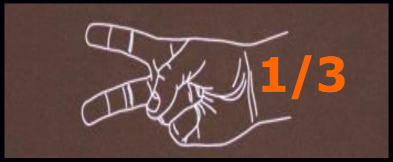


Game Theory

- *central design ?*
- *cooperative components ?*
- *rich theory ?*

we will study (and sometimes question) the algorithmic foundations of this theory

Game Theory

	 1/3	 1/3	 1/3
 1/3	0,0	-1,1	1,-1
 1/3	1,-1	0,0	-1 , 1
 1/3	-1,1	1 , -1	0,0

Game Theory

Games are thought experiments to help us learn how to ***predict rational behavior*** in ***situations of conflict.***

Rational Behavior: a player will always prefer to improve her station regardless of the consequences.

- Encoded via a utility function.
- Players want to maximize their own expected utility.
- Much of the “irrationality” can also be incorporated ...
- Assume other people are as smart or smarter than you ...

Game Theory

Games are thought experiments to help us learn how to ***predict*** ***rational behavior*** in ***situations of conflict.***

Situation of conflict: Everybody's actions affect others.

- In traditional optimization, the goal is optimizing a single objective.
- In game theory, every player has a utility function to optimize.
- The output depends on the player's own choice and the choices of the others.

Game Theory

Games are thought experiments to help us learn how to ***predict*** ***rational behavior*** in ***situations of conflict.***

Predict: We want to know what happens in a game. Such predictions are called solution concepts (e.g., Nash equilibrium).

Algorithmic Game Theory

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

Mechanism Design: How can we design a system that will be launched and used by competitive users to optimize our objectives ?

?

Can we predict what will happen in a large system?

Game theory says yes, through its prediction tools (solution concepts).

Can we ***efficiently*** predict what will happen in a large system?

Are the predictions of Game Theory ***plausible*, *i.e. likely to arise***?

An overview of the class

- *Administration*
- *Solution Concepts*
- *Equilibrium Computation*
- *Price of Anarchy*
- *Mechanism Design*

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Administrivia

TA: Liana Yepremyam (Office Hours: consult webpage)

Attendance/Evaluation:

Everybody is welcome

Requirements, if registered for credit):

- Class attendance and participation (play a few games in class)
- Scribe one lecture
- Solve problem sets: 3-4 problems sets, two weeks to solve.
- Final exam or Project: let me know by mid-October
- Project Survey or Research;

  Encouraged to do original research, apply ideas from this class to your own area of interest, work in groups;

  Project proposal due in mid-October.

If just auditing: - Become a listener. Why not?

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Battle of the Sexes

	Theater!	Football fine
Theater fine	1, 5	0, 0
Football!	0, 0	5, 1

Think of this game as a **metaphor** of real-life examples, not necessarily in the context of a couple's decision making, not necessarily about football vs theater, and not necessarily with exactly these numerical values associated to different outcomes.

Nash Equilibrium: A pair of strategies (deterministic or randomized) such that the strategy of the row player is a *Best Response* to the strategy of the column player and vice versa.

Aims to capture the behavioral norms to which a society where this game occurs may converge.

Battle of the Sexes

		5/6	1/6
		Theater!	Football fine
1/6	Theater fine	1, 5	0, 0
5/6	Football!	0, 0	5, 1

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(Theater fine, Theater!) *Matriarchic society*

(Football!, Football fine) *Patriarchic society*

In medias res: There is always an odd number of Nash Equilibria

Remark:

Matrix-form games intend to model repeated occurrences of the same conflict, provided that there are no strategic correlations between different occurrences of the game. If such correlations exist, we exit the realm of matrix-form games, entering the realm of repeated games.

How can repeated occurrences happen without inter-occurrence correlations?

Imagine a population of blue players (these are the ones preferring football) and orange players (these are those preferring theater). Members of the blue population encounter members of the orange population at random and need to decide whether to watch football or theater.

What do the Nash equilibria represent?

The Nash equilibria predict what types of behaviors and (in the case of randomized strategies) at what proportions will arise in the two populations at the steady state of the game.

