# Managing the currency risk of a futures portfolio

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# Outline

# Introduction

Problem: Global investors often need to post collateral in multiple currencies, while their performance is measured in one currency, creating exchange rate risk.

Two competing incentives:

- Keep posted collateral *low* to minimize exchange rate risk
- Keep posted collateral *high* to minimize margin calls

What collateral levels in each different currencies optimally balance these two opposing forces?

# Introduction

Optimal collateral

# Introduction

Similar problems:

- Equity portfolio hedging -
  Minimizing currency risk while minimizing insurance costs
- Transaction costs -
  Maximizing risk/return while minimizing transaction costs
- Inventory management -
  Maximizing sales while minimizing shipping costs
- Staff dispatch -
  Minimizing travel time while minimizing expenses

# Introduction

A good solution needs to properly forecast the underlying prices and exchange rates, accounting for the higher moments and comoments of their respective time-series.

What we did:

- Select a few candidate models for the dynamics of the underlyings' prices and exchange rates
- Assess and compare their goodness-of-fit
- Optimize the "portfolio" of posted collateral based on the chosen model

# Outline

# Model

We chose a copula-based multivariate GARCH framework as advocated in Xiaohong and Yanqin [2006], Patton [2006] and Rémillard [2010]:

$$X_{i,t} = \mu_t(\boldsymbol{\theta}_i) + h_t(\boldsymbol{\theta}_i)^{1/2}\ \epsilon_{i,t} \tag{1}$$

where $i = 1, \dots, D$ and innovations $\epsilon_{1,t}, \dots, \epsilon_{D,t}$ are *i.i.d.* with continous multivariate distribution function

$$K(x_1, \dots, x_D) = C_{\boldsymbol{\theta}}(F_1(x_1), \dots, F_D(x_D)) \tag{2}$$

where the $F_i$ are the cumulative distribution functions of the marginal distributions $X_i$ and $C_{\boldsymbol{\theta}}$ is the copula function with parameter(s) $\boldsymbol{\theta}$.

# Model

Two steps:

(i) Find appropriate univariate process for each random variable (e.g. AR(1)-GARCH(1,1), eGARCH, GJR-GARCH, etc) for

$$X_{i,t} = \mu_t(\boldsymbol{\theta}_i) + h_t(\boldsymbol{\theta}_i)^{1/2} \, \epsilon_{i,t}$$

(ii) Find appropriate copula to capture the dependence between the standardized residuals (e.g. Gaussian, Student, Clayton, Frank, Gumbel, etc) for

$$C_{\boldsymbol{\theta}}(F_1(x_1), \ldots, F_D(x_D))$$

# Outline

# Goodness-of-fit

How do you choose between the different models and once a model is chosen, how do you know it is statistically correct?

$\Rightarrow$ parametric bootstrapping

# Goodness-of-fit

$H_0$: Dataset belongs to said distribution
$H_1$: Dataset does not belong to said distribution

# Parametric bootstrapping

General procedure:

(i) Estimate the parameters of the chosen parametric distribution that best fit the dataset

(ii) Calculate a distance $S_T$ between the empirical distribution and the parametric distribution (good candidate: Cramèr-von Mises statistic)

(iii) Generate a large number N of "bootstrapped" samples of the same size as the dataset from the parametric distribution

(iv) For each of these bootstrapped samples $k = 1, \ldots, N$,

   (a) Estimate the parameters of the chosen parametric distribution that best fit the bootstrapped sample

