Assignment 3 – COMP 426: Automated Reasoning

Fall 2007
Due Oct 5 2007

Exercise 1 : Subject reduction proof [**40 pts**]

Consider the rules for natural numbers on page 44 of the Constructive Logic course notes. We localize the use of assumptions by writing $\Gamma \vdash M \in \tau$ where Γ denotes all the assumptions available to prove that indeed M has type τ. The corresponding reduction rules are given in the notes.

$$\frac{}{\Gamma \vdash \mathbf{0} \in \mathbf{nat}}\ \mathbf{nat}I_0 \qquad\qquad \frac{\Gamma \vdash x \in \mathbf{nat}}{\Gamma \vdash \mathbf{s}(\mathbf{x}) \in \mathbf{nat}}\ \mathbf{nat}I_s$$

$$\frac{\Gamma \vdash n \in \mathbf{nat} \qquad \Gamma \vdash t_0 \in \tau \qquad \Gamma, x \in \mathbf{nat} \vdash t_s \in \tau}{\Gamma \vdash \mathbf{case}\ n\ \mathbf{of}\ \mathbf{0} \Rightarrow t_0 \mid \mathbf{s}(x) \Rightarrow t_s \in \tau}\ \mathbf{nat}E^x$$

$$\frac{\Gamma \vdash t \in \mathbf{nat} \qquad \Gamma \vdash t_0 \in \tau \qquad \Gamma, x \in \mathbf{nat}, f(x) \in \tau \vdash t_s \in \tau}{\Gamma \vdash \mathbf{rec}\ t\ \mathbf{of}\ f(\mathbf{0}) \Rightarrow t_0 \mid f(\mathbf{s}(x)) \Rightarrow t_s \in \tau}\ \mathbf{nat}E^{f,x}$$

Prove subject reduction for this tiny extension of the language:
If $\cdot \vdash t \in \tau$ and $t \Longrightarrow t'$ then $\cdot \vdash t' \in \tau$.

Exercise 2 : Reductions [**30 pts**]

Give reductions for the following terms and show their type. Check with Tutch that if indeed the original term M has some type A then the normal form of M (i.e. the term we obtain by reducing M) has the same type A.

Reduction 1 : $(\lambda x.\lambda y.y)\ (\lambda x.x)\ (\lambda x.\lambda y.x)$

Reduction 2: $(\lambda x.x)\ ((\lambda x.\mathsf{inl}\ x)\ ((\lambda x.\lambda y.x)\ (\lambda x.\lambda y.x\ y)\ ()))$

Exercise 3: Primitive recursion [**30 pts**]

Write specifications and implementations for factorial, and exponentiation. Use plus, times, and minus which are given in the notes.