Battle of the Sexes

Suppose now that the blue player removes a strategy from his set of strategies and introduces another one:

	Theater!	Football fine
~~Theater fine~~	~~1, 5~~	~~0, 0~~
Football!	0, 0	5, 1
Theater great, I'll invite my mom	2, -1	0, 0

unique Equilibrium

(Football!, Football fine)

Moral of the story: The player who knows game theory managed to eliminate her unwanted Nash equilibrium from the game.

Back to Rock-Paper-Scissors

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

The unique Nash Equilibrium is the pair of uniform strategies.

Contrary to the battle of the sexes, in RPS randomization is necessary to construct a Nash equilibrium.

Rock-Paper-Scissors Championship

The behavior observed in the RPS championship is very different from the pair of uniform strategies; indeed, the matrix-form version of RPS did not intend to capture the repeated interaction between the same pair of players---recall earlier remark; rather the intention is to model the behavior of a population of, say, students in a courtyard participating in random occurrences of RPS games

Guess Two-Thirds of the Average

- k players $p_1, p_2, p_3, \ldots, p_k$
- each player submits a number in [0,100]

$$x_1, x_2, \ldots, x_k$$

- compute

$$\bar{x} := \frac{1}{k}\sum_{i=1}^{k} x_i$$

- find x_j, closest to $\frac{2}{3}\bar{x}$
- player p_j wins \$100, all other players win nothing

Let's Play!

Guess Two-Thirds of the Average

Is it rational to play above $\frac{2}{3} \cdot 100$?

A: no (why?)

Given that no rational player will play above $\frac{2}{3} \cdot 100$ is it rational to play above $(2/3)^2 \cdot 100$?

A: no (same reasons)

$\vdots$

All rational players should play 0.

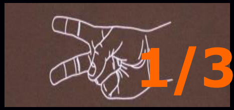
The all-zero strategy is the only Nash equilibrium of this game.

Rationality versus common knowledge of rationality

historical facts: 21.6 was the winning value in a large internet-based competition organized by the Danish newspaper Politiken. This included 19,196 people and with a prize of 5000 Danish kroner.

OK, Nash equilibrium makes sense and is stable, but does it always exist?

2-player Zero-Sum Games

	 1/3	 1/3	 1/3
 1/3	0,0	-1,1	1,-1
 1/3	1,-1	0,0	-1 , 1
 1/3	-1,1	1 , -1	0,0

von Neumann ’28:
In two-player zero-sum games, it always exists.

[original proof used analysis]

Danzig ’47

$\Longleftrightarrow$ **LP duality**

Poker

von Neuman’s predictions are in fact accurate in predicting players’ strategies in two-player poker!

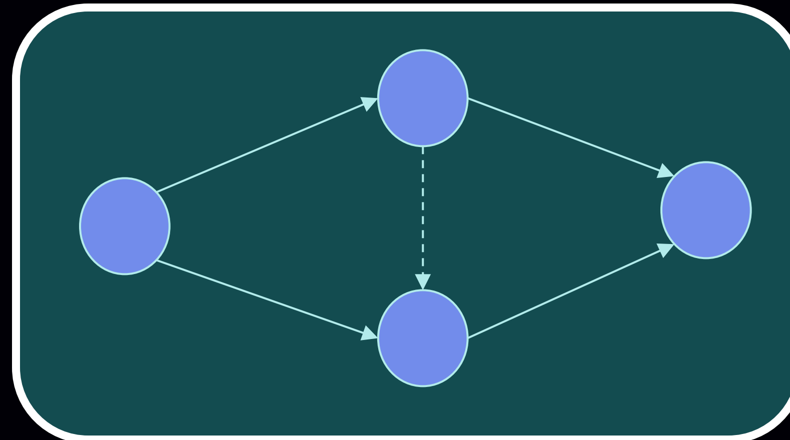
Poker

von Neuman’s predictions are in fact accurate in predicting players’ strategies in two-player poker!

But what about larger systems (more than 2 players) or systems where players do not have directly opposite interests?