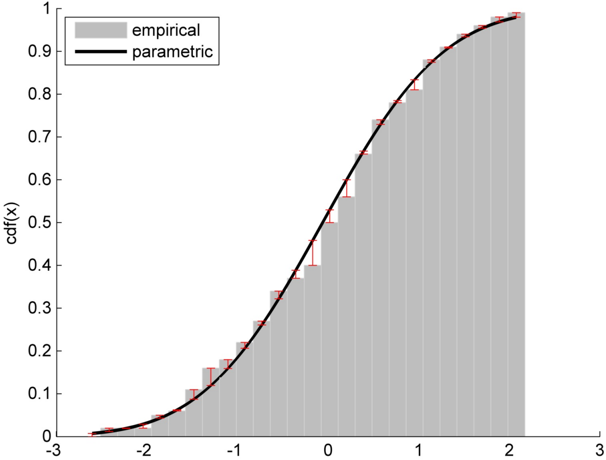
   (b) Calculate a distance $S_T^{(k)}$ between their empirical distribution and the parametric distribution

(v) The $p$-value for the test is given by the fraction of the $S_T^{(k)}$ bigger than $S_T$

# Cramèr-von Mises statistic

For univariate distributions, the Cramèr-von Mises statistic is given by

$$S_T = \sum_{t=1}^{T} \frac{1}{T} (F_T(x_t) - F_\theta(x_t))^2$$



# Cramèr-von Mises statistic

For copulas, the Cramèr-von Mises statistic is given by

$$S_T = \sum_{t=1}^{T} \frac{1}{T}\left(C_T(\hat{u}_{1,t}, \ldots, \hat{u}_{D,t}) - C_\theta(\hat{u}_{1,t}, \ldots, \hat{u}_{D,t})\right)^2$$

where $\hat{u}_{1,t}, \ldots, \hat{u}_{D,t}$ are the normalized ranks

$$\hat{u}_{i,t} = \frac{1}{T-1} \sum_{k=1}^{T} \mathbb{1}(x_{i,t} \geq x_{i,k}),$$

$C_T$ is the empirical copula

$$C_T(u_{1,t}, \ldots, u_{D,t}) = \frac{1}{T-1} \sum_{k=1}^{T} \mathbb{1}(\hat{u}_{1,t} \geq u_{i,k}, \ldots, \hat{u}_{D,t} \geq u_{D,k})$$

and $C_\theta$ is the parametric copula chosen.

# Rosenblatt transform

Unfortunately, $C_\theta$ do not often have a closed form and numerical approximations are computationally impractical when the number of dimensions gets high. Fortunately an alternative is proposed in Genest et al. [2009] using Rosenblatt's transform:

$$\mathbf{U} \sim C \;\Leftrightarrow\; \mathcal{T}(\mathbf{U}) \sim C_\perp$$

# Rosenblatt transform

$\mathcal{T}(u_1, \ldots, u_D) = (e_1, \ldots, e_D)$ given by $e_1 = u_1$ and

$$e_i = \frac{\frac{\delta^{i-1}}{\delta u_1 \ldots \delta u_{i-1}} C(u_1, \ldots, u_i, 1, \ldots, 1)}{\frac{\delta^{i-1}}{\delta u_1 \ldots \delta u_{i-1}} C(u_1, \ldots, u_{i-1}, 1, \ldots, 1)} \tag{3}$$

[Rosenblatt, 1952].

The recipe to compute the Rosenblatt transform for both meta-elliptical and archimedean copulas can be found in Rémillard et al. [2011].

# Parametric bootstrapping - Copula-based Multivariate GARCH model

(i) Estimate the parameters of each univariate marginal process

(ii) Estimate the parameter(s) of the chosen copula on the standardized residuals $\epsilon_t$ obtained in step (i)

(iii) Compute the normalized ranks $\mathbf{u}_t = u_{1,t}, \ldots, u_{D,t}$:

$$u_{i,t} = \frac{1}{T-1} \sum_{k=1}^{T} \mathbb{1}(\epsilon_{i,t} \geq \epsilon_{i,k})$$

# Parametric bootstrapping - Copula-based Multivariate GARCH model

(iv) Compute Rosenblatt transforms $\mathbf{e}_t = e_{1,t}, \ldots, e_{D,t}$, $t = 1, \ldots, T$ using equation (3)

