Comp 230: Logic and Computability

Fall 2017

DIRK SCHLIMM

Lectures: Tuesdays and Thursdays, 4:05–5:25pm

McConnell Engineering Building 13

Website: http://www.cs.mcgill.ca/~cs230 and myCourses

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TA: Clara Lacroce

Office hours: Wednesdays, 4:30–6:30pm, McConnell 106.

TA: David Gaber

Office hours: Fridays, 12:30–2:30pm, Leacock 934

TA: Julien Ouellette-Michaud

Office hours: Mondays, 1:00–3:00pm, Leacock 934.

For any questions related to the course, please use the myCourses **discussion board**. In this way, everybody else in the course can also profit from your question!

Course Description

Summary. The aim of this course is to introduce students to the theoretical foundations of computer science. Since students are not expected to have taken a logic course before, the course will include an introduction to propositional and predicate calculus. Other topics are proof systems, computability, Turing machines, Church-Turing thesis, unsolvable problems, completeness, incompleteness, Tarski semantics, uses and misuses of Gödel's theorem.

Prerequisites. High-school level mathematics. Not open to students who have done PHIL-310 or MATH-498.

Textbook. The following textbook is required for this course; available at The Word Bookstore, 469 Milton St. (5 mins. from the University Street Gates; no credit cards).

• Douglas R. Hofstadter, *Gödel, Escher, Bach*, 20th Anniversary Edition, Basic Books, 1999.

Additional reading materials will be made available online, or handed out in class.

Requirements & grading. Students are expected to attend and participate in class, do the assigned readings, complete homework assignments, and write a midterm and a final exam.

Usually a set of problems will be assigned every two weeks. The assignments will be posted on myCourses. It is your responsibility to obtain the assignment if you miss class. Assignments are due at the beginning of class on the date mentioned in the assignment. Assignments have to be handed in, but will not be graded. Instead, you can compare your work with the sample solutions.

The final grade depends on homework assignments (not graded) (5%), three in-class quizzes (10% each), a midterm (25%), and a final exam (40%).

Late day policy: Every student can take up to two 'late days' during the semester. Otherwise, late homework will *not* be accepted (except in cases of documented emergencies; any documentation must be submitted *before* the due date).

Plagiarism and other university regulations. "McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/students/srr/honest for more information)."

"Instructor generated course materials (e.g., handouts, notes, summaries, exam questions, etc.) are protected by law and may not be copied or distributed in any form or in any medium without explicit permission of the instructor. Note that infringements of copyright can be subject to follow up by the University under the Code of Student Conduct and Disciplinary Procedures."

"In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded."

"In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change."

Course Overview

• Part I: Fundamentals and Formal Systems

- 1. Introduction to the course. $\sqrt{2}$ is irrational
- 2. Definition by recursion. Mathematical induction
- 3. Sets: Relations, functions, cardinality
- 4. Denumerable and non-denumerable sets: Diagonalization
- 5. Syntax. A simple formal system. The MIU system
- 6. Semantics. Interpretations. Infinitude of primes (Euclid)
- 7. R.e. sets vs. recursive sets
- 8. Multiple interpretations, non-Euclidean geometry, consistency, completeness

• Part II: Logic and Formal Arithmetic

- 1. Propositional logic: Semantics
- 2. Propositional logic: Natural deduction
- 3. Propositional logic: Soundness and completeness
- 4. First-order logic: Expressions
- 5. First-order logic: Semantics
- 6. First-order logic: Substitutions, completeness
- 7. First-order arithmetic
- 8. Gödel numbering. Solution to MU puzzle

• Part III: Computability and Incompleteness

- 1. Primitive recursive functions (BlooP)
- 2. Recursive functions (FlooP)
- 3. Turing machines. Halting problem
- 4. Gödel's First Incompleteness Theorem
- 5. Gödel's Second Incompleteness Theorem
- 6. Church-Turing thesis. Tarski's Theorem

See the course website http://www.cs.mcgill.ca/~cs230 for more details.