Markets

Routing in Networks



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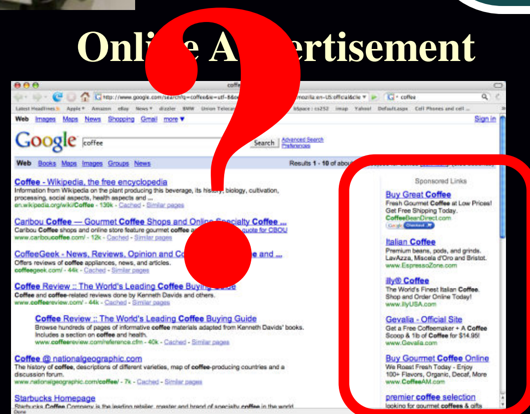
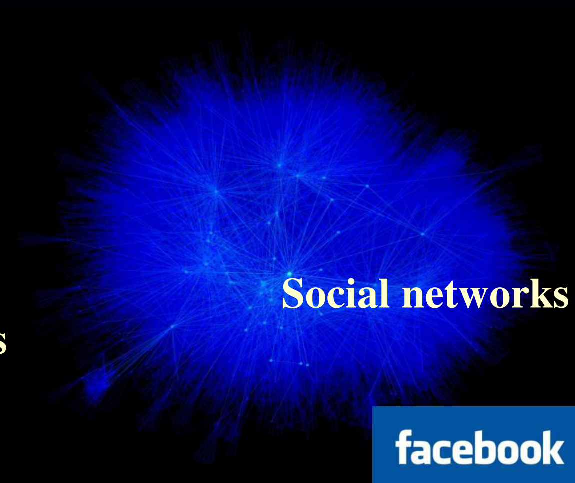


Evolution

Social networks



Elections

Modified Rock Paper Scissors

Not zero-sum any more

	25%	50%	25%
33%	0,0	-1, 1	2,-1
33%	1,-1	0,0	- 1 , 1
33%	- 2, 1	1 , -1	0,0

Is there still an equilibrium?

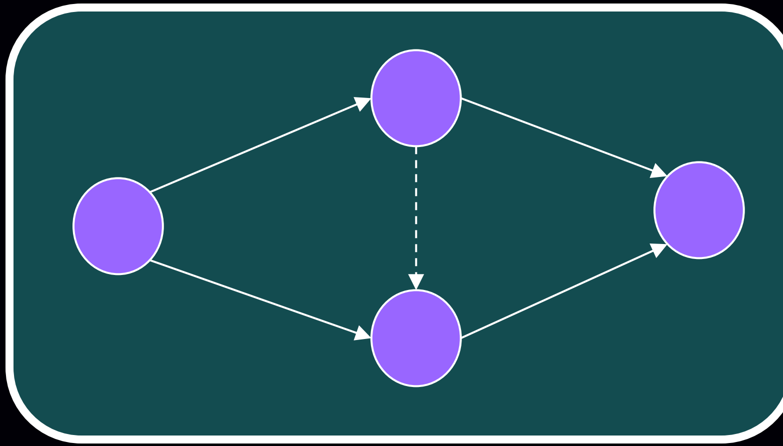
[that is a pair of randomized strategies so that no player has incentive to deviate given the other player's strategy ?]

John Nash '51:
There always exists a Nash equilibrium, regardless of the game's properties.