(v) Compute Cramér-von Mises statistic

$$S_T = T \int_{[0,1]^D} \{F_T(\mathbf{u}) - C_\perp(\mathbf{u})\}^2 d\mathbf{u}$$

$$= \frac{T}{3^D} - \frac{1}{2^{D-1}} \sum_{t=1}^{T} \prod_{i=1}^{D} \left(1 - e_{i,t}^2\right) + \frac{1}{T} \sum_{t=1}^{T} \sum_{k=1}^{T} \prod_{i=1}^{D} (1 - \max(e_{i,t}, e_{i,k}))$$

# Parametric bootstrapping - Copula-based Multivariate GARCH model

(vi) For some large integer $N$, repeat the following steps for each $k$ in $(1, \ldots, N)$:
   (a) Generate random trajectories of the processes with parameters found in (i) and (ii) of the same length as the original dataset
   (b) Repeat steps (i) to (v) on trajectories generated in (a) to obtain $S_T^{(k)}$.

(vii) The approximate $p$-value for the test is given by

$$p = \frac{1}{N} \sum_{k=1}^{N} \mathbb{1}(S_T^{(k)} > S_T)$$

# Outline

# Collateral optimization

The optimization objective:

Minimize the exchange rate risk on the posted collateral (as measured by the tracking error, Value-at-Risk or Tail Conditional Expectation) subject to a given tolerance on the probability of a margin call

# Collateral optimization

In mathematical terms:

$$\min_{\lambda_{1,t},\ldots,\lambda_{D,t}} R^{\alpha}\left(\sum_{i=1}^{D}\left(\lambda_{i,t}-\lambda_{i,t}^{*}\right)\times Y_{i,t+1}\right) \tag{4}$$

subject to

$$\lambda_{i,t}^{*} \leq \lambda_{i,t} \leq \infty,\;\; i=1,\ldots,D$$

and

$$\mathbb{P}\left(\left(\prod_{i=1}^{D}\mathbb{1}\left(\lambda_{i,t}+\mathrm{PnL}_{i,t+1}\geq\lambda_{i,t}^{*}\right)\right)=0\right)\leq P_{\mathrm{tol}} \tag{5}$$

# Collateral optimization

where

$$R^{\alpha}(X) = \begin{cases} \mathbb{E}\left[|X|\right] & \text{for expected tracking error} \\ -x^{(\alpha)}(X) & \text{for Value-at-Risk} \\ \mathbb{E}[X|X \leq x^{(\alpha)}] & \text{for Tail Conditional Expectation} \end{cases} \tag{6}$$

and

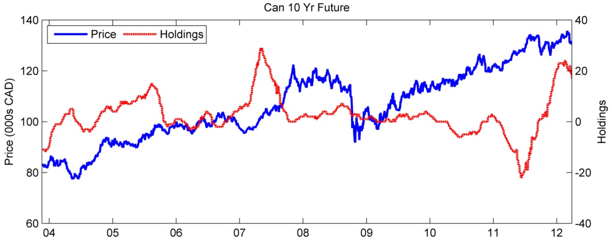
$$\mathrm{PnL}_{i,t+1} = \sum_{j=1}^{n_{i,t}} {}_i\omega_{j,t} \times {}_iW_{j,t+1},$$

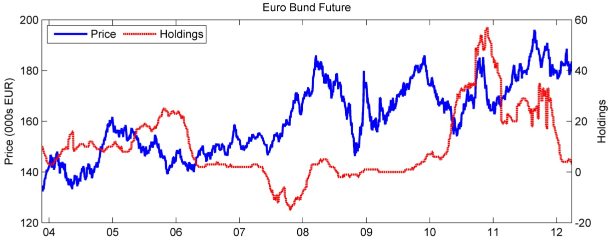
# Outline

# Dataset

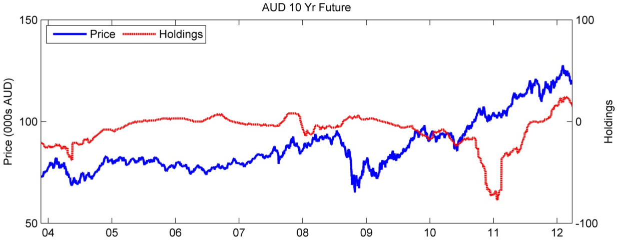
The data set consists of daily holdings of five futures contracts denominated in four non-USD currencies from november 2003 to march 2012 for a total of 1941 observations.

