

Algo FindMin(A, n)

Input: Array A of n integers

Output: Return the smallest element in A

```
i ← 1
m ← A[0]
while (i < n)
  if (A[i] < m) then m ← A[i]
  i ← i + 1
return m
```

Loop invariant: $\boxed{\text{At iteration } i, m = \min\{A[0], \dots, A[i-1]\}}$

Goal: Prove the loop invariant holds

① Initialization: Before the start of the loop,
LI holds

$$i = 1, m = A[0] = \min\{A[0], \dots, A[i-1]\}$$

② Maintenance: $\left[\begin{array}{l} \text{Assume L.I. holds at beginning} \\ \text{of } \text{loop}, \text{ an iteration of loop} \\ \text{We must show that L.I. holds} \\ \text{at the end of that iteration} \end{array} \right.$

Assume $m = \min\{A[0], \dots, A[i-1]\}$

If $(A[i] < m)$, then replacing m with $A[i]$
results in $m = \min\{A[0], \dots, A[i]\}$

If $(A[i] \geq m)$, then m remains unchanged,
 m is now $\min\{A[0], \dots, A[i]\}$

After increasing i by one: $\boxed{m = \min\{A[0], \dots, A[i-1]\}}$
L.I.

③ Termination

3.1. Algo will stop, because counter variable i get increased by one at each iteration. So, it will eventually reach n .

3.2 When loop terminates, ~~the~~

$$m = \min \{ A[0], \dots, A[n-1] \}$$

Loop stops when $i = n$

Loop invariant says $m = \min \{ A[0], \dots, A[i-1] \}$
 $\Leftrightarrow m = \min \{ A[0], \dots, A[n-1] \}$