Nobel 1994, due to its large influence in understanding systems of competitors…

Markets

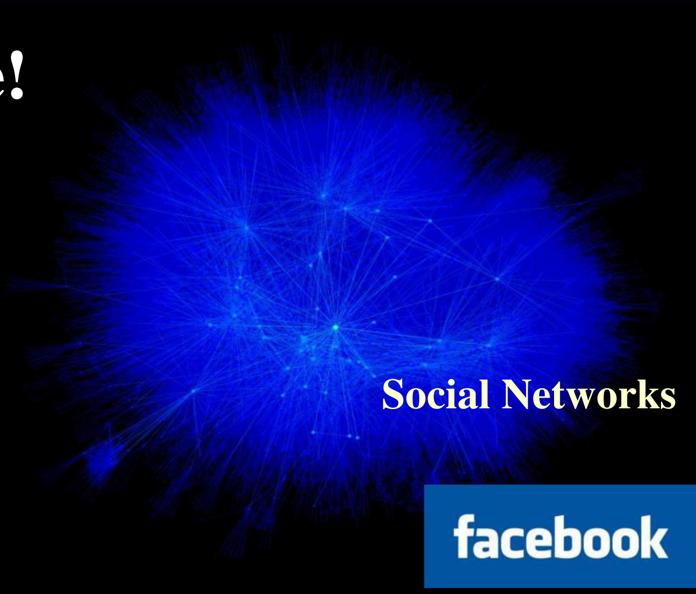
Routing in Networks



and every other game!

Evolutionary Biology

Elections



Social Networks

Applications…

game =

market	→	price equilibrium
Internet	→	packet routing
roads	→	traffic pattern
facebook, twitter, linkedin, …	→	structure of the social network

Modified Rock Paper Scissors

Not zero-sum any more

	25%	50%	25%
33%	0,0	-1, 1	2,-1
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Highly Non-Constructive

Is there still an equilibrium?

[that is a pair of randomized strategies so that no player has incentive to deviate given the other player's strategy ?]

John Nash '51:
There always exists a Nash equilibrium, regardless of the game's properties.

Brouwer's Fixed Point Theorem

Nobel 1994

How can we compute a Nash equilibrium?

- if we had an algorithm for equilibria we could predict what behavior will arise in a system, before the systems is launched

- in this case, we can easily compute the equilibrium, thanks to gravity!

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2-player zero-sum vs General Games

1928 Neumann:

- existence of min-max equilibrium in *2-player*, *zero-sum* games;

- proof uses analysis;

+ Danzig '47: equivalent to LP duality;

+ Khachiyan'79: poly-time solvable;

+ a multitude of distributed algorithms converge to equilibria.

1950 Nash:

- existence of an equilibrium in *multiplayer*, *general-sum* games;

- Proof uses Brouwer's fixed point theorem;

- intense effort for equilibrium computation algorithms:

  Kuhn '61, Mangasarian '64, Lemke-Howson '64, Rosenmüller '71, Wilson '71, Scarf '67, Eaves '72, Laan-Talman '79, etc.

- Lemke-Howson: simplex-like, works with LCP formulation;

no efficient algorithm is known after 50+ years of research.

hence, also no efficient dynamics …

the Pavlovian reaction

"Is it NP-complete to find a Nash equilibrium?"

Why should we care about the complexity of equilibria?

• First, if we believe our equilibrium theory, efficient algorithms would enable us to make predictions:

Herbert Scarf writes…

"[Due to the non-existence of efficient algorithms for computing equilibria], general equilibrium analysis has remained at a level of abstraction and mathematical theoretizing far removed from its ultimate purpose as a method for the evaluation of economic policy."

The Computation of Economic Equilibria, 1973

• More importantly: If equilibria are supposed to model behavior, computational tractability is an important modeling *prerequisite*.

"If your laptop can't find the equilibrium, then how can the market?"

Kamal Jain, EBay

N.B. computational intractability implies the non-existence of efficient dynamics converging to equilibria; how can equilibria be universal, if such dynamics don't exist?

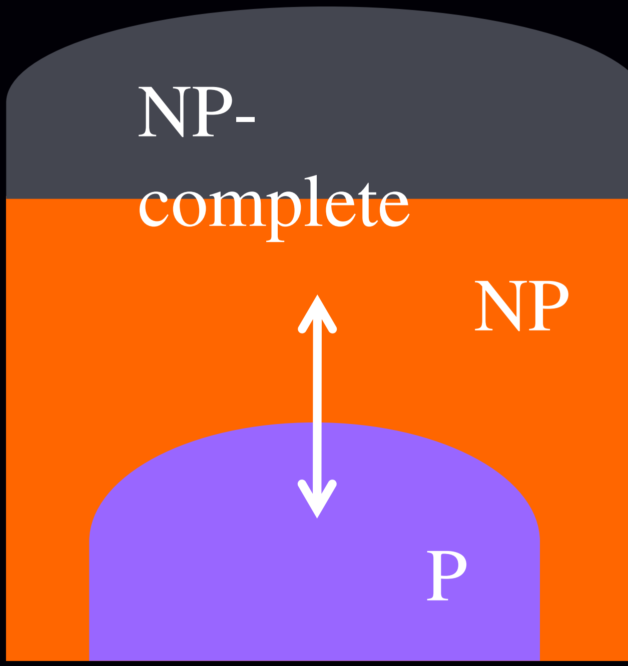
the Pavlovian reaction

"Is it NP-complete to find a Nash equilibrium?"