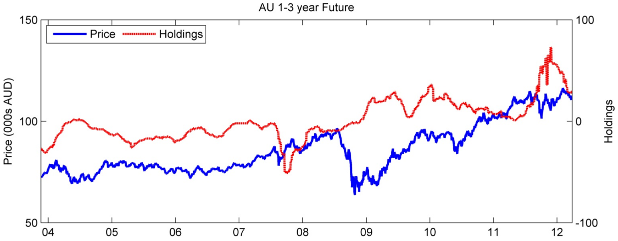
# Dataset


Can 10 Yr Future
Price
Holdings
Price (000s CAD)
Holdings
140
120
100
80
60
40
20
0
-20
-40
04
05
06
07
08
09
10
11
12

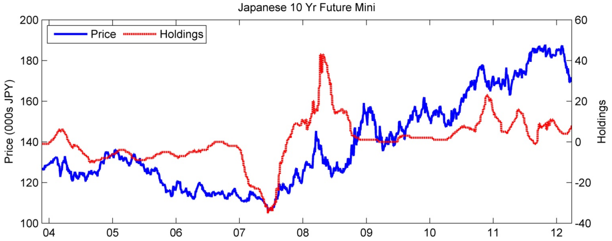

Euro Bund Future
Price
Holdings
Price (000s EUR)
Holdings
200
180
160
140
120
60
40
20
0
-20
04
05
06
07
08
09
10
11
12

# Dataset


AUD 10 Yr Future
Price
Holdings
150
100
50
100
0
-100
Price (000s AUD)
Holdings
04
05
06
07
08
09
10
11
12

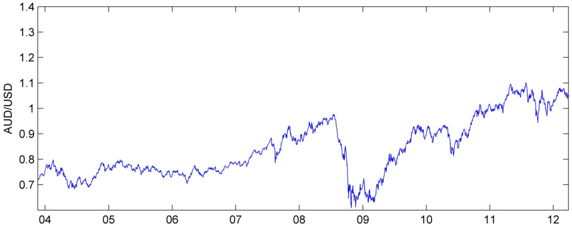

AU 1-3 year Future
Price
Holdings
150
100
50
100
0
-100
Price (000s AUD)
Holdings
04
05
06
07
08
09
10
11
12

# Dataset

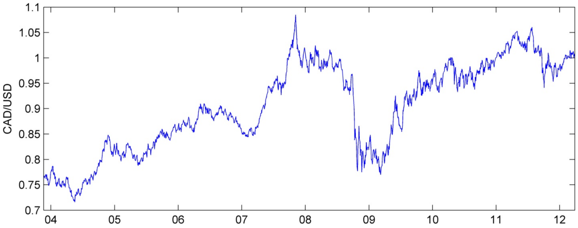


Japanese 10 Yr Future Mini
Price
Holdings
Price (000s JPY)
Holdings
200
180
160
140
120
100
60
40
20
0
-20
-40
04
05
06
07
08
09
10
11
12

