SUMMARY OF CONCENTRATION INEQUALITIES
FOR THE SUM OF K-WISE INDEPENDENT RANDOM VARIABLES

The following was proved by Bellare and Rompel (Randomness-Efficient Oblivious Sampling, FOCS, 1994).

Theorem 1. *Let k be an even integer, and let X be the sum of n k-wise independent random variables taking values in $[0,1]$. Let $\mu = \mathbb{E}[X]$ and $a > 0$. Then we have*

$$\mathbf{Pr}[|X-\mu| > a] < 1.1\left(\frac{nk}{a^2}\right)^{k/2}$$
$$\mathbf{Pr}[|X-\mu| > a] < 8\left(\frac{k\mu + k^2}{a^2}\right)^{k/2}.$$

The following were proved by Schmidt, Siegel, and Srinivasan (Chernoff-Hoeffding Bounds for Applications with Limited Independence, SIAM Journal on Discrete Mathematics, 1995).

Lemma 2. *Let X be the sum of n k-wise independent binary random variables and let $\mu = \mathbb{E}[X]$. For any $a > 0$,*

$$\mathbf{Pr}[X > a] < \frac{\mu^k \binom{n}{k}}{n^k \binom{a}{k}}.$$

Theorem 3. *If X is the sum of k-wise independent random variables taking values in $[0,1]$, and $\mu = \mathbb{E}[X]$, then*

$$\mathbf{Pr}(|X-\mu| > \epsilon\mu) < \exp(-\lfloor k/2 \rfloor) \qquad \forall \epsilon \le 1, k \le \lfloor \epsilon^2 \mu e^{-1/3} \rfloor$$
$$\mathbf{Pr}(|X-\mu| > \epsilon\mu) < \exp(-\lfloor \epsilon^2 \mu/3 \rfloor) \qquad \forall \epsilon \le 1, k \ge \lfloor \epsilon^2 \mu e^{-1/3} \rfloor$$
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$$\mathbf{Pr}(|X-\mu| > \epsilon\mu) < \exp(-\epsilon \ln(1+\epsilon)\mu/2) < \exp(-\epsilon\mu/3) \qquad \forall \epsilon \ge 1, k \ge \lceil \epsilon\mu \rceil$$