- the theory of NP-completeness does not seem appropriate;

- in fact, NASH seems to lie below NP-complete;

- Stay tuned! we are going to answer this question later this semester



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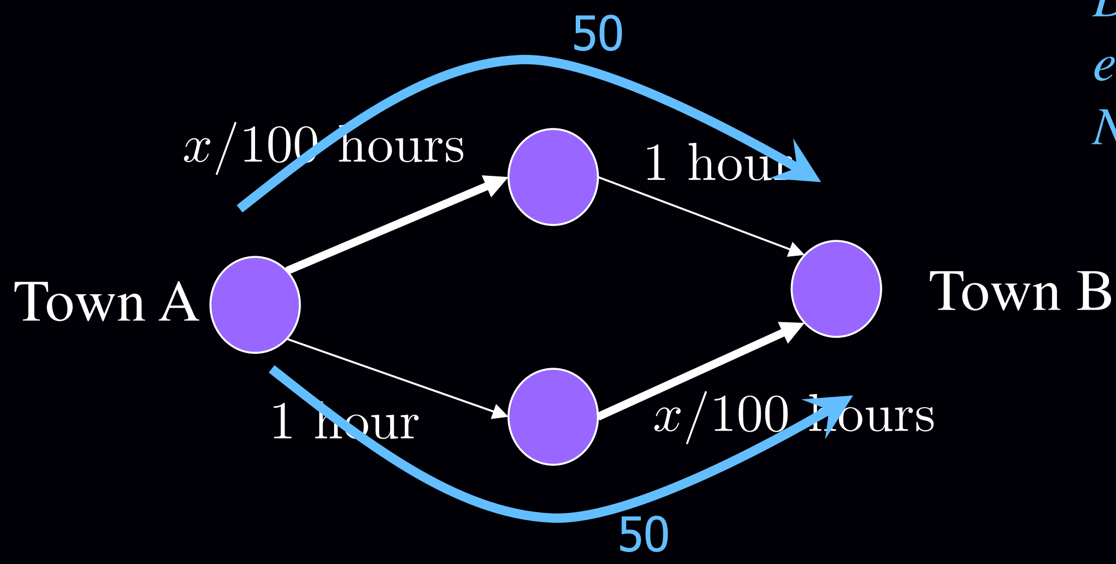
Traffic Routing



Delay is 1.5 hours for everybody at the unique Nash equilibrium

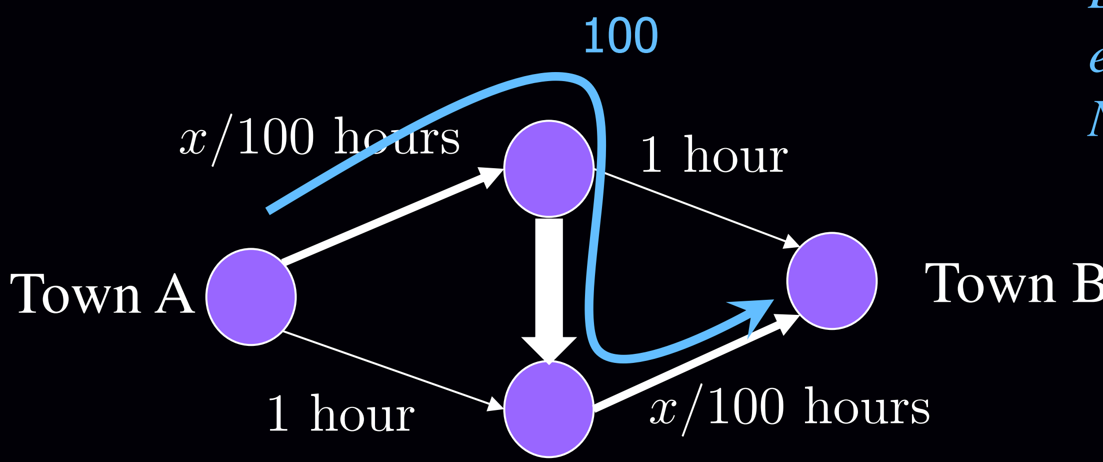
Suppose 100 drivers leave from town A towards town B.