# Dataset

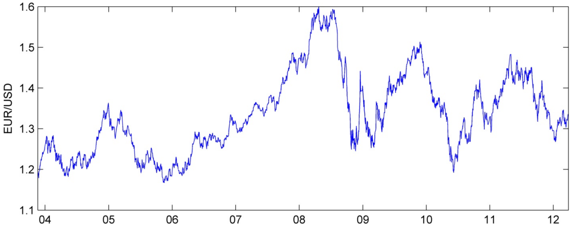


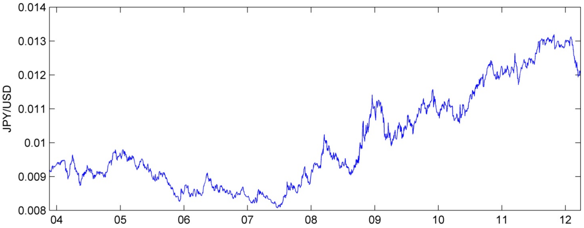
AUD/USD
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
04
05
06
07
08
09
10
11
12

CAD/USD
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
04
05
06
07
08
09
10
11
12

# Dataset


1.6
1.5
1.4
1.3
1.2
1.1
EUR/USD
04
05
06
07
08
09
10
11
12


0.014
0.013
0.012
0.011
0.01
0.009
0.008
JPY/USD
04
05
06
07
08
09
10
11
12

# Dataset

| | Japanese 10 Yr Future Mini | Can 10 Yr Future | Euro Bund Future | AUD 10 Yr Future | AU 1-3 year Future | AUD-USD | CAD-USD | EUR-USD | JPY-USD |
|---|---|---|---|---|---|---|---|---|---|
| **AR(1)-GARCH(1,1), gaussian innovations** | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **AR(2)-GARCH(2,2), gaussian innovations** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| **AR(1)-GARCH(1,1), student innovations** | 0.56 | 0.69 | 0.37 | 0.58 | 0.50 | 0.45 | 0.60 | 0.54 | 0.52 |

Table : $p$-values from the goodness-of-fit tests on marginal processes

# Dataset

| | p values |
|---|---|
| **MV Gaussian** | 0.00 |
| **AR(1)-GARCH(1,1) & gaussian copula** | 0.01 |
| **AR(1)-GARCH(1,1) & student copula** | 0.12 |
| **AR(1)-GARCH(1,1) & Clayton copula** | 0.00 |
| **AR(1)-GARCH(1,1) & Frank copula** | 0.00 |
| **AR(1)-GARCH(1,1) & Gumbel copula** | 0.00 |

Table : $p$-values from the goodness-of-fit tests on copula-based MV GARCH models

# Backtesting

- Two alternative strategies:
  (i) Naive: Always post as collateral 2x the minimum margins requirements
  (ii) Model the nine time series with a multivariate Gaussian
- 500 days buffer left at beginning of sample for calibration
- Daily recalibration
- GOF tests run every year
- $P_{tol} = 0.05$, $\alpha = 0.05$