Every driver wants to minimize her own travel time.

What is the traffic on the network?

In any unbalanced traffic pattern, all drivers on the most loaded path have incentive to switch their path.

Traffic Routing



Delay is 2 hours for everybody at the unique Nash equilibrium

A benevolent mayor builds a superhighway connecting the fast highways of the network.

What is now the traffic on the network?

No matter what the other drivers are doing it is always better for me to follow the zig-zag path.

Traffic Routing

Adding a fast road on a road-network is not always a good idea!

Braess's paradox

$4/3$

In the RHS network there exists a traffic pattern where all players have delay 1.5 hours.

$$\text{PoA} = \frac{\text{performance of system in worst Nash equilibrium}}{\text{optimal performance if drivers did not decide on their own}}$$

Price of Anarchy: measures the loss in system performance due to free-will

Traffic Routing

Obvious Questions:

What is the worst-case PoA in a system?

How do we design a system whose PoA is small?

In other words, what incentives can we provide to induce performance that is close to optimal?

E.g. tolls?

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Mechanism Design

- Understand how to design systems with strategic participants that have good performance guarantees

- Let’ start with a poorly designed system...

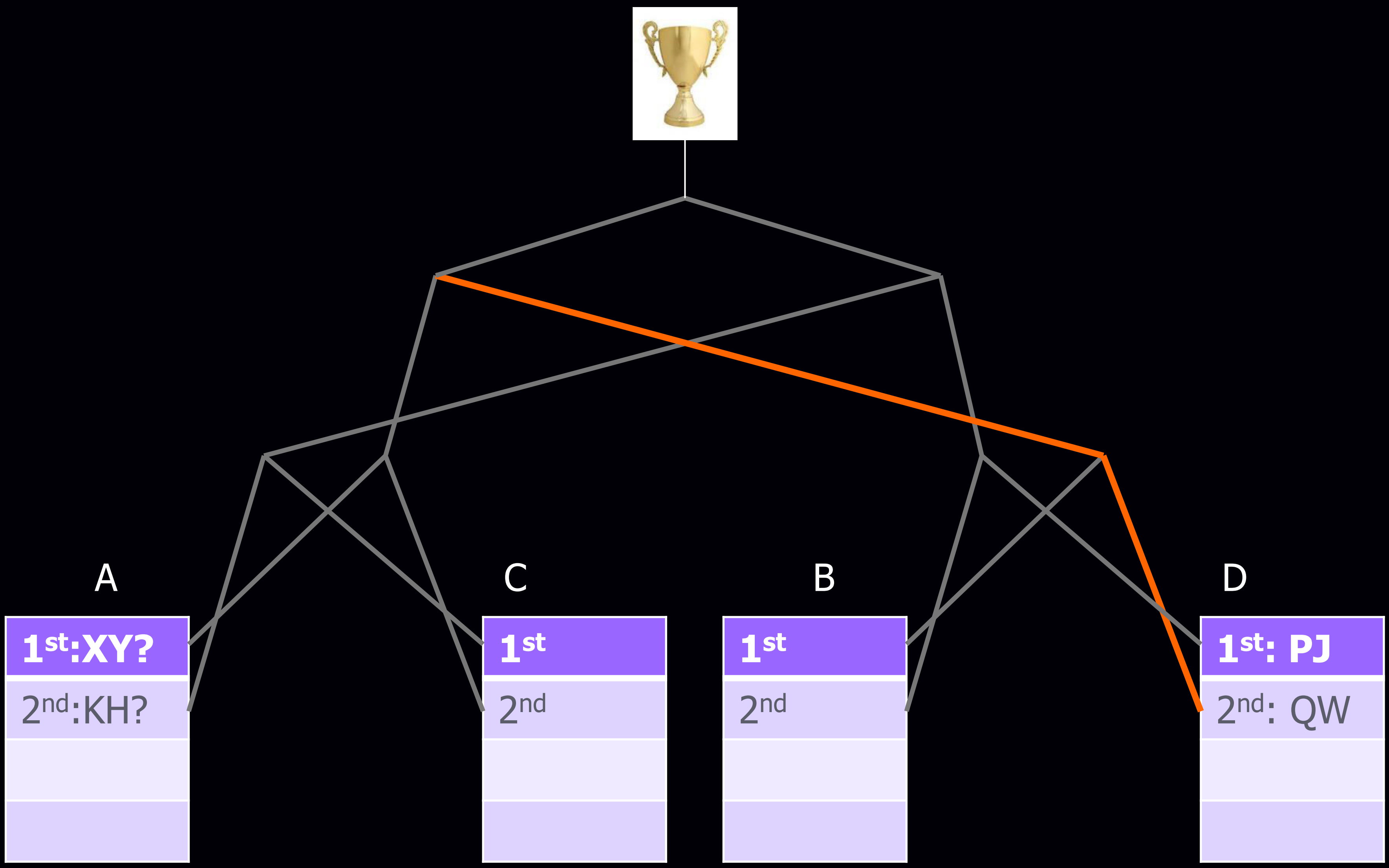
- Biggest scandal in 2012 London Olympics