# Backtesting

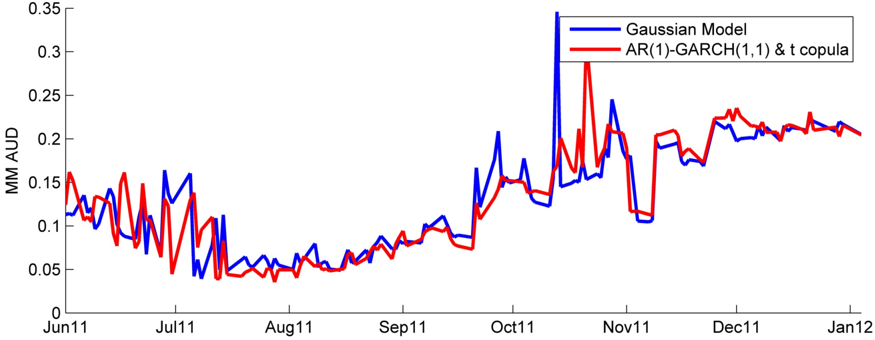
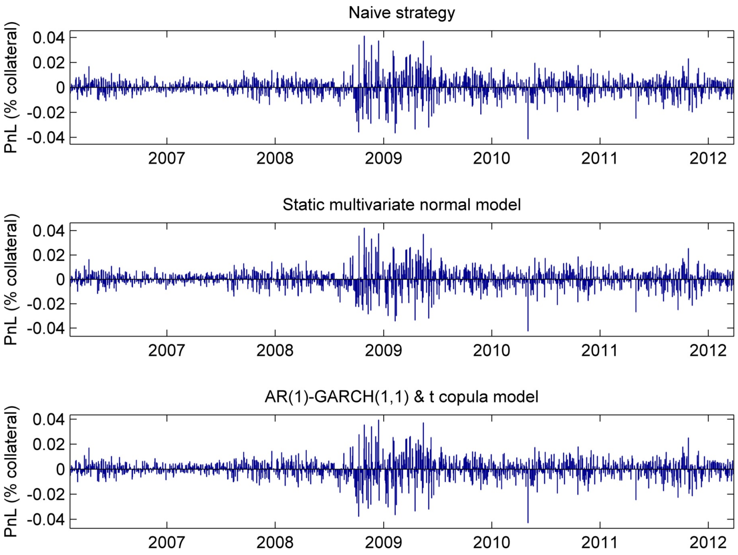
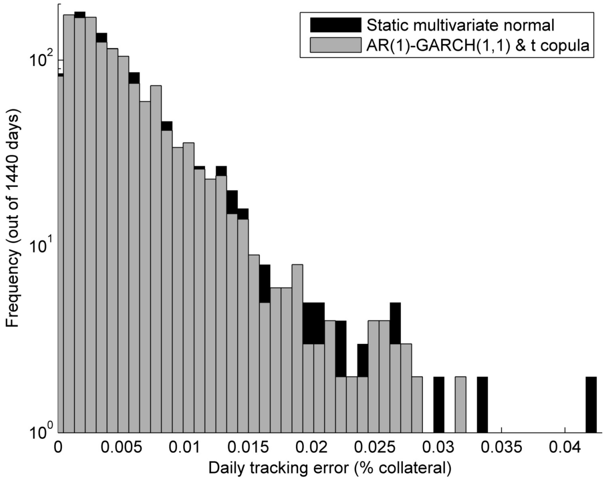



Figure : Optimal posted collateral in JPY, June 2011 - January 2012

# Backtesting



# Backtesting



# Results

| | Naive | MV Gaussian | AR-GARCH & t-copula |
|---|---|---|---|
| **Avg. daily tracking error (% collateral)** | 0.54 | 0.55 | 0.55 |
| **# of margin calls (out of 1440 days)** | 104 | 94 | 71 |
| **Frequency of margin call** | 0.0722 | 0.0653 | 0.0493 |

Table : Objective: Minimize the daily tracking error while keeping the probability of a margin call under 0.05

# Results

| | Naive | MV Gaussian | AR-GARCH & t-copula |
|---|---|---|---|
| **Realized daily VaR (% collateral)** | 1.19 | 1.21 | 1.24 |
| **# of margin calls (out of 1440 days)** | 104 | 105 | 78 |
| **Frequency of margin call** | 0.0722 | 0.0729 | 0.0542 |

Table : Objective: Minimize the Value-at-Risk while keeping the probability of a margin call under 0.05

# Results

| | Naive | MV Gaussian | AR-GARCH & t-copula |
|---|---|---|---|
| **Avg. daily tail loss (% collateral)** | -1.83 | -1.85 | -1.85 |
| **# of margin calls (out of 1440 days)** | 104 | 100 | 71 |
| **Frequency of margin call** | 0.0722 | 0.0694 | 0.0493 |

Table : Objective: Minimize the Tail Conditional Expectation while keeping the probability of a margin call under 0.05

# Outline

# Conclusion

Copula-based GARCH:

- are amongst the best model available for multivariate financial time series
- have absolute goodness-of-fit tests now available (parametric bootstrapping)
- provides for better and more robust portfolio engineering and risk management

# Outline

# References

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