  - Women’s badminton double

  - No failed drug test

  - Failed tournament design

2012 London Olympics Scandal



2012 London Olympics Scandal

- Four teams are disqualified, due to “not using one’s best effort to win a game”
- Ironically, they are using their best effort to win a Olympic medal
- System rule matters: poorly designed systems suffer from unexpected and undesirable results
- To quote Hartline and Kleinberg:

  “The next time we bemoan people exploiting loopholes to subvert the intent of the rule makers, instead of asking ’What’s wrong with these people?” let’s instead ask, ’What’s wrong with the rules?’ and then adopt a scientifically principled approach to fixing them.”

Auctions

- Suppose we have one item for sale;

- k parties (or *bidders*) are interested in the item;

- party i has value v_i for the item, which is private, and our objective is to give the item to the party with the largest value (alternatively make as much revenue as possible from the sale);

- we ask each party for a bid, and based on their bids $b_1, b_2, \ldots, b_k$ we decide who gets the item and how much they pay;

-if bidder i gets the item and pays price p, her total utility is $v_i - p$ (quasilinear)

Auctions

First Price Auction: Give item to bidder with largest b_i, and charge him b_i

clearly a bad idea to bid above your value (why?)

but you may bid below your value (and you will!)

e.g. two bidders with values $v_1 = \$5$, $v_2 = \$100$

Nash equilibrium = $(b_1, b_2) = (\$5, \$5.01)$ (assume bids are in increments of cents)

Let's Play it!

- bidders want to place different bids, depending on their opponents' bids, which they don't know a priori; hence cycling may occur while they are trying to learn/guess them, etc.

- it is non-obvious how to play

- in the end, the auctioneer does not learn people's true values

Frist Price Auction Game

- Assume your value v_i is sampled from U[0,1].

- You won't overbid, so you will discount your value. Your strategy is a number d_i in [0,1] which specifies how much you want to discount your value, e.g. $b_i = (1-d_i)\, v_i$

- Game 1: What will you do if you are playing with only one student (picked random) from the class?

- Game 2: Will you change your strategy if you are playing with two other students? If yes, what will it be?

In conclusion

- *We are going to study and question the algorithmic foundations of Game Theory*

  - *Mechanism Design*

    auctions

  - *Complexity of finding equilibria*

    NP-completeness theory not relevant, new theory below NP…

  - *Models of strategic behavior*

    dynamics of player interaction:

    e.g. best response, exploration-exploitation,…

  - *Price of Anarchy*

    